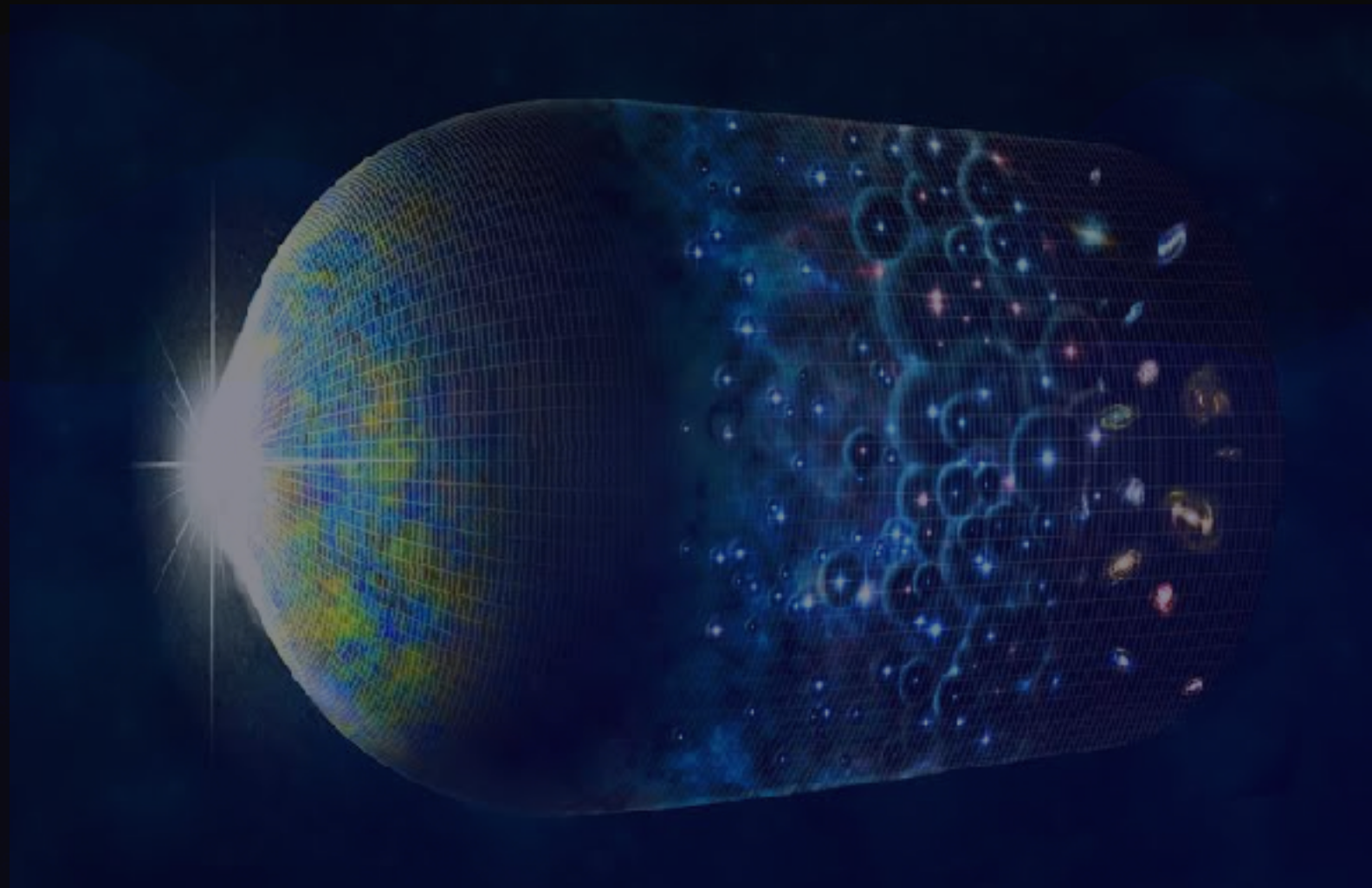


# Cosmology: Observables and surveys



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Physics Institute  
University of São Paulo





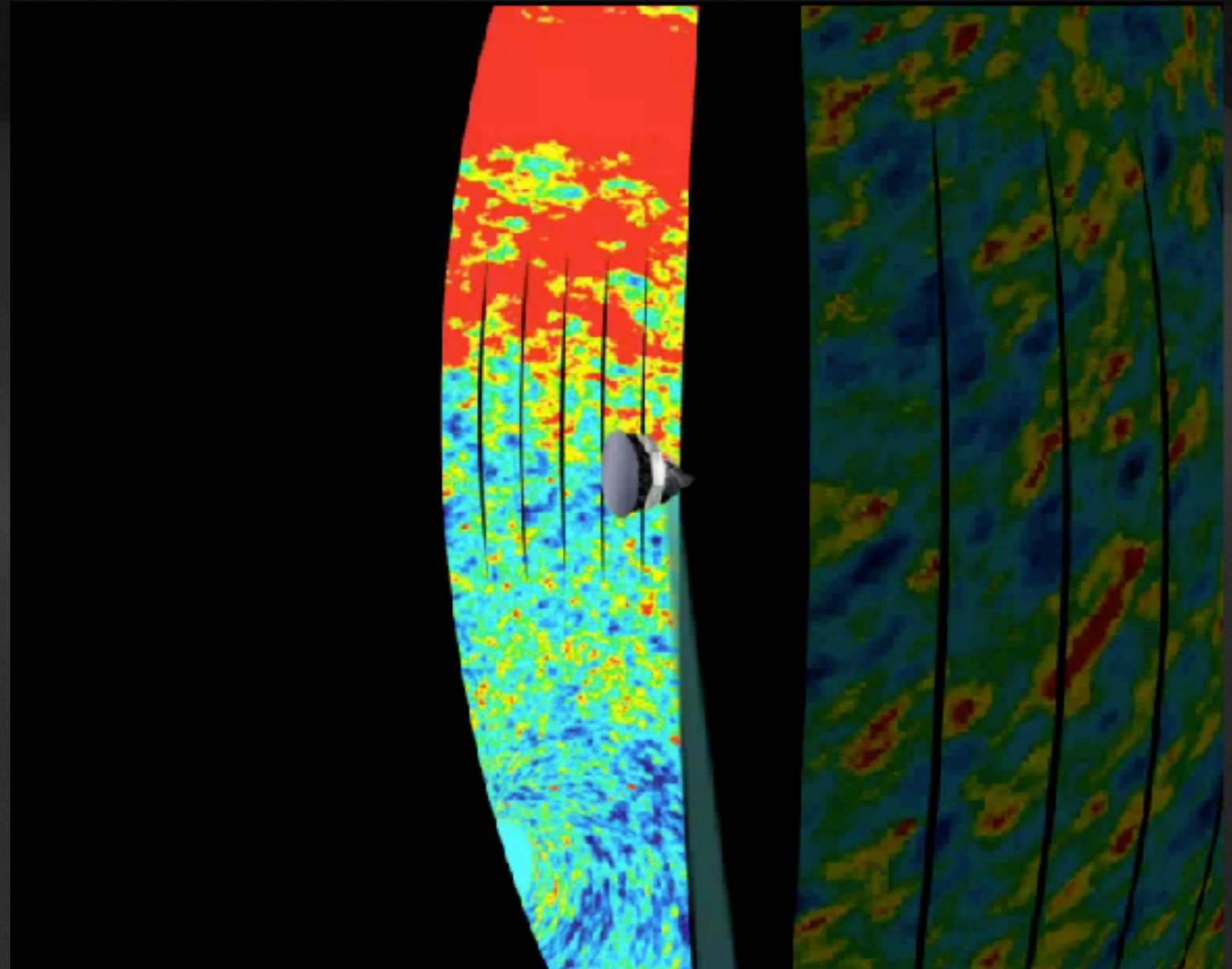
# The cosmic microwave background

## Message from the edge of the Universe

- The CMB tells us that the Universe was much simpler at  $z \sim 10^3$ , when  $t \sim 4. \times 10^5$  yrs.
- Radiation and matter were in thermodynamic equilibrium
- Temperature and density fluctuations were very small:

$$\frac{\delta T}{T} \sim \frac{\delta \rho}{\rho} \sim 10^{-4}$$

- The CMB is a picture of what the Universe looked like at that time: it gives us a sort of template for the initial conditions.

