Stellar evolution and compact objects & GWs: Observations and implications

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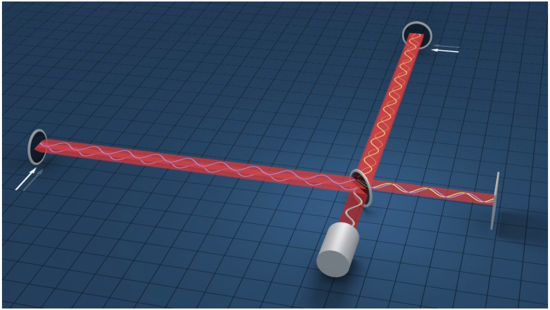
GWs ♡ compact objects

Outline

Very precise rulers



Robert Hurt (Caltech)

Light intensity I depends light travel difference in perpendicular arms

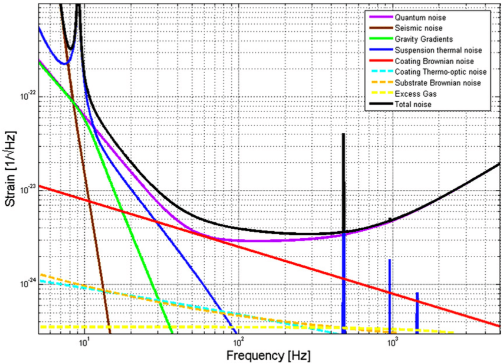
Tiniest measurable phase shift: $\Delta\phi \sim 10^{-8}$

$I \propto \sin\left(\omega_{GW} L_{intf}\right) \implies L_{opt} = \frac{1}{4 f_{gw}} \sim 750\mathrm{km}\left(\frac{100\mathrm{Hz}}{f_{gw}}\right)^{-1}$

Effective optical path increased by factor $N \sim 500$ via Fabry-Perot cavities

$\phi \sim \Delta L/\lambda \to \Delta L_{min} \sim 10^{-15}$ m, $h_{min} \sim \frac{\delta L}{N \times L} \sim 10^{-19}/N$

Noise budget



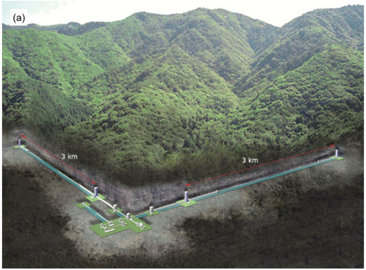
Barriga et al., CQG (2013) 084005

Why $Hz^{-1/2}$? Detector's FoM is *noise spectral density* $S_n(f)$:

$$\langle \tilde{n}(f)\tilde{n}(f')\rangle = S_n(f)\delta(f-f')$$

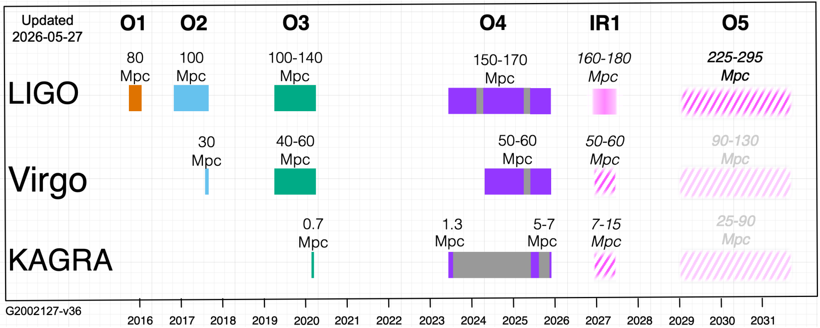
i.e. $S_n(f_i) \sim |\tilde{n}(f_i)|^2 \Delta f$.