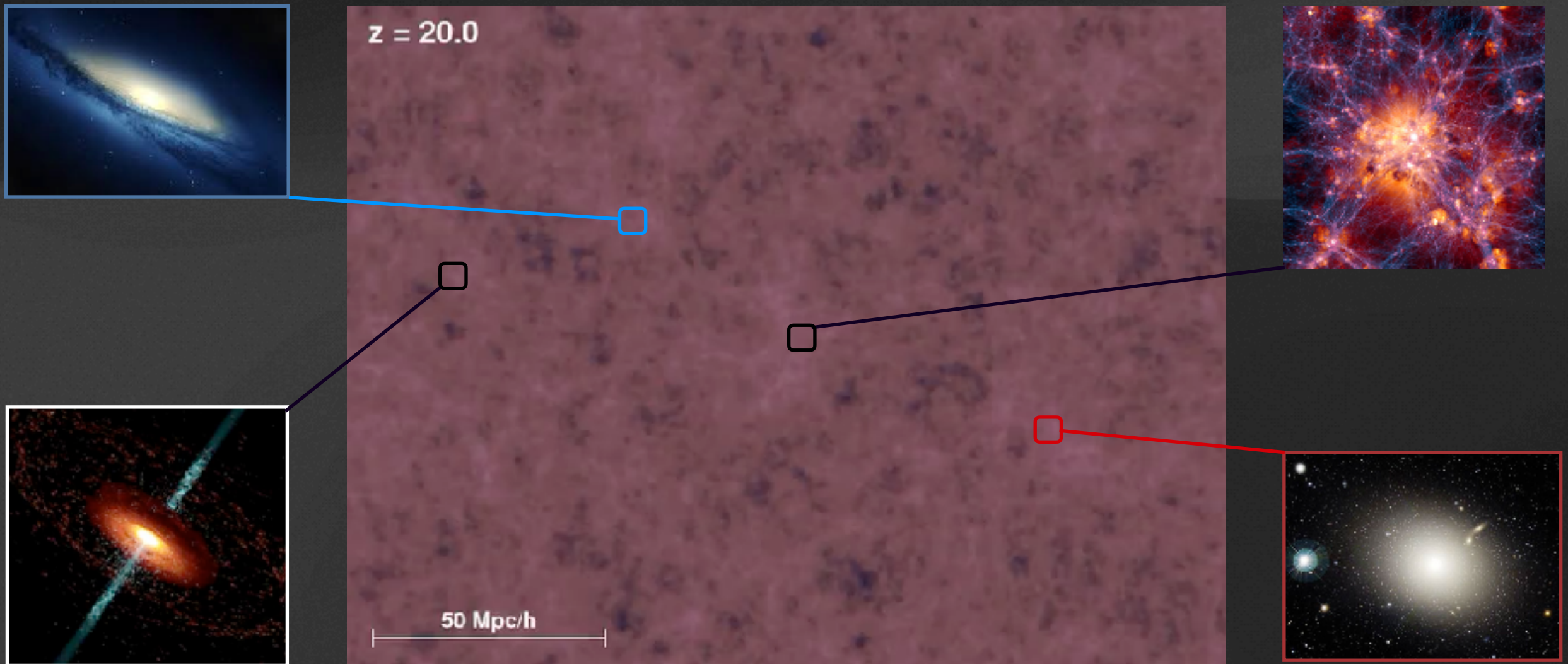




# The cosmic microwave background

## Message from the edge of the Universe

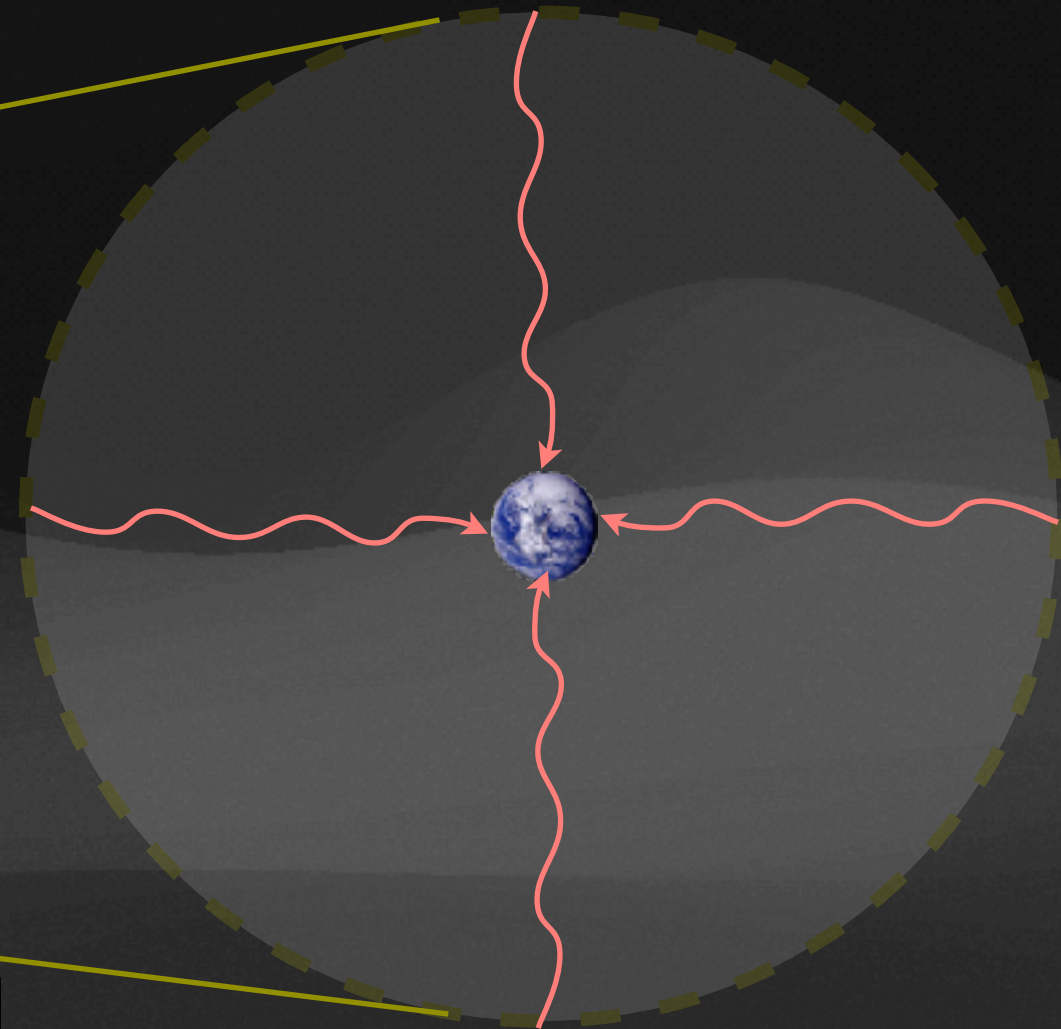
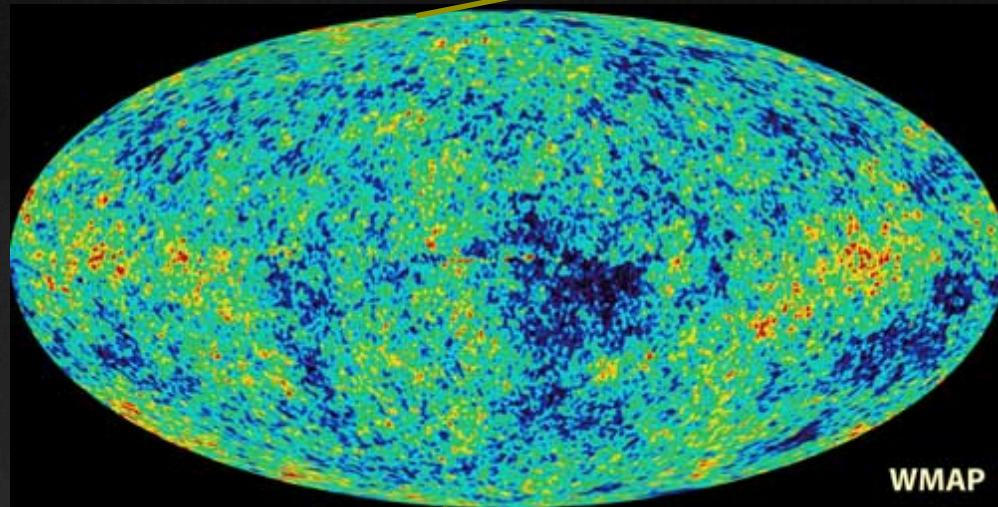
- The CMB tells us that the Universe started out in a very simple state, with tiny inhomogeneities.
- Over the course of billions of years of evolution, gravity acted to create the structures we see now.





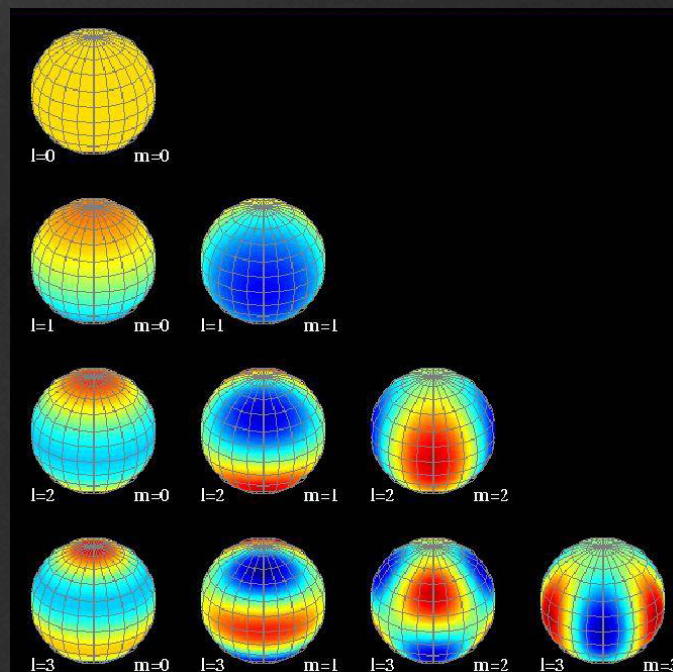
# The physics of the CMB

Astonishingly simple



$$\delta T(\theta, \varphi) = \sum_{\ell, m} a_{\ell m} Y_{\ell}^m(\theta, \varphi)$$

$$C_{\ell} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$



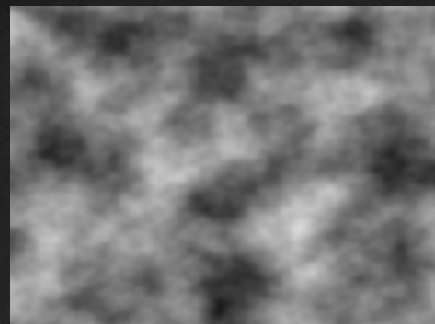
Angular power spectrum:  
decomposition in multipoles



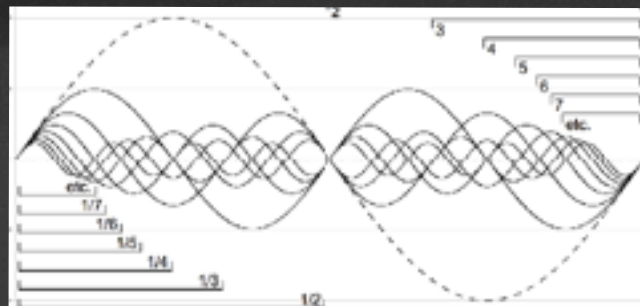
# The physics of the CMB

Astonishingly simple

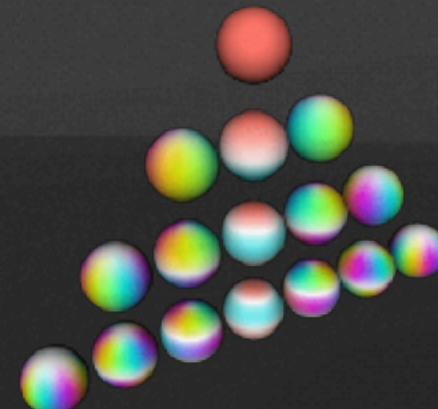
- Spherical harmonics and Fourier transforms: two of a kind



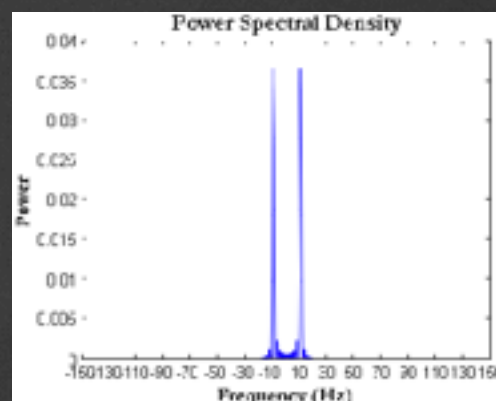
$$f(x, y) \longleftrightarrow f(\theta, \phi)$$



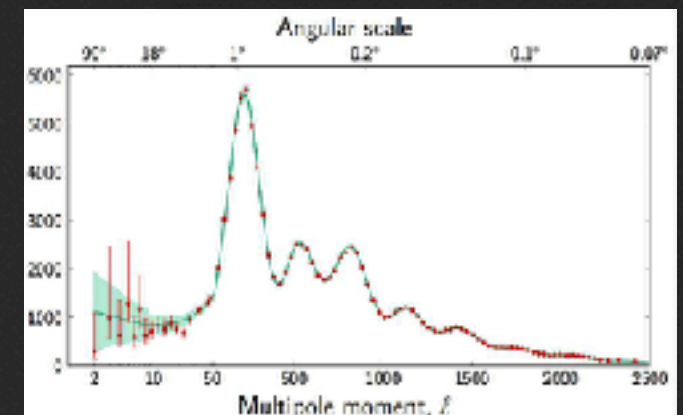
$$e^{i(k_x x + k_y y)} \longleftrightarrow Y_{\ell m}(\theta, \phi)$$



$$f(x, y) = \sum_{\vec{k}} \tilde{f}_{\vec{k}} e^{i(k_x x + k_y y)} \longleftrightarrow f(\theta, \phi) = \sum_{m=-\ell}^{\ell} \sum_{\ell=0}^{\infty} a_{\ell m} Y_{\ell m}(\theta, \phi)$$



$$P_k = |\tilde{f}_{\vec{k}}|^2 \longleftrightarrow C_{\ell} = \langle |a_{\ell m}|^2 \rangle$$