LIGO H, L, Virgo, KAGRA



- Observation run **O1**: 12/9/15– 19/1/16 $\sim$ 130 days, with 49.6 days of actual data, PRX (2016) 4, 041014, 2 dets, 3BBH
- **O2** 30/11/16 – 31/7/17 2 dets + 1/8– 25/8/17 3 dets
3(+1) BBH + 1BNS in triple coinc, 122.2 d
- **O3a**: 3 detectors, 1/4 - 1/10 2019, 39 detections, 149.6 d
- **O3b**: 1/11/19 – 27/3/20 $\rightarrow$ 90 detections
(69 BBH + 4 NSBH + 2 BNS with FAR$< yr^{-1}$), 124.6 d
- **O4a**: 24/5/2023 $\rightarrow$ 16/1/2024 128 (86 BBH, including 2 NSBH + GW230529)
KAGRA until 20/6/23, 126.5 d
- **O4b**: 10/4/2024 $\rightarrow$ 28/1/2025, 141.9 d (229 $\rightarrow$ 106 BBH)
- **O4c**: 28/1/2025 $\rightarrow$ 18/11/2025 (break 1/4–11/6), 126.5d

LIGO/Virgo/KAGRA arxiv:2508.18082/18083


Updated 2026-05-27
O1
O2
O3
O4
IR1
O5
LIGO
80 Mpc
100 Mpc
100-140 Mpc
150-170 Mpc
160-180 Mpc
225-295 Mpc
Virgo
30 Mpc
40-60 Mpc
50-60 Mpc
50-60 Mpc
90-130 Mpc
KAGRA
0.7 Mpc
1.3 Mpc
5-7 Mpc
7-15 Mpc
25-90 Mpc
G2002127-v36
2016
2017
2018
2019
2020
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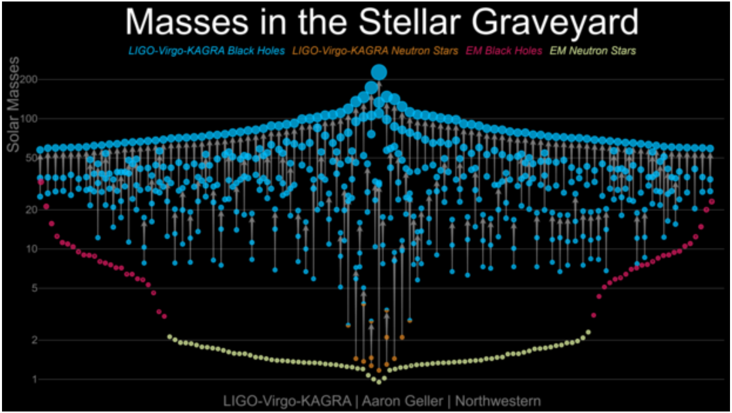
Outline

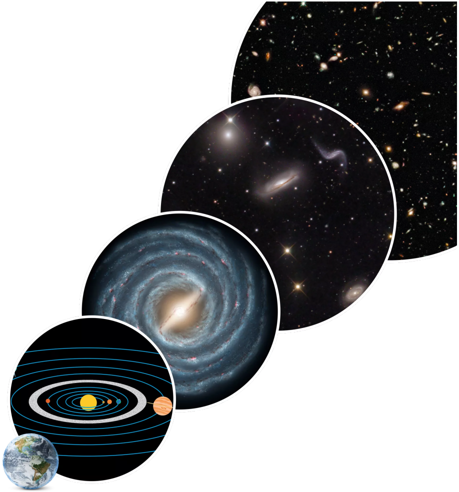
Stellar ($\lesssim 100 M_\odot$) compact object with known masses



Frequency 10-10^3 Hz determines size of sources

Remnant of various GW events represent first Intermediate Mass Black Holes ($> 10^2 M_\odot$) – SuperMassive BHs $\gtrsim 10^5 M_\odot$ (up to $10^9 M_\odot$)

Cosmic ladder



$$6\times10^3\ \text{km} \overset{10^6}{\rightarrow} 10^{10}\ \text{km} \overset{10^7}{\rightarrow} 15\ \text{kpc} \overset{10^5}{\rightarrow} 1\ \text{Mpc} \overset{10^3}{\rightarrow} 3\text{Gpc}$$

$$1\text{kpc} \simeq 3\times10^{16}\ \text{km},\ 3\text{kpc} \sim 10^4\ \text{light years}$$

Outline

Wave equation, TT gauge

Linearised Einstein equations ($\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h,\ h \equiv \eta^{\mu\nu}h_{\mu\nu}$):

$$\Box\bar{h}_{\mu\nu} + \eta_{\mu\nu}\bar{h}_{\alpha\beta}{}^{,\alpha\beta} - \bar{h}_{\mu\alpha,\nu}{}^{,\alpha} - \bar{h}_{\nu\alpha,\mu}{}^{,\alpha} = -16\pi G_N T_{\mu\nu}$$

With coordinate transformation

$$x^\mu \to x'^\mu = x^\mu + \xi^\mu \implies \bar{h}_{\mu\nu} \to \bar{h}'_{\mu\nu} = h_{\mu\nu} + \xi_{\mu,\nu} + \xi_{\nu,\mu} - \eta_{\mu\nu}\xi^\alpha{}_{,\alpha}$$

one can set $\bar{h}_{\mu\nu}{}^{;\nu} = 0$ leaving 6 d.o.f.: 2 are radiative 4 are longitudinal
The vacuum e.o.m.s are

$$\Box\bar{h}_{\mu\nu} = 0 \to \Box\bar{h}'_{\mu\nu} = \Box\bar{h}_{\mu\nu} + \Box\left(\xi_{\mu,\nu} + \xi_{\mu,\nu} - \eta_{\mu\nu}\xi^\alpha{}_{,\alpha}\right)$$

4 more d.o.f. can be killed *in vacuum* keeping the Lorentz condition

$$\partial^\mu\bar{h}_{\mu\nu} = 0 \to \partial^\mu\bar{h}'_{\mu\nu} = \Box\xi_\nu = 0$$

$h_{\mu\nu} \overset{\xi^\mu, h_\nu{}^{;\mu}_\mu}{\to} h_{ij}(h = 0 = h_{ij}{}^{,j})$, for $z-$propagating waves means one is left with 2 radiative d.o.f.

$$h^{TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \Lambda_{ij,kl}(\hat{z})h_{ij}$$

$$\Lambda_{ij,kl}(\hat{n}) = P_{ik}P_{jl} - 1/2 P_{ij}P_{kl} \qquad P_{ij}(\hat{n}) = \delta_{ij} - \hat{n}_i\hat{n}_j$$

GW EM tensor

Averaged Einstein equations

$$\langle R^{(1)}_{\mu\nu}\rangle = 8\pi G_N\left(T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T\right)-\langle R^{(2)}_{\mu\nu}\rangle$$

Background (slowly varying) + waves (high frequency):

$$T^{(GW)} \equiv -\frac{1}{8\pi G_N}\langle R^{(2)}_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}R^{(2)}\rangle$$

and the EM tensor (energy density)

$$T^{(GW)}_{\mu\nu}=\frac{1}{32\pi G_N}\langle\partial_\mu h^{TT}_{\alpha\beta}\partial_\nu h^{(TT)\alpha\beta}\rangle \to \rho_{GW}=\frac{1}{16\pi G_N}\langle\dot h^2_+ + \dot h^2_\times\rangle$$

and flux

$$\frac{dE_V}{dt}=\int_V d^3x\,\dot T^{(GW)00}=-\int_{\partial V} n_i T^{(GW)0i}=-\int_{\partial V} T^{(GW)00}$$

Quadrupole formula

$$E=-\frac{1}{2}\eta M v^2 \qquad \eta\equiv m_1 m_2/M^2\,, \qquad F=\frac{1}{32\pi G}\Lambda_{ij,kl}(\hat n)\dot h_{ij}\dot h_{kl}$$

$$\begin{aligned} h_{ij}(t,x) &= -16\pi G\frac{1}{\Box}T_{ij}=4G\int d^3x\frac{T_{ij}(t_r=t-|x-x'|,x')}{|x-x'|}\\ &\simeq \frac{4G}{r}\int d^3x T_{ij}(t-r,x')+\dot T_{ij}(t-r,x')\hat n\cdot x'+\frac{1}{2}\ddot T_{ij}(t-r,x')\hat n^k\hat n^l x'^k x'^l+\ldots\end{aligned}$$

expansion parameter $\dot T_{ij}x'/T_{ij}\sim\omega x'\sim v$.
By using repeastdly EM-conservation:

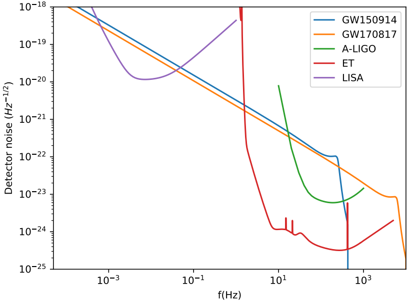
$$\begin{aligned}\int(T^{0k}x^ix^j)_{,k}=0=-\int\dot T^{00}x^ix^j+T^{0i}x^j+T^{0j}x^i\,,\\ \int(T^{ik}x^j+T^{jk}x^i)_{,k}=0=-\int\dot T^{i0}x^j+\dot T^{j0}x^i+2T^{ij}\,,\end{aligned}$$

$$F=\frac{\dot h_{ij}\dot h_{kl}}{32\pi G}\int d\Omega\Lambda_{ij,kl}(\hat n)=\frac{G}{5}\dddot Q^{TT}_{ij}\dddot Q^{(TT)}_{kl}\overset{circ}{=}\frac{32}{5}\eta^2\frac{v^{10}}{G}$$

where $Q^{(TT)}_{ij}\equiv\Lambda_{ij,kl}(\hat n)\int T^{00}x_ix_j\sim\Lambda_{ij,kl}(\hat n)\,\eta M x^ix^j \implies$

$$-\eta M v\frac{dv}{dt}=\frac{dE}{dt}=-\frac{32}{5}\eta^2\frac{v^{10}}{G}\implies\Delta t_{i\to f}=\frac{5GM}{256\eta}\left(\frac{1}{v_i^8}-\frac{1}{v_f^8}\right)$$

Sensitivities and duration



$$\tilde{h}(f) \sim \frac{f^{-7/6}(GM_c)^{5/6}}{D_L} e^{i\psi(f)},\ v^3 = GM\pi f,\ M_c \equiv \eta^{3/5} M$$

Signal duration controlled by $\frac{5}{256\pi}(\pi M_c f_i)^{-5/3} = 30 \times \frac{1}{\eta}\left(\frac{M}{20M\odot}\right)^{-5/3}\left(\frac{f_i}{20\mathrm{Hz}}\right)^{-5/3}$

$$\Delta t_{i\to f} \sim \frac{5}{256\pi}(\pi G M_c)^{-5/3}\left(f_i^{-8/3} - f_m^{-8/3}\right) \to$$

$$\frac{5}{256\pi}(\pi M_c f_i)^{-5/3} \times \begin{cases} \frac{1}{f_i} & \Delta t < t_{exp} \\ \frac{\Delta f}{f_i^2} & \Delta t > t_{exp} \end{cases}$$

Wave generation: localized sources

Einstein formula relates h_{ij} to the source quadrupole moment Q_{ij}

$$
\begin{aligned}
Q_{ij} &= \int d^3x\rho\left(x_ix_j - \frac{1}{3}\delta_{ij}x^2\right), \qquad v^2 \simeq G_N M/r\,, \quad \eta \equiv m_1m_2/M^2 \\
h_{ij} &\sim g(\theta_{\hat{n}L})\frac{2G_N}{D}\frac{d^2Q_{ij}}{dt^2} \simeq \frac{2G_N\eta M v^2}{D}\cos(2\phi(t)) \\
f &= 2\mathrm{kHz}\left(\frac{r}{30\mathrm{Km}}\right)^{-3/2}\left(\frac{M}{3M_\odot}\right)^{1/2} < f_{Max} \simeq 12kHz\left(\frac{M}{3M_\odot}\right)^{-1} \\
v &= 0.3\left(\frac{f}{1\mathrm{kHz}}\right)^{1/3}\left(\frac{M}{M_\odot}\right)^{1/3} < \frac{1}{\sqrt{6}}
\end{aligned}
$$

Geometric factor $g(\theta_{\hat{n}L})$ takes account of transversality projection $\Lambda_{ij,kl}(\hat{n})$ (angular momentum L of the binary, observation direction $\hat{n}$)

$$
\begin{aligned}
h_+ &\sim \frac{1+\cos^2(\theta_{\hat{n}L})}{2}\eta\frac{Mv^2}{D}\cos\phi(t_s/M,\eta,S_i^2/m_i^4,\ldots) \\
h_\times &\sim \cos(\theta_{\hat{n}L})\,\eta\frac{Mv^2}{D}\sin\phi(t_s/M,\eta,\ldots)
\end{aligned}
$$