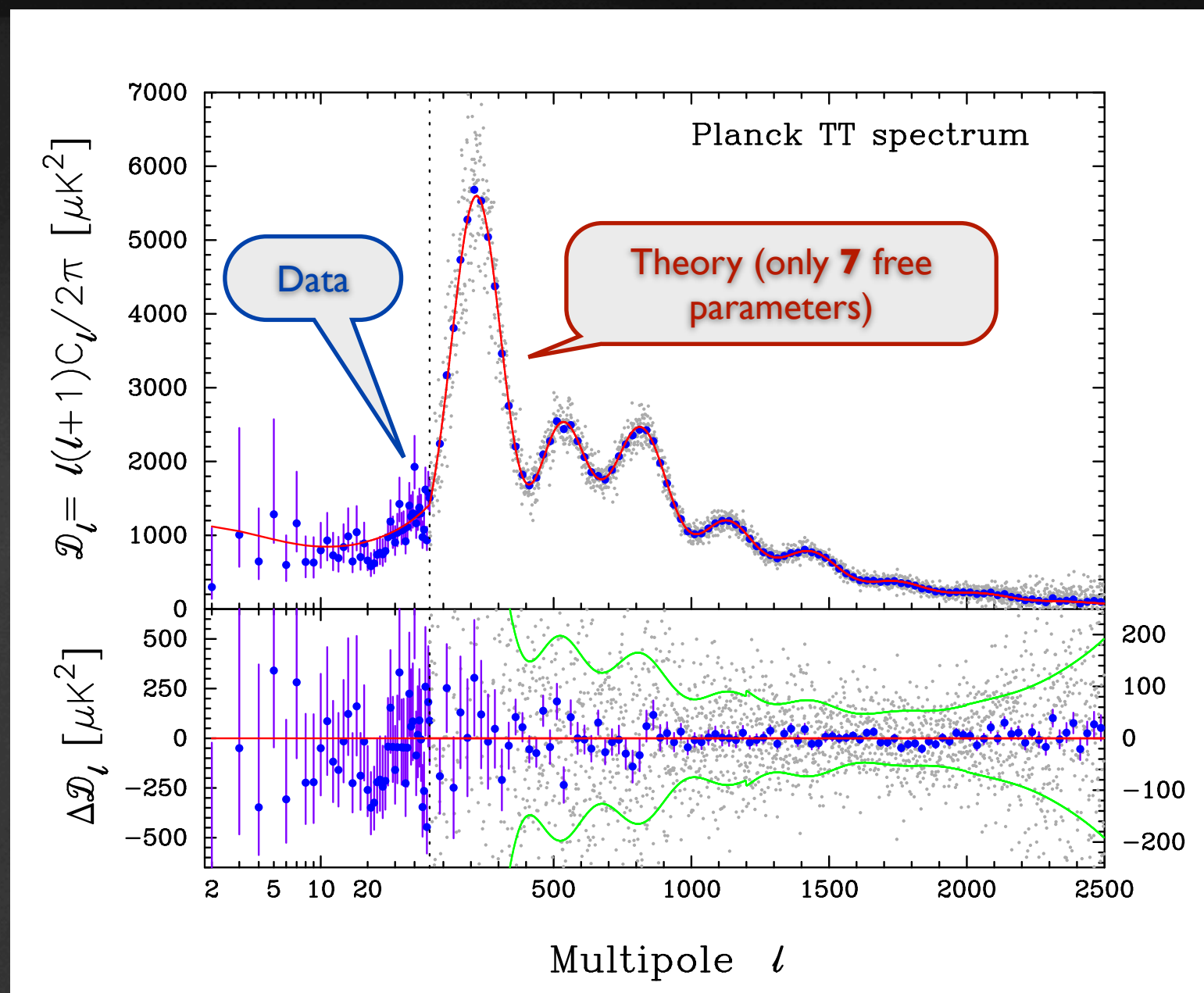




# The physics of the CMB

Astonishingly simple

- Theory v. data





# The physics of the CMB

Astonishingly simple

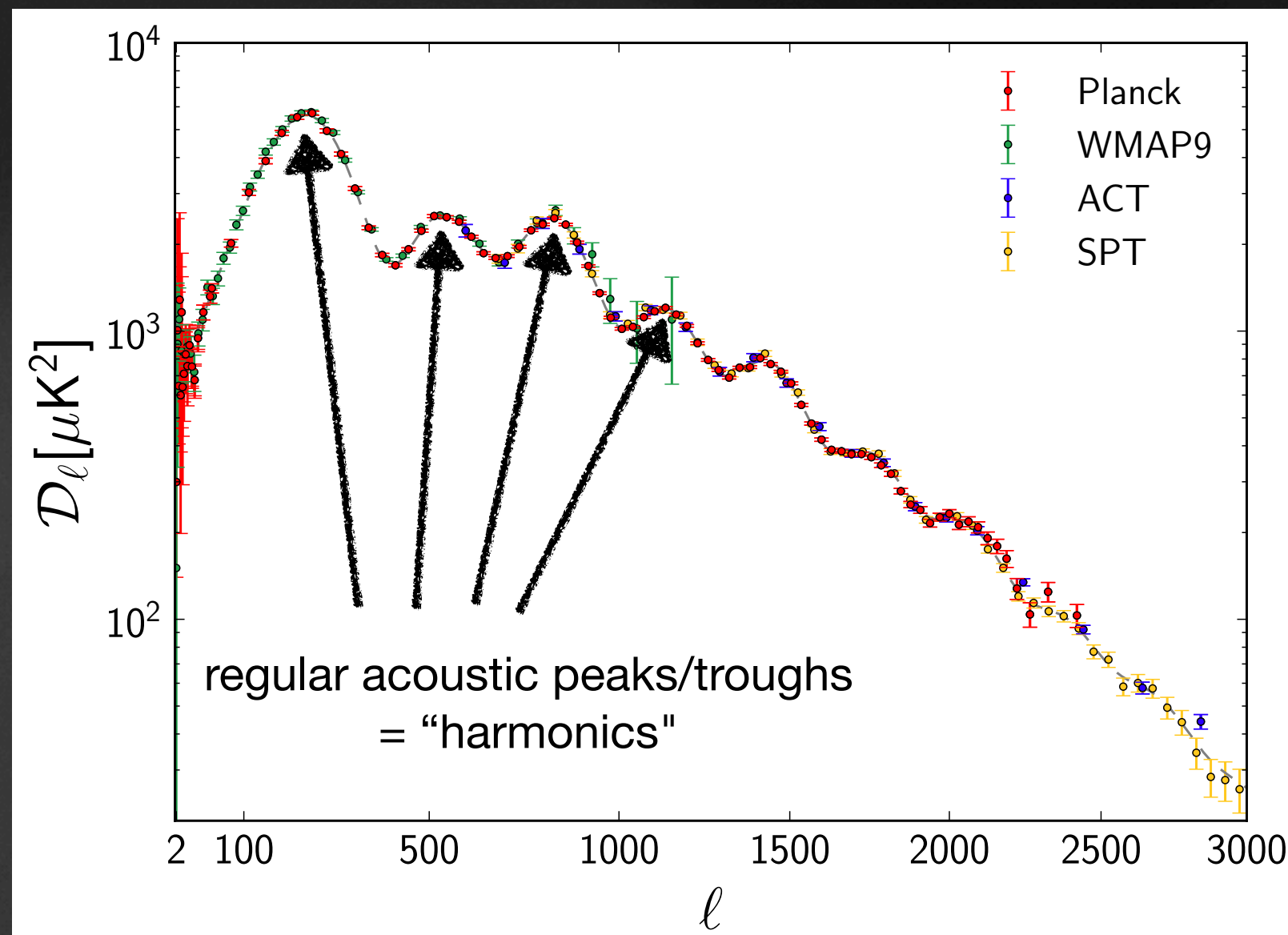
- Theory v. data

| Parameter                           | <i>Planck</i> (CMB+lensing) |                       | <i>Planck</i> +WP+highL+BAO |                       |
|-------------------------------------|-----------------------------|-----------------------|-----------------------------|-----------------------|
|                                     | Best fit                    | 68 % limits           | Best fit                    | 68 % limits           |
| $\Omega_b h^2$ . . . . .            | 0.022242                    | $0.02217 \pm 0.00033$ | 0.022161                    | $0.02214 \pm 0.00024$ |
| $\Omega_c h^2$ . . . . .            | 0.11805                     | $0.1186 \pm 0.0031$   | 0.11889                     | $0.1187 \pm 0.0017$   |
| $100\theta_{\text{MC}}$ . . . . .   | 1.04150                     | $1.04141 \pm 0.00067$ | 1.04148                     | $1.04147 \pm 0.00056$ |
| $\tau$ . . . . .                    | 0.0949                      | $0.089 \pm 0.032$     | 0.0952                      | $0.092 \pm 0.013$     |
| $n_s$ . . . . .                     | 0.9675                      | $0.9635 \pm 0.0094$   | 0.9611                      | $0.9608 \pm 0.0054$   |
| $\ln(10^{10} A_s)$ . . . . .        | 3.098                       | $3.085 \pm 0.057$     | 3.0973                      | $3.091 \pm 0.025$     |
| $\Omega_\Lambda$ . . . . .          | 0.6964                      | $0.693 \pm 0.019$     | 0.6914                      | $0.692 \pm 0.010$     |
| $\sigma_8$ . . . . .                | 0.8285                      | $0.823 \pm 0.018$     | 0.8288                      | $0.826 \pm 0.012$     |
| $z_{\text{re}}$ . . . . .           | 11.45                       | $10.8^{+3.1}_{-2.5}$  | 11.52                       | $11.3 \pm 1.1$        |
| $H_0$ . . . . .                     | 68.14                       | $67.9 \pm 1.5$        | 67.77                       | $67.80 \pm 0.77$      |
| Age/Gyr . . . . .                   | 13.784                      | $13.796 \pm 0.058$    | 13.7965                     | $13.798 \pm 0.037$    |
| $100\theta_*$ . . . . .             | 1.04164                     | $1.04156 \pm 0.00066$ | 1.04163                     | $1.04162 \pm 0.00056$ |
| $r_{\text{drag}}$ . . . . .         | 147.74                      | $147.70 \pm 0.63$     | 147.611                     | $147.68 \pm 0.45$     |
| $r_{\text{drag}}/D_V(0.57)$ . . . . | 0.07207                     | $0.0719 \pm 0.0011$   |                             |                       |

# The physics of the CMB

The music of the spheres cosmos

- Acoustic oscillations of the photon-baryon fluid (+ dark matter!)

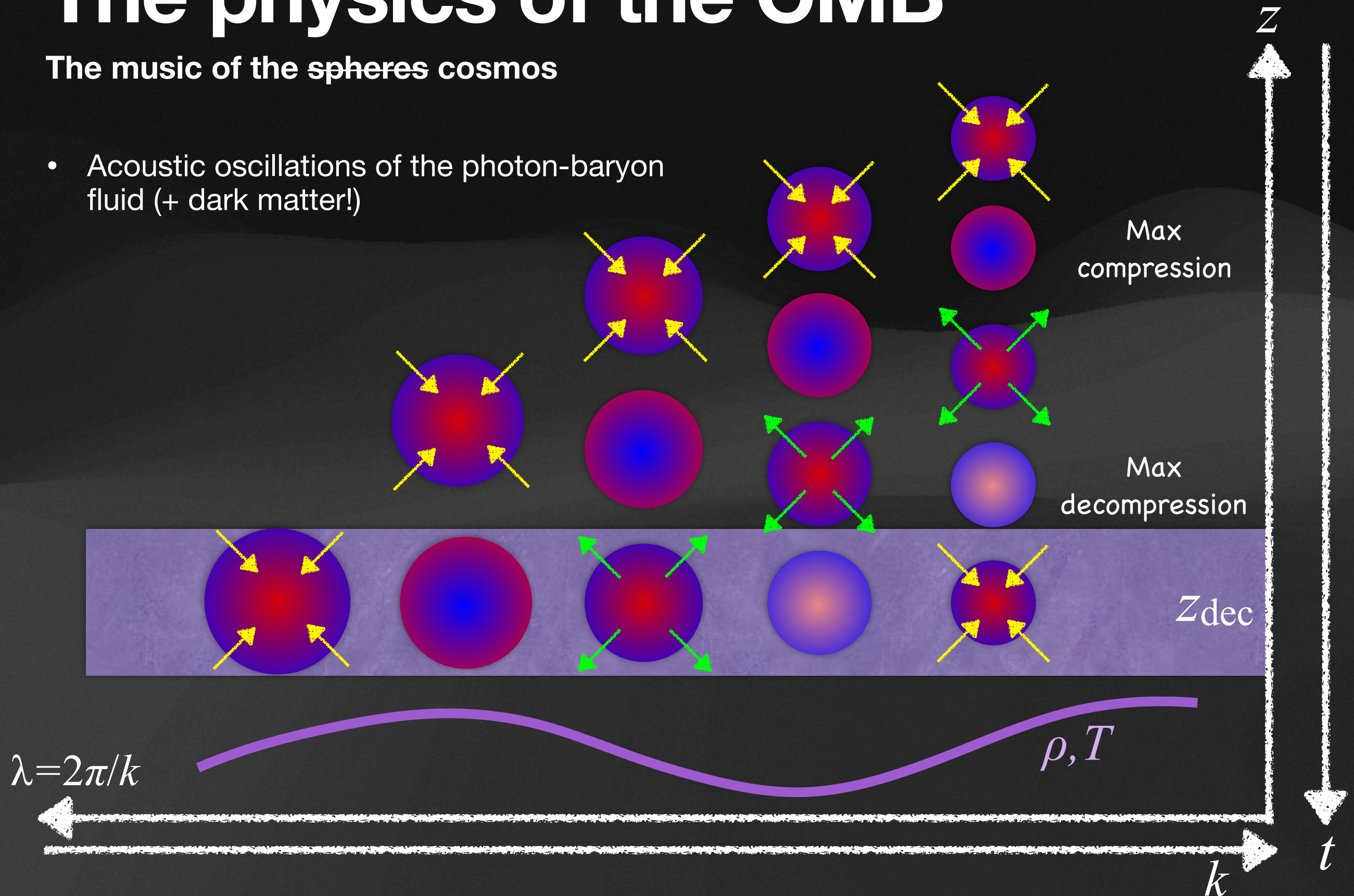




# The physics of the CMB

## The music of the spheres cosmos

- Acoustic oscillations of the photon-baryon fluid (+ dark matter!)