Amplitudes of 2 polarizations modulated by $\theta_{\hat{n}L}$ ($h\nearrow$ for $\theta_{\hat{n}L}\searrow_0$), never both vanishing unlike dipolar motion for the electromagnetic case

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h_+ &\sim \frac{1+\cos^2(\theta_{\hat{n}L})}{2}\,\eta\frac{M(1+z)v^2}{D(1+z)}\cos\phi(t_O/(M(1+z)),\eta,S_i^2/m_i^4,\ldots) \\
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h sensitive to red-shifted masses $M\to M(1+z)\equiv\mathcal{M}$

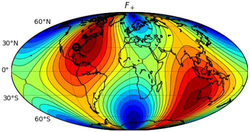
Almost omnidirectional detectors

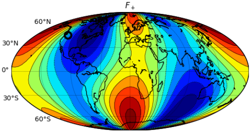
Detectors measure h_{det}: linear combination $F_+ h_+ + F_\times h_\times$

L H

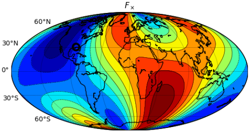
-1 0 1

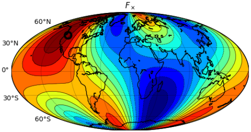
F_+





$F_\times$





$h_{+,\times}$ depend on source

pattern functions $F_{+,\times}$ depend on orientation source/detector

Almost omnidirectional detectors

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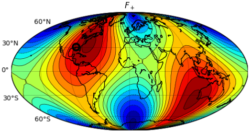
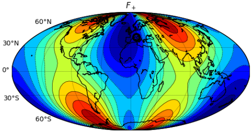
L V

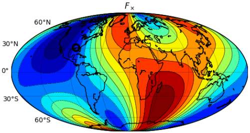
-1 0 1

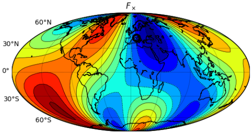
F_+





$F_\times$

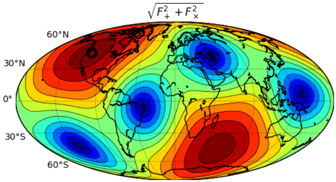


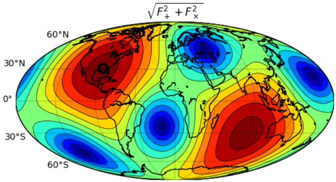


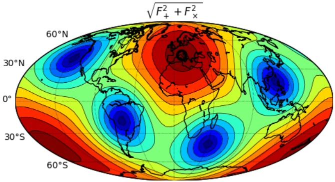
$h_{+,\times}$ depend on source

pattern functions $F_{+,\times}$ depend on orientation source/detector

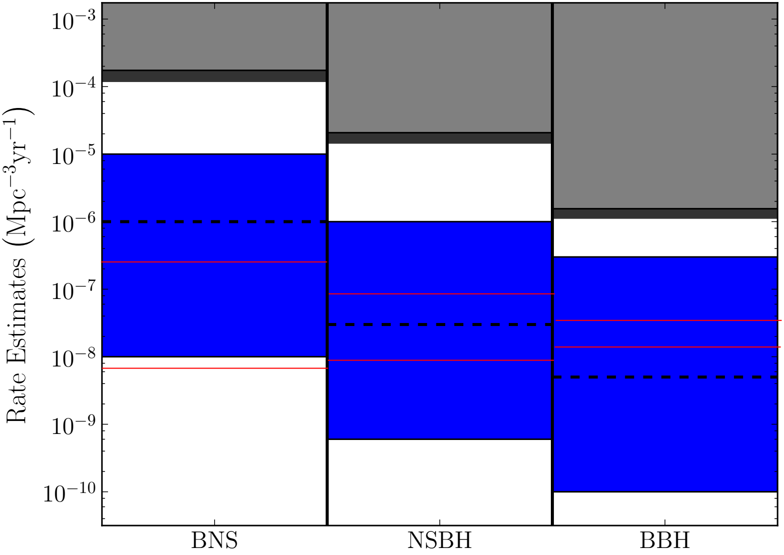
Pattern functions: $\sqrt{F_+^2 + F_\times^2}$





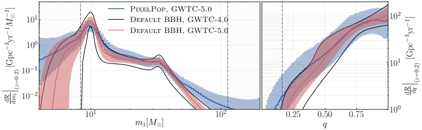


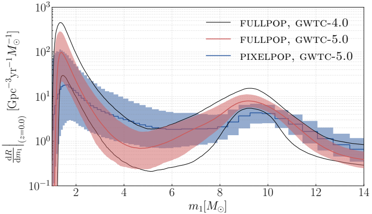
Outline



Astro predictions, measures from O1/O2/O3. Galaxy density $\sim 2\times10^{-2}\mathrm{Mpc}^{-3}$

LIGO/Virgo CQG ('10) 27 173001, PRX ('16), APJ (2016), PRL 119 ('17)





Features at $\frac{M}{M_\odot} \sim 10, 30$ (and 20?), power law steepening for $M > 35 M_\odot$

Focus on supposed gap at $2 \leq \frac{M}{M_\odot} < 5$ (stellar collapse)

Bailyn+,arxiv:9708032

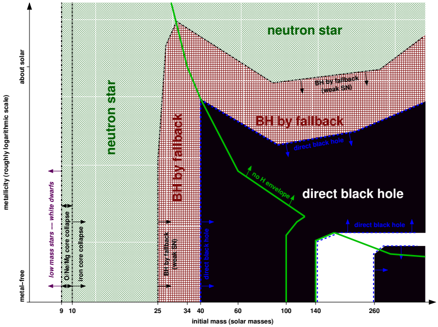
and $50 < \frac{M}{M_\odot} < 130$ (PPISN from star with $70 < M/M_\odot < 140$)

Woosley arxiv:1608.08939

LIGO/Virgo/KAGRA, arxiv:2508.18083, 2605.27226

Solar Mass Black Holes

1st Mass gap: $2M_\odot < M_{BH} \lesssim 5M_\odot$: SN explosion prevents BH formation
2nd Mass gap: $50M_\odot < M_{BH} \lesssim 150M_\odot$
Pair Instability Super Nova ($\gamma \to e^+e^-$ drops pressure):
Are the (would be) gaps populated?



Heger+, Astrophys.J. 591 (2003) 288-300

Equilibrium of a degenerate star

Modeling a star with $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$, assuming spherical symmetry ($ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2 d^2\Omega$) one has

$$\frac{dp}{dr} = -G\frac{\left(m + 4\pi r^3 p\right)(\rho + p)}{r(r-2m)} \to -G\frac{m\rho}{r^2}$$

For a *degenerate* star the particle density from Fermi-Dirac distribution

$$n = 2\int_0^\infty \frac{1}{e^{(E(p)-\mu)/T} + 1}\frac{d^3p}{(2\pi\hbar)^3} \overset{T\to 0}{\to} \frac{P_F^3}{3\pi^2\hbar^3}$$

with $P_F = \mu(T=0)$. To find the equation of state $rho(p)$ assume free ultra relativistic particles:

$$\frac{E_{ur}}{V} = \int_0^{P_F} P\frac{d^3P}{(2\pi\hbar)^3} = \frac{3}{4}(3\pi^2)^{1/3}\hbar n^{4/3}$$

Applied to a white dwarf (relativistic electrons) with 2 nucleons per electrons:

$$p_{ur} = \frac{1}{4}(3\pi^2)^{1/3}\hbar\left(\frac{\rho}{2m_p}\right)^{4/3}$$

leading to

$$\begin{aligned}
&\frac{1}{3}(3\pi^2)^{2/3}\hbar(2m_p)^{-4/3}\rho^{1/3}\frac{d\rho}{dr} = -Gm(r)\frac{\rho}{r^2}\\
&M_*^{2/3}\rho^{1/3}\frac{d\rho}{dr} = -m\frac{\rho}{r^2}\,, \qquad M_*^{2/3} \equiv \frac{1}{3}(3\pi^2)^{2/3}\frac{\hbar}{G}(2m_p)^{-4/3}
\end{aligned}$$

Under rescaling $r \to \lambda r$, $\rho \to \lambda^a \rho$, $M \to \lambda^{a+3} M$

$$M_*^{2/3}\lambda^{\frac{4}{3}a-1}\rho^{1/3}\frac{d\rho}{dr} = -\lambda^{2a+1} m\frac{\rho}{r^2}$$

the eq. is invariant for $a = -3$

Solutions are parametrized by 1 initial conditions $\rho_c \equiv \rho(r = 0)$, from the scaling