# The physics of the CMB

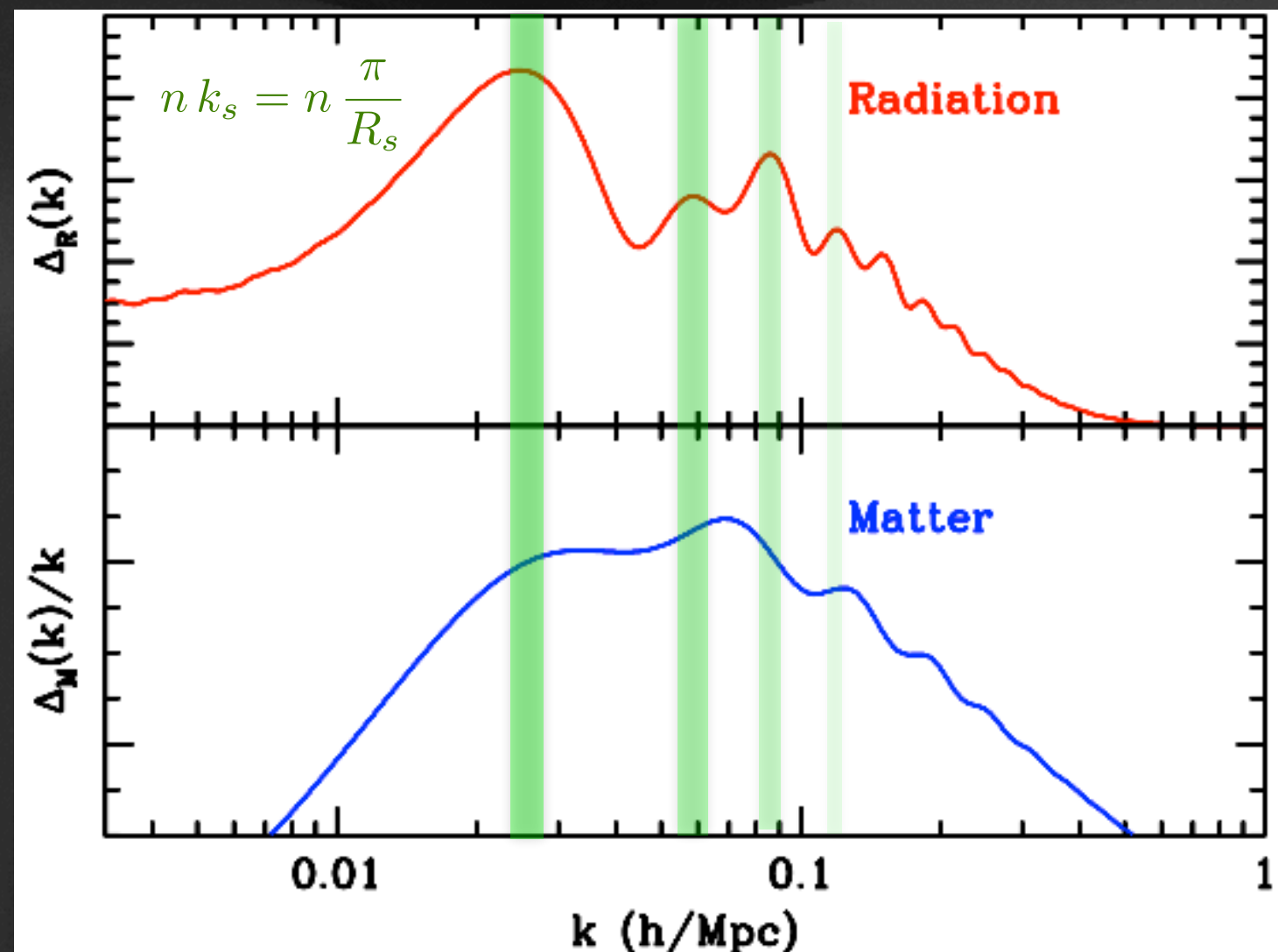
## The music of the spheres cosmos

- The **characteristic scale** at matter/radiation decoupling is the **distance** that the plasma sound wave can **travel** up to that moment.

- This is also called the **sound horizon at decoupling**, and is simple to compute:

$$\begin{aligned} R_s &= \int_0^{t_{dec}} \frac{c_s dt}{a(t)} \\ &= \int_0^{a_{dec}} \frac{c_s da}{H a^2} \\ &= \int_{z_{dec}}^{\infty} \frac{c_s dz}{H(z)} \end{aligned}$$

- The peaks are simply the **harmonics** of that fundamental scale!

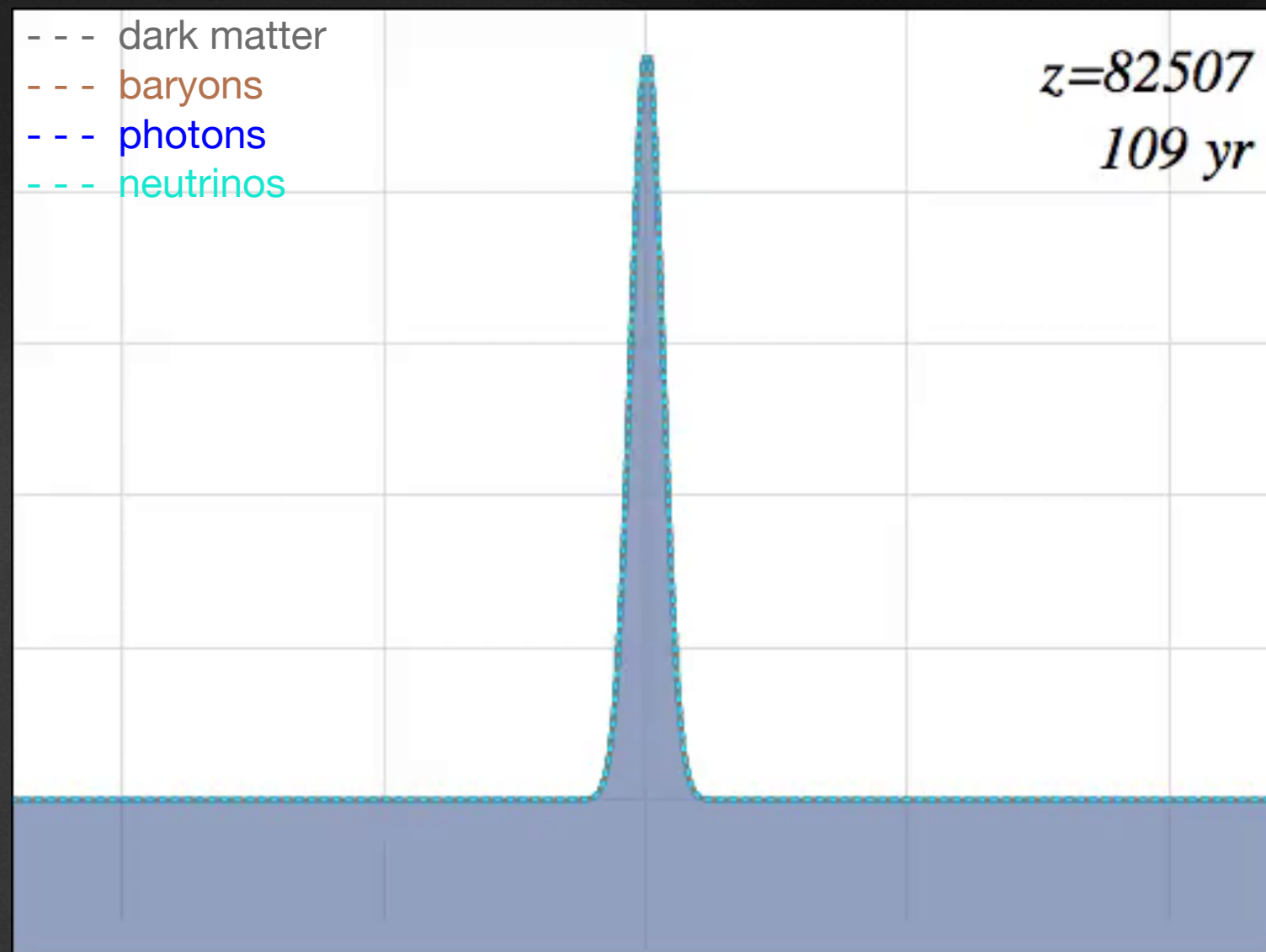




# The physics of the CMB

## The music of the spheres cosmos

- A complementary super-simplified picture emerges when we look at an individual “clump”, a region that for any reason had a high temperature and a large concentration of matter at very early times.



<https://www.youtube.com/watch?v=L7ws68XmgKo>

# The physics of the CMB

## The music of the spheres cosmos

- Observations from the Planck satellite constrain many cosmological parameters, but in particular this **acoustic horizon** is one of the most accurately measured:

$$\theta_{\star} = (1.04162 \pm 0.00056) \times 10^{-3} \quad , \quad R_s(z_{dec}) = (146.8 \pm 1.8) \text{ Mpc}$$

- We can measure the angle in the sky that corresponds to this scale at the last scattering surface. Remember that, since this is an **angle**, the relevant distance to take into account is the **angular diameter distance**, which we introduced in the last class:

$$\theta_s = \frac{R_{s,ph}(z_{dec})}{D_a(z=0 \rightarrow z_{dec})} = \frac{a_{dec} R_s(z_{dec})}{a_0 D_a(z_{dec})} = \frac{R_s(z_{dec})}{(1+z_{dec}) D_a(z_{dec})}$$

- But recall that the angular diameter distance is given in terms of the FLRW metric by:

$$D_a(z) = (1+z) \frac{1}{\sqrt{K}} \sin \left[ \sqrt{K} \chi(z) \right] \quad , \quad \text{where} \quad \chi = \int \frac{cdt}{a(t)}$$

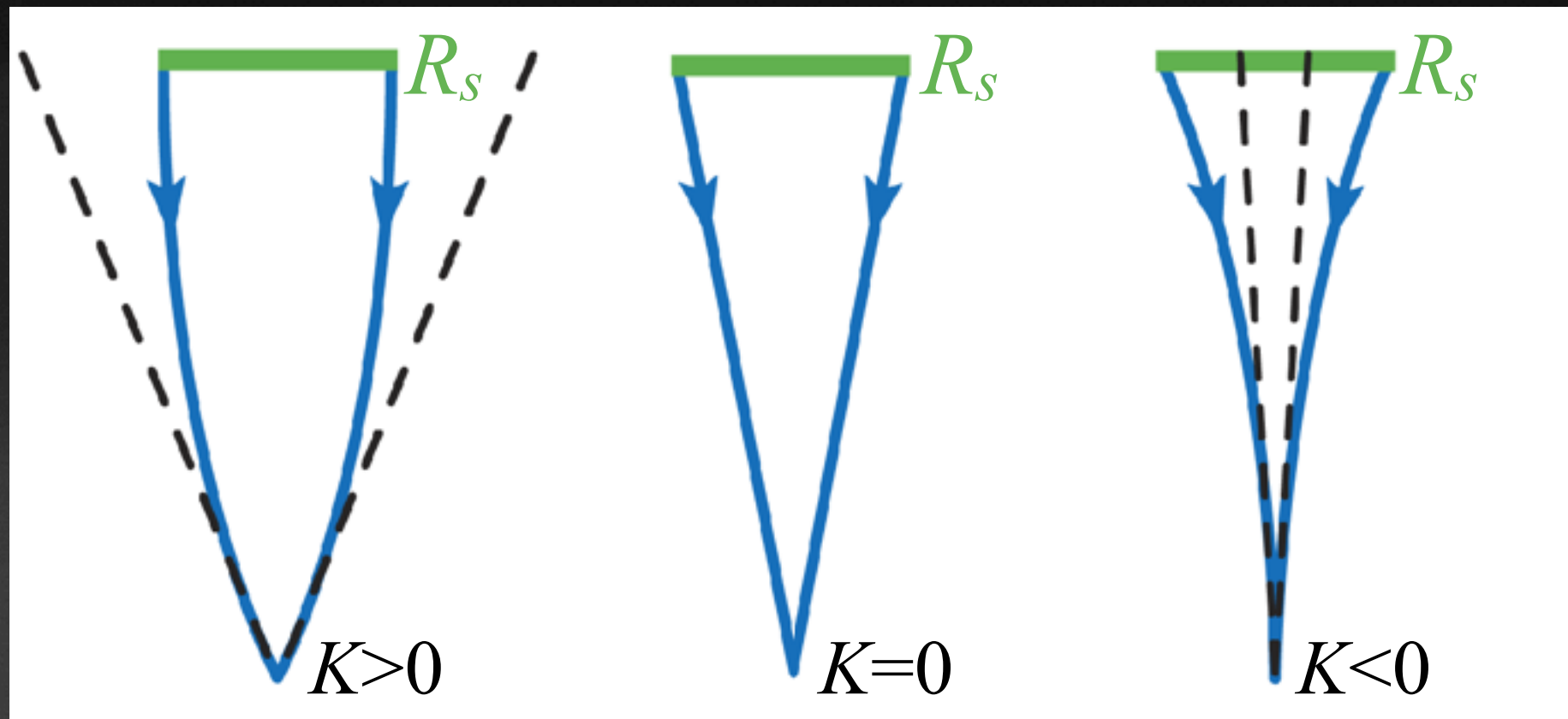
- This means that  $\theta_s$  is really sensitive to the spatial curvature!



# The physics of the CMB

## Strength from simplicity

- Schematically, spatial curvature is directly constrained from  $R_s$



$$\Omega_m^0 = 0.31, \Omega_\Lambda^0 = 0.7, \Omega_k^0 = -0.01 \quad \Rightarrow \quad D_a(z = 1100) = 8.442 h^{-1} \text{ Mpc}$$

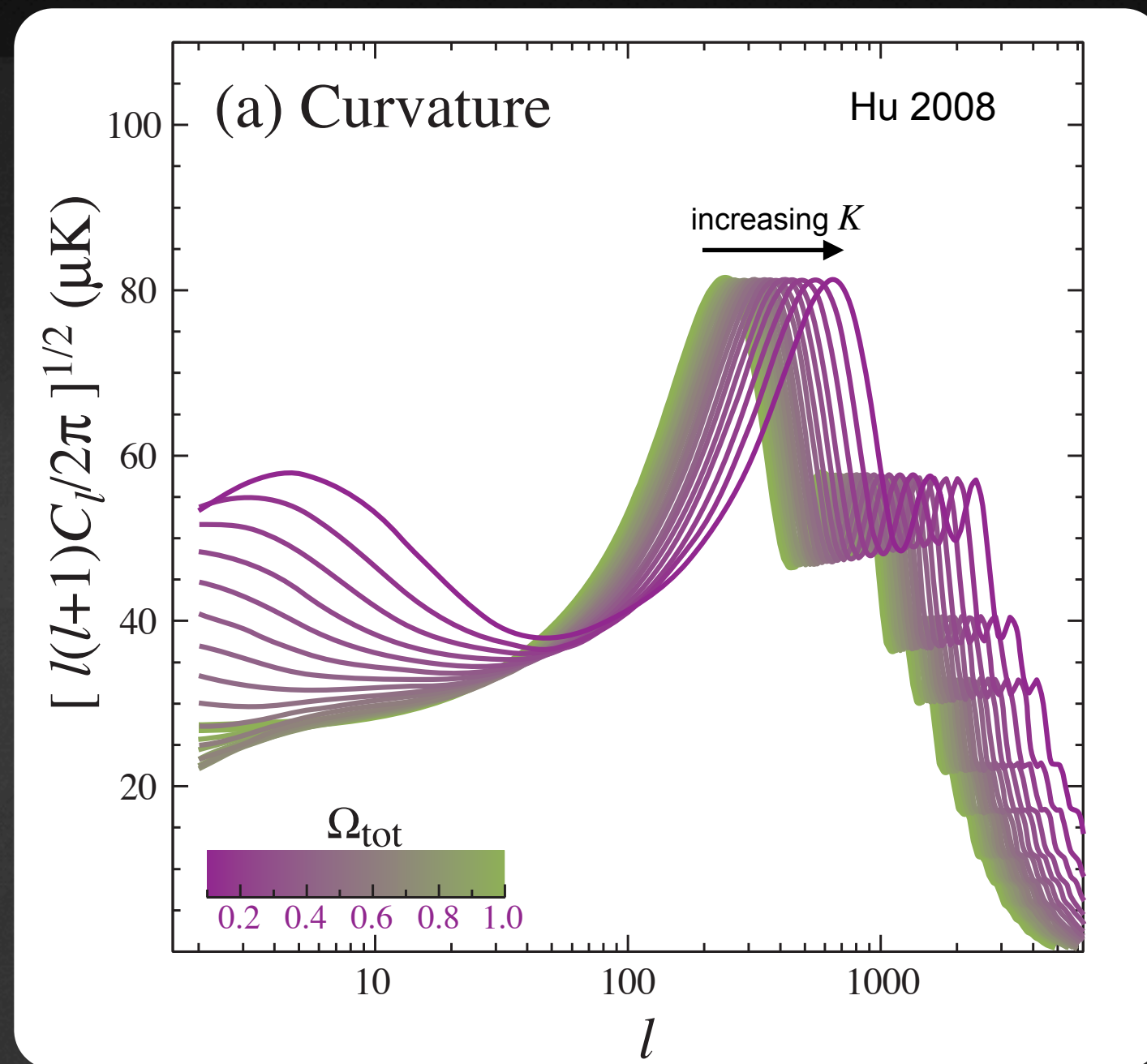
$$\Omega_m^0 = 0.3, \Omega_\Lambda^0 = 0.7, \Omega_k^0 = 0.0 \quad \Rightarrow \quad D_a(z = 1100) = 8.676 h^{-1} \text{ Mpc}$$

$$\Omega_m^0 = 0.29, \Omega_\Lambda^0 = 0.7, \Omega_k^0 = 0.01 \quad \Rightarrow \quad D_a(z = 1100) = 8.924 h^{-1} \text{ Mpc}$$

# The physics of the CMB

## Strength from simplicity

- The result is that spatial curvature has a huge impact on the CMB angular power spectrum

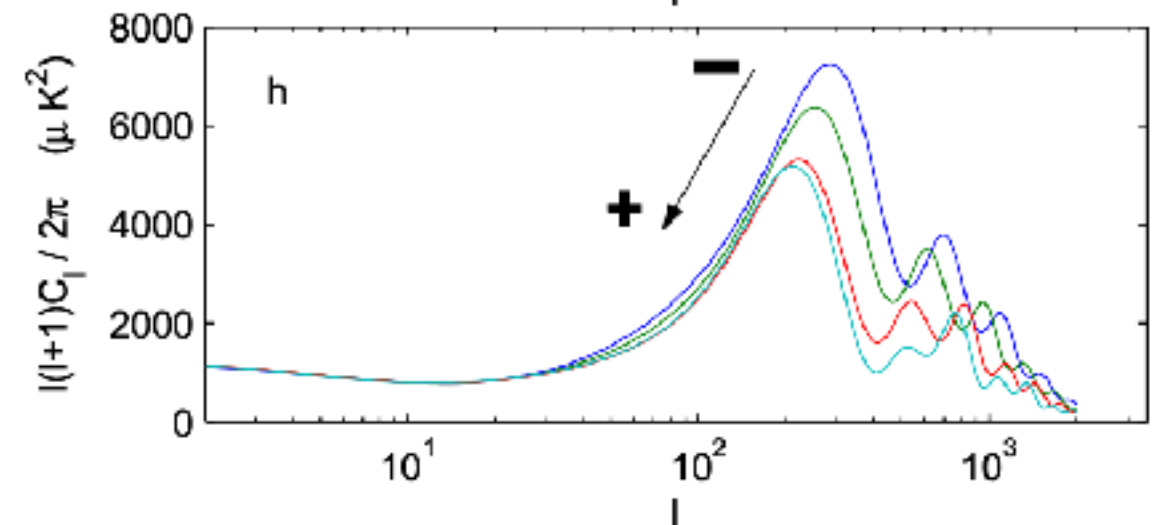
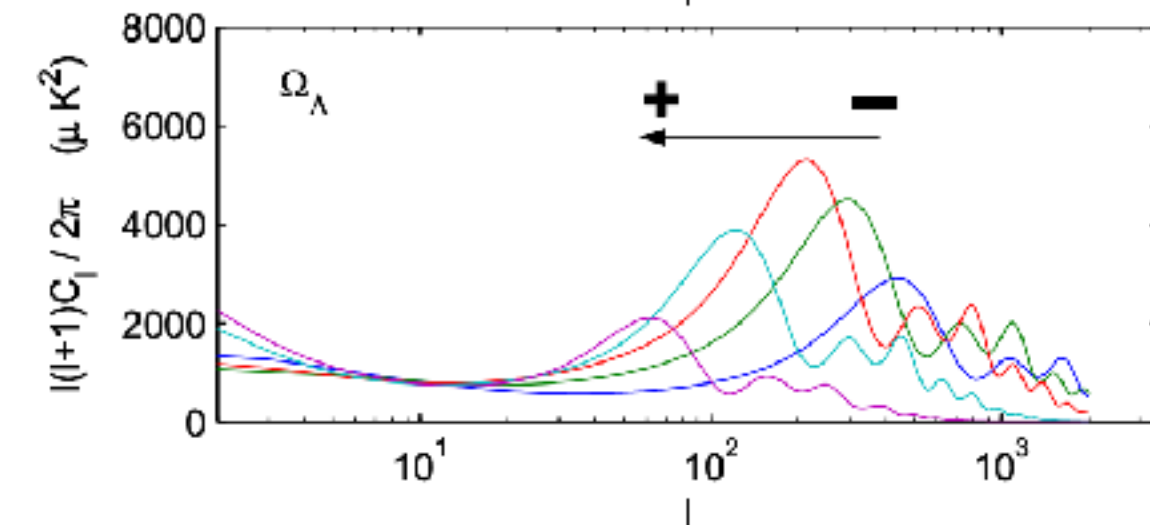
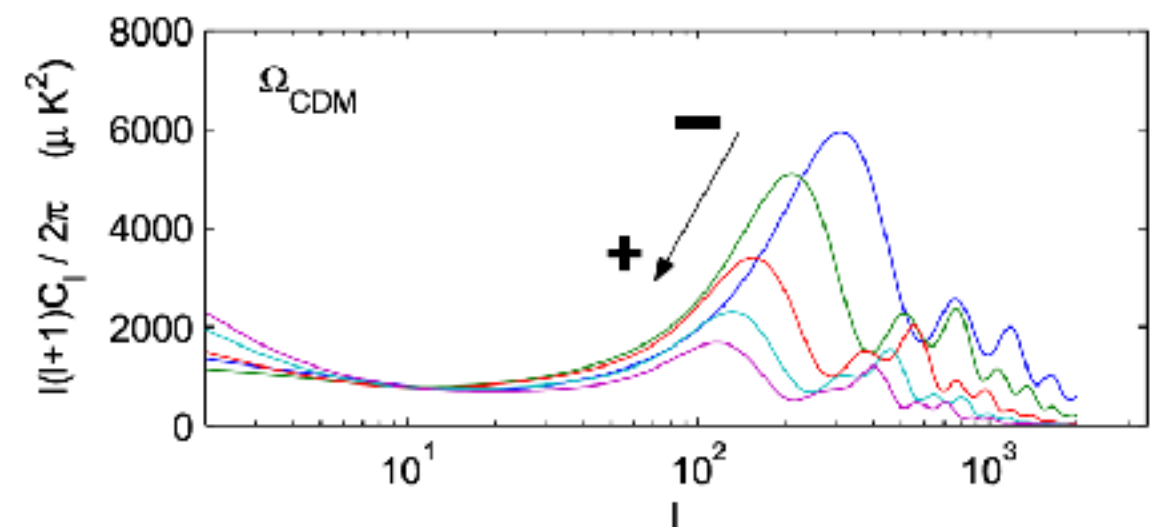
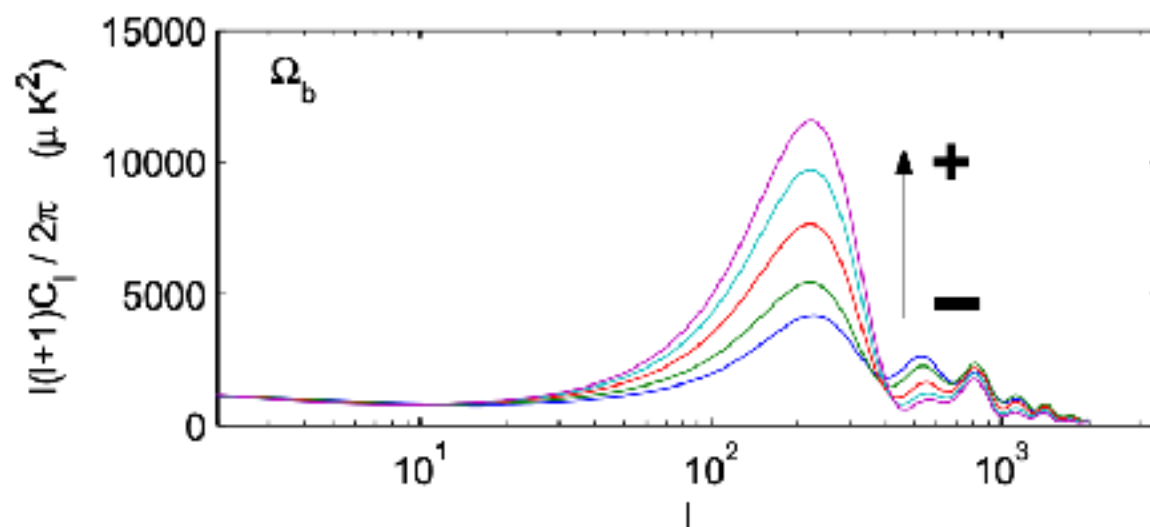




# The physics of the CMB

## Strength from simplicity

- Several other parameters have a significant impact on the  $C_\ell$ , making it a **very sensitive** probe of some cosmological parameters



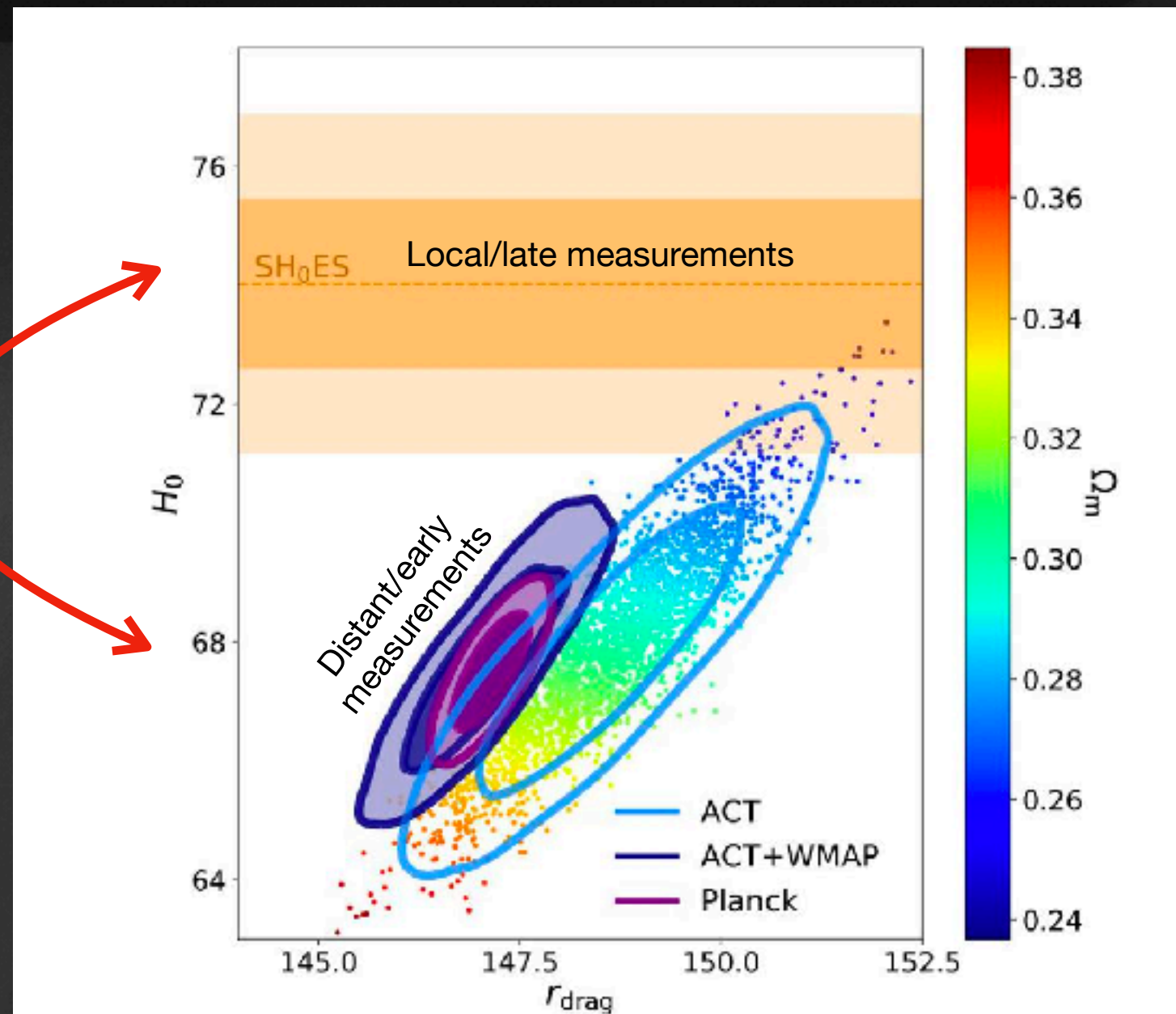
E. Martínez-González, arxiv: astro-ph/0610162

# The physics of the CMB

## Strength from simplicity

- However, things are not quite so pretty after all...