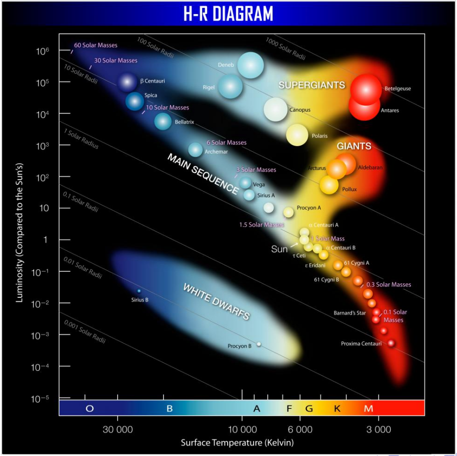
$$\begin{aligned}
&M \sim M_*\\
&R \propto \rho_c^{-1/3} M_*^{1/3}
\end{aligned}$$

with

$$M_* \sim \left(\frac{\hbar}{G}\right)^{3/2} m_p^{-2} \equiv \frac{m_{Planck}^3}{m_p^2}$$

Analogously for the neutron stars

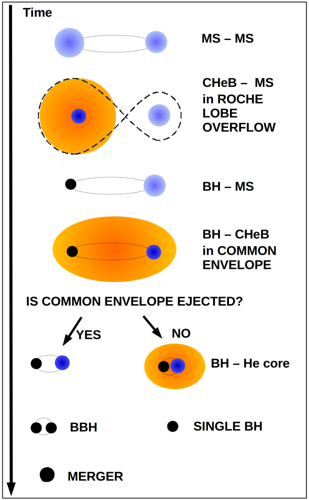
Astrophysical *magnitude* defined as $M_1 - M_2 = -2.5\log_{10}(L_1/L_2)$
E.g. $M_* \sim -21 - 2.5\log_{10}(L_*/L_{MW})$ (for ref.
$L_{MW} \simeq 5\times 10^{36}\mathrm{W} \simeq 2\times 10^{10} L_\odot$) Stars (unlike BHs) are very well observed and they happen to fall into regions of the Luminosity - Temperature diagram (Hertzsprung-Russell)



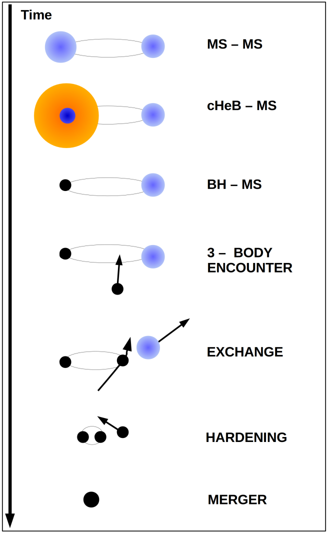
Codes: **O**h, **B**e **A** **F**ine **G**uy/irl **K**iss **M**e (alphabetical order os spectral lines)

- O: hottest blue stars, blue-violet $> 3 \times 10^4$ K
- B: blue-white, $1 - 3 times 10^4$ K
- A: white, $7.5 - 10 \times 10^3$ K
- F: yellow (metal line appears), $6 - 7.5 \times 10^3$ K
- G: yellow (sun-like stars), $5.2 - 6 \times 10^3$ K
- K: orange (cooler metal/molecular bands) $3.7 - 5.2 \times 10^3$ K
- M: (coolest red stars) $2 - 3.7 \times 10^3$ K

Field binaries vs. clusters



common envelope



dynamical encounter

Strange cases

- GW150914: 30+30 $M_\odot$ unseen before in BH-star
- "Low" Mass gap
  - GW190704 7 3.1 $M_\odot$,
  - GW190814 23 2.6 $M_\odot$,
  - GW190821 8 3.9 $M_\odot$,
  - GW190910 34 3.2 $M_\odot$,
  - GW190920 7 2.8 $M_\odot$,
  - GW200210 24.1 2.8 $M_\odot$,
  - GW230529 3.6 1.4 $M_\odot$,
- GW191219 31.1 1.17 $M_\odot$, highest mass ratio
- GW231123 137 101 $M_\odot$, highest total mass