Latest:  
 $\sim 4 - 5\sigma$   
tension between  
the different  
measurements



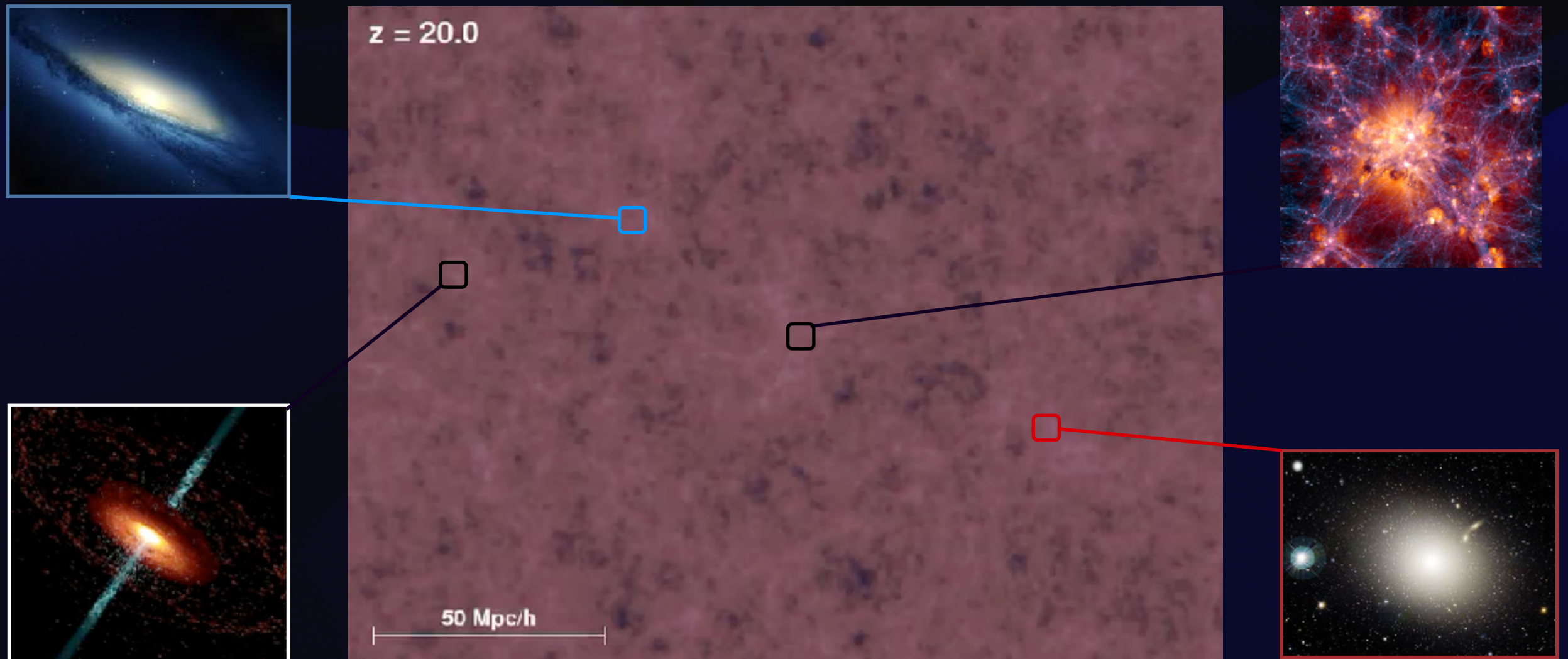
ACT, 2021



# The perturbed Universe

## On growing old and getting wrinkles

- The initial conditions of our Universe are imprinted on the CMB
- How do we get from that ( $\delta\rho/\rho \sim 10^{-4} - 10^{-5}$  at  $z \sim 10^3$ ) to the stars, galaxies, supernovas and black holes we see today ( $\delta\rho/\rho \gg 1$  at  $z \sim 0$ ) ?





# The perturbed Universe

## On growing old and getting wrinkles

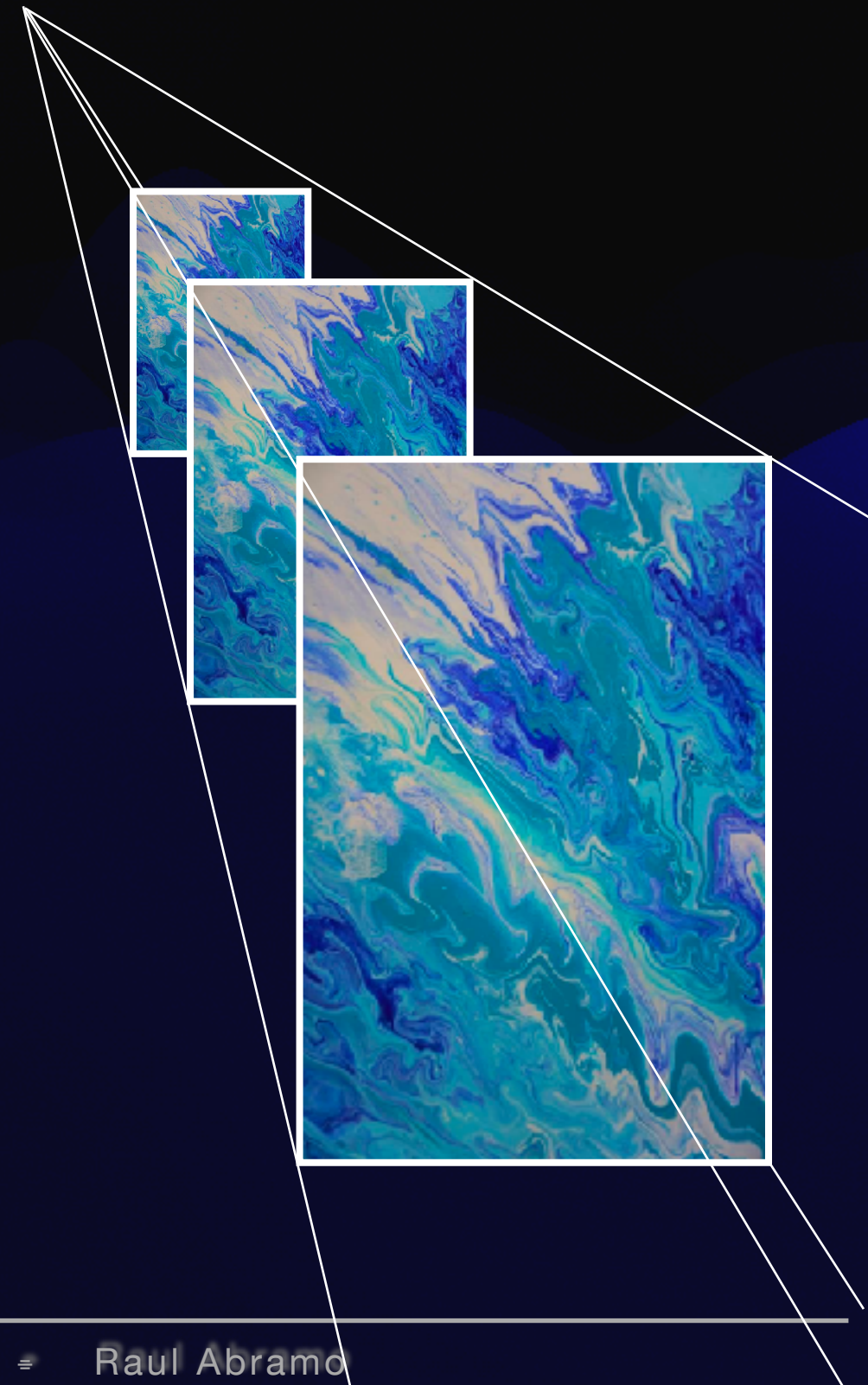
- In order to understand this process we shall study how the perturbations of a fluid behave in an expanding Universe.
- Let's say that the fluid is described by its density, pressure and a velocity such that, using the **physical** distances and velocities we have:

$$\rho(\vec{x}, t) = \bar{\rho}(t) + \delta\rho(\vec{x}, t)$$

$$p(\vec{x}, t) = \bar{p}(t) + \delta p(\vec{x}, t)$$

$$\vec{v}(\vec{x}, t) = H(t)\vec{x} + \delta\vec{v}(\vec{x}, t)$$

where  $H(t)\vec{x}$  expresses the Hubble flow of cosmic expansion





# The perturbed Universe

## On growing old and getting wrinkles

- You may have already encountered these fluid equations in other contexts:

Continuity equation/energy conservation  $\dot{\rho} + \rho \vec{\nabla} \cdot \vec{v} = 0$

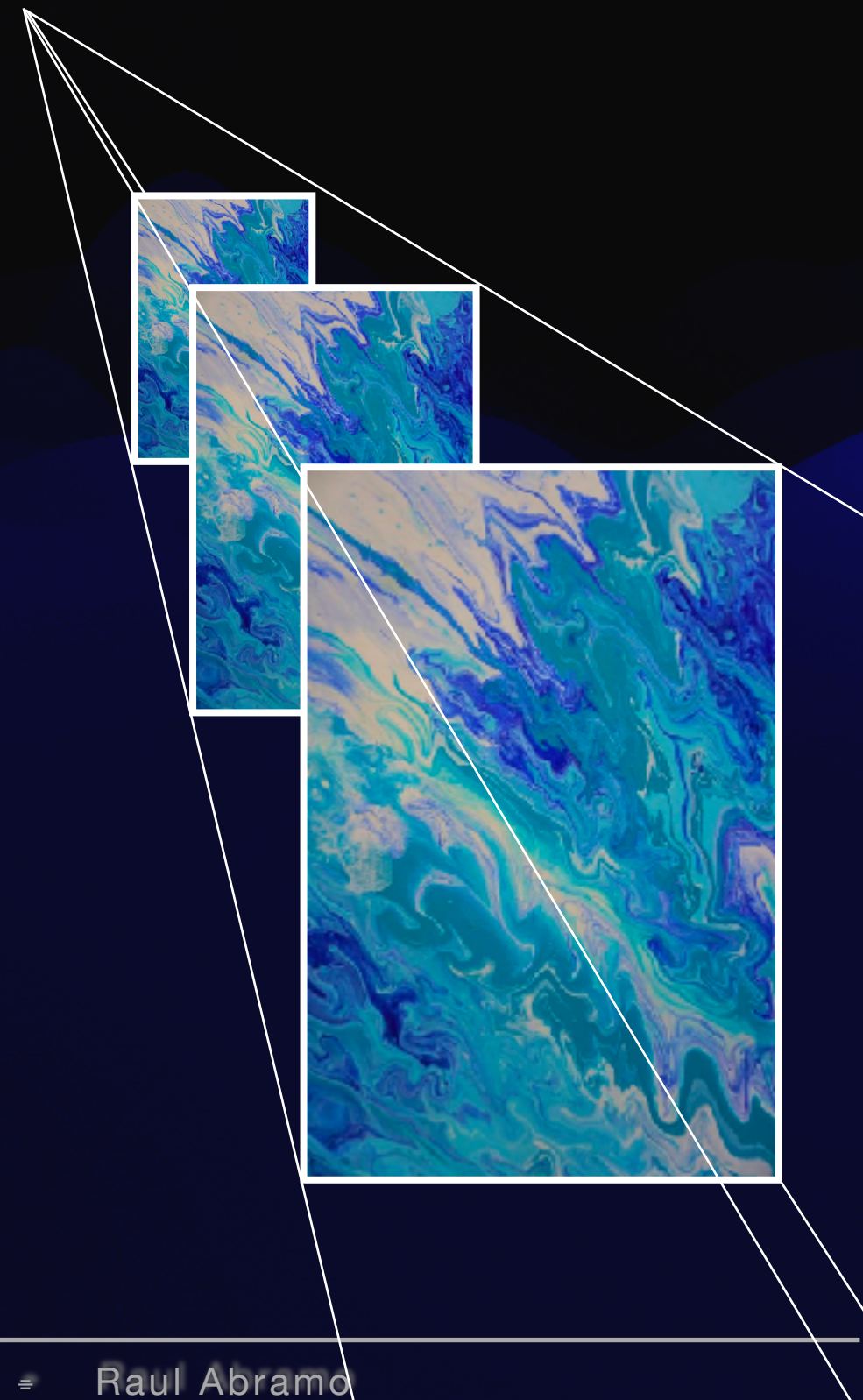
Euler equation/momentum conservation  $\vec{v} + \frac{\vec{\nabla} p}{\rho} + \vec{\nabla} \Phi = 0$

Poisson equation/gravity  $\nabla^2 \Phi = 4\pi G \rho$

- The background equations, for the mean density ( $\bar{\rho}$ ) and pressure ( $\bar{p}$ ), are simply the **Friedmann equations** (we will assume a matter-dominated Universe).
- In order to compute the dynamics of perturbations it is useful to write these equations in terms of the **comoving** coordinates and velocities:

$$\vec{x} = a(t) \vec{r}$$

$$\delta \vec{v} = a(t) \vec{u}$$





# The perturbed Universe

## Structure formation

- Opening up these equations and keeping only the **linear terms** we get:

Continuity equation:  $\delta\dot{\rho} + \vec{\nabla} \cdot \vec{u} = 0$

Euler equation:  $\vec{u} + 2H\vec{u} = -\vec{\nabla}\Phi - \frac{1}{a}\frac{\vec{\nabla}\delta p}{\bar{\rho}}$

Poisson equation:  $\nabla^2\Phi = 4\pi G \delta\rho$

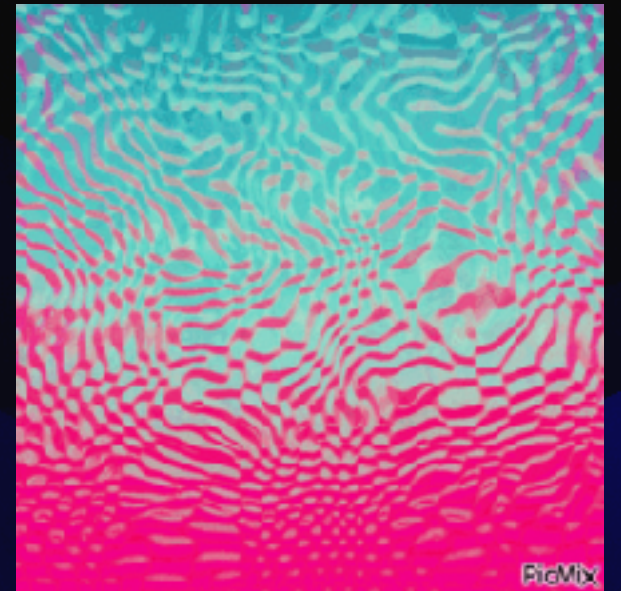
- We can simplify these equations by assuming that we are in a (cold) matter-dominated Universe, so  $\bar{p} = 0$  ,  $\delta p \rightarrow 0$  .
- In that case we combine the three equations above and rewrite them in terms of the **density contrast**:

$$\delta_\rho = \frac{\rho(\vec{x}, t) - \bar{\rho}(t)}{\bar{\rho}(t)} = \frac{\delta\rho(\vec{x}, t)}{\bar{\rho}(t)}$$

- In the linear regime we then get a **second-order** equation:

$$\ddot{\delta}_\rho + 2H\dot{\delta}_\rho = 4\pi G\bar{\rho} \delta_\rho = \frac{3H^2}{2}\delta_\rho$$

Question: Arrive at this equation from the previous ones





# The perturbed Universe

## Structure formation

- Let's appreciate this last equation again for a moment: this is a very simple equation for  $\delta_\rho(\vec{x}, t)$  alone, and which does not involve any spatial operators (gradient, laplacian):

$$\ddot{\delta}_\rho + 2H\dot{\delta}_\rho - \frac{3H^2}{2}\delta_\rho = 0$$

- Recall that in a Universe dominated by cold matter ("dust") the scale factor is  $a(t) \sim t^{2/3}$ , and  $H(t) = 2/(3t)$ . We then get the equation:

$$\ddot{\delta}_\rho + \frac{4}{3t}\dot{\delta}_\rho - \frac{2}{3t^2}\delta_\rho = 0$$

- This linear, second-order equation has of course two linearly independent solutions. The form of the derivatives leads us to believe that a power-law solution is possible, something like

$\delta_\rho(\vec{x}, t) = \delta_\rho(\vec{x}, t_0) \times (t/t_0)^\beta$ . Using that *ansatz* we obtain :

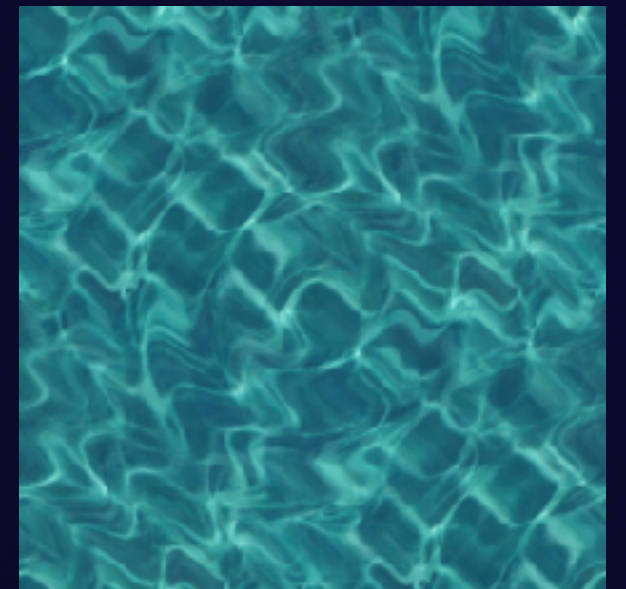
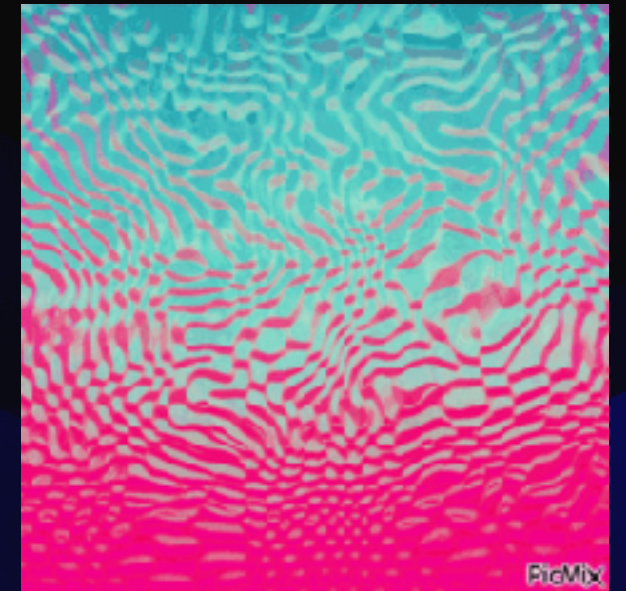
$$\beta(\beta - 1) + \frac{4}{3}\beta - \frac{2}{3} = 0$$

- There are two solutions:  $\beta_+ = 2/3$  and  $\beta_- = -1$ . Therefore:

$$\delta_\rho(\vec{x}, t) = \delta_\rho(\vec{x}, t_0) \times [c_+(t/t_0)^{2/3} + c_-(t/t_0)^{-1}]$$

- Clearly, the solution with  $\beta_- = -1$  decays with time, so given some "random" initial conditions, only the growing solution ( $\beta_+ = 2/3$ ) survives. Therefore, the dominant solution is:

$$\delta_\rho(\vec{x}, t) \rightarrow \delta_\rho(\vec{x}, t_0) \times (t/t_0)^{2/3} = a(t) \delta_\rho(\vec{x}, t_0)$$





# The perturbed Universe

## Slowly at first, then rapidly

- We conclude, therefore, that at least for a while, when the Universe was dominated by matter ( $z \sim 10^4 \rightarrow z \sim 0.6$ ), and while the perturbations were small ( $\delta_\rho \ll 1$ ), the density fluctuations grow in the **same way as the scale factor**,

$$\delta_\rho(\vec{x}, t) = \frac{a(t)}{a(t_i)} \delta_\rho(\vec{x}, t_i)$$

In the absence of expansion, this growth is exponential

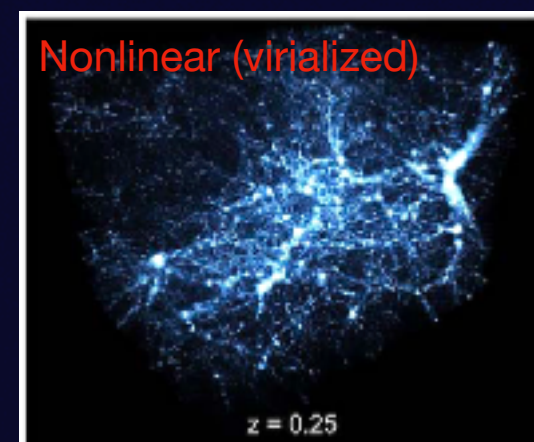
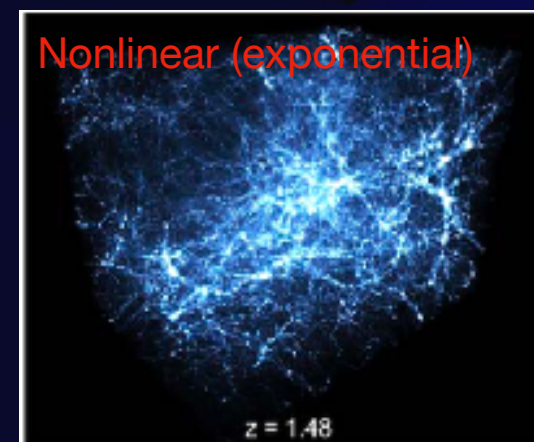
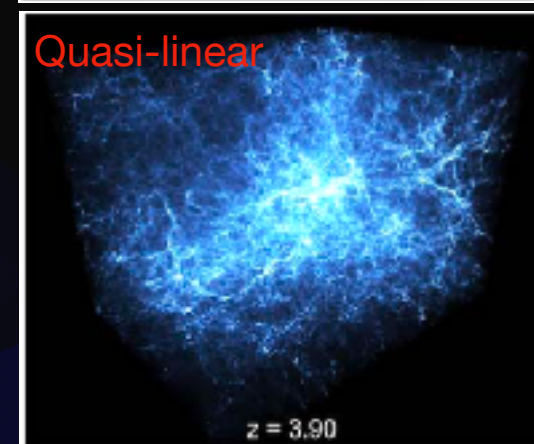
- Equivalently, we can say that the density contrast grows with the redshift:

$$\delta_\rho(\vec{x}, z) = \frac{1+z_i}{1+z} \delta_\rho(\vec{x}, t_i)$$

- Let's take the initial redshift of the formation of the CMB,  $z_i \sim 10^3$ , and let's assume that the initial density contrast at that time was  $|\delta_\rho(z_i)|_{rms} \sim \sqrt{\langle \delta_\rho^2(z_i) \rangle} \sim 10^{-4}$  – a good approximation for many scales. We then obtain:

$$|\delta_\rho(z)|_{rms} = \frac{10^3}{1+z} |\delta_\rho(z_i)|_{rms} \sim \frac{10^3}{1+z} 10^{-4} = 0.1 (1+z)^{-1}$$

- Now it is clear what is happening: we needed to wait from  $z_i \sim 10^3$  until a redshift  $z \sim 0$  (i.e., billions of years) for the density contrast to grow enough that it could become  $\sim 1$ .
- Only after that period of growth the Universe was able to start forming stars, planets, galaxies... life!!!

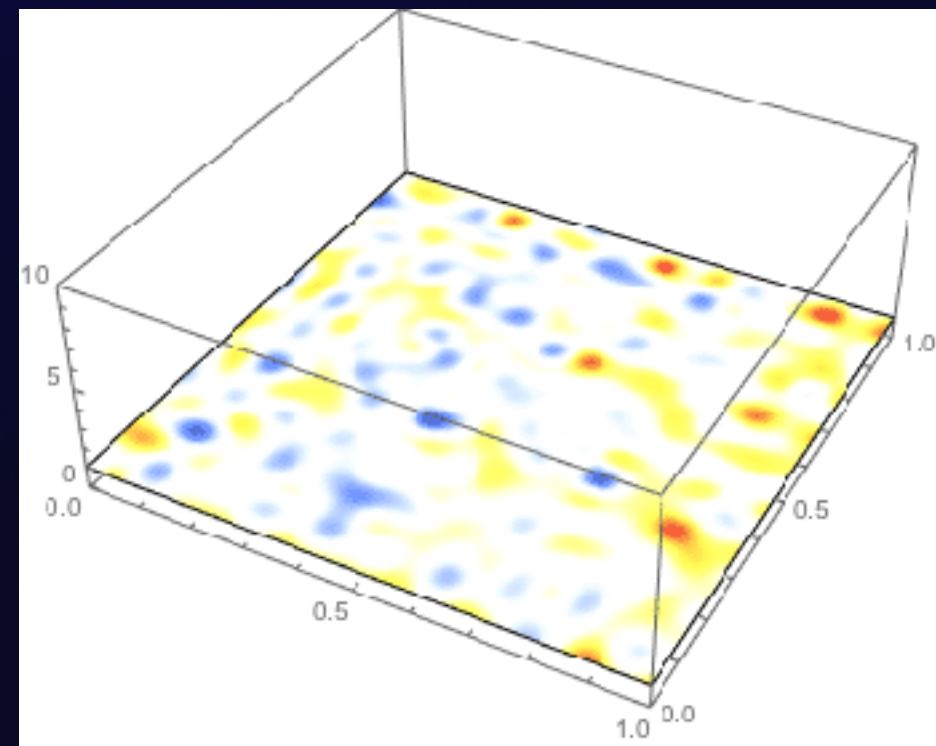
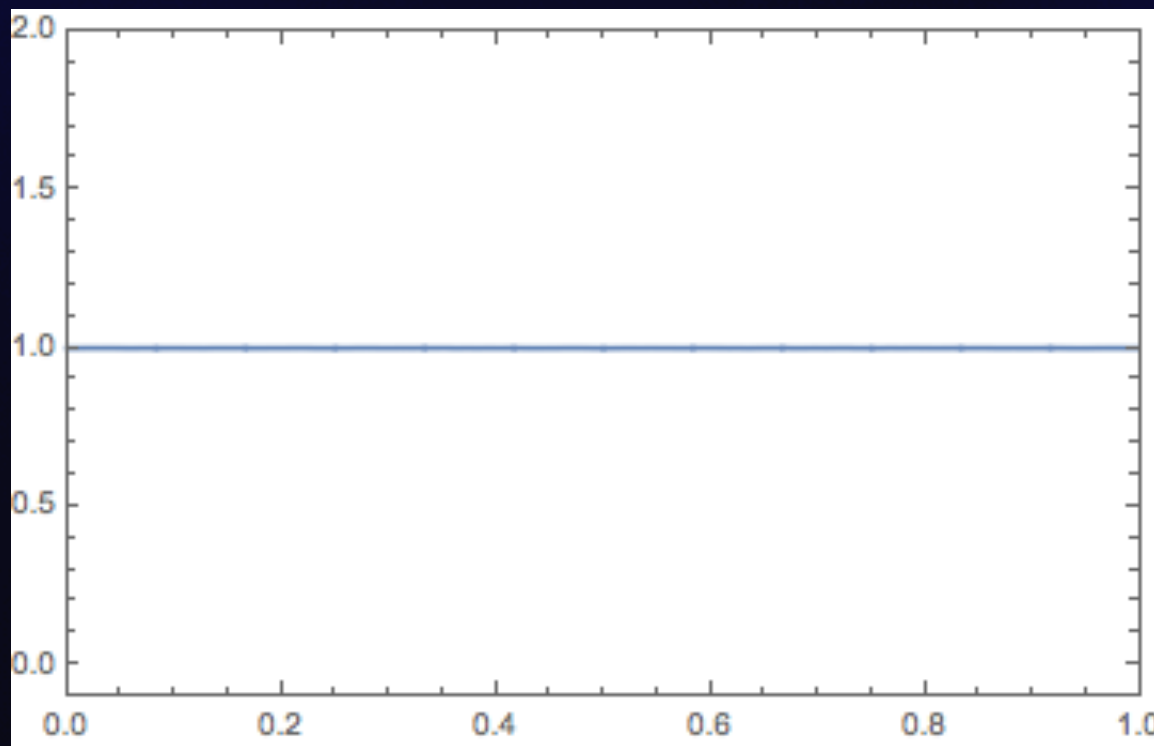




# The perturbed Universe

## Beyond linear theory

- However, bear in mind that the equations we used until now are only valid in the **linear regime**, when the density contrast is very small,  $\delta\rho/\rho \ll 1$ .
- In the linear regime the initial density profile is unchanged: only its **amplitude** changes with time, and the peaks/troughs don't move around.
- This is of course a **poor approximation**: as the gravity between two overdensities pulls them together, they start to get closer



# The perturbed Universe

## Statistical description

- The initial density field is very well described by a **Gaussian Random Field**. It is time we defined this better, as well as the **summary statistics** that we use to characterize this field.

- As we saw, the density field can be written as:

$$\rho(\vec{x}) = \bar{\rho} [1 + \delta(\vec{x})] ,$$

with the density contrast being a "random" field with zero mean, i.e.,  $\langle \delta \rangle = 0$

- It is best to decompose this spatial field in terms of Fourier modes:

$$\tilde{\delta}(\vec{k}) = \int d^3x e^{i\vec{k}\cdot\vec{x}} \delta(\vec{x})$$

- The modes  $\vec{k}$  of this field,  $\tilde{\delta}(\vec{k})$ , are **complex random numbers** that follow a normal (Gaussian) **probability distribution** with variance given by a **power spectrum**  $P(k)$ , i.e.:

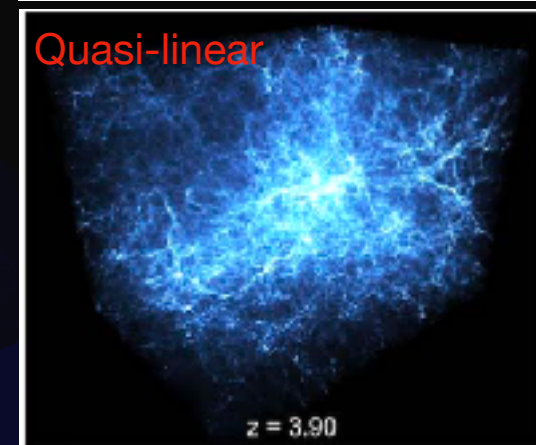
$$\mathcal{P}[\tilde{\delta}(\vec{k})] \sim \exp \left[ -\frac{|\tilde{\delta}(\vec{k})|^2}{2 P(k)} \right] , \quad \text{and conversely we get } \left\langle |\tilde{\delta}(\vec{k})|^2 \right\rangle \rightarrow P(k)$$

- In a **homogeneous** and **isotropic** Universe, this spectrum does not depend on the **direction** of the modes  $\vec{k}$ . But this function can still depend on the **scale** as given by  $k = 2\pi/\lambda$ .

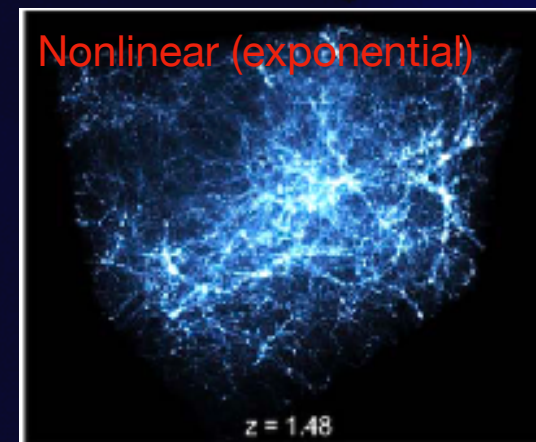
Linear



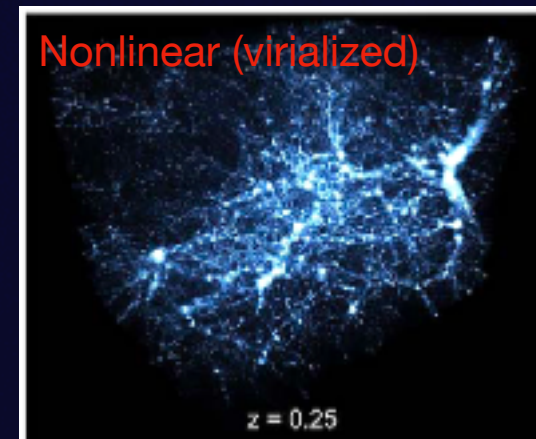
Quasi-linear



Nonlinear (exponential)



Nonlinear (virialized)





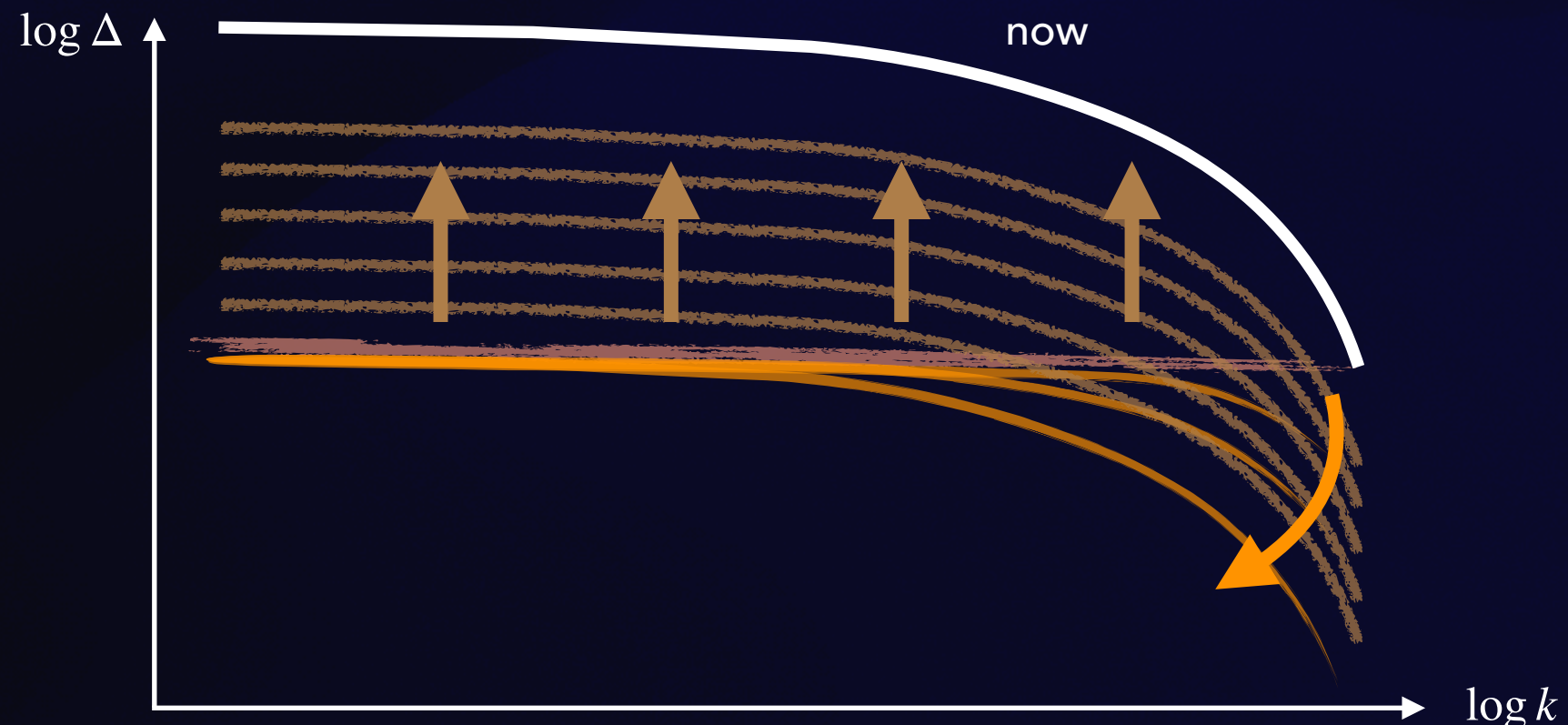
# The perturbed Universe

## Statistical description

- Inflation predicts a nearly scale-invariant initial spectrum of perturbations. Mathematically this statement is that the mass spectrum

$$\Delta(k) = k^3 P(k) \quad , \quad \text{is nearly scale-invariant, i.e., } \Delta(k) \simeq A_0 \left( \frac{k}{k_0} \right)^{-\alpha} \quad \text{with } \alpha \simeq 0 \quad .$$

- However, different physical process (in particular, radiation pressure) act to “equilibrate” the density fluctuations during the radiation-dominate era, which damp the small-wavelength modes.
- When we enter the matter era, the amplitude of the modes grow as  $\sim a(t)$  , and therefore the spectrum also grows in time as  $P(k, t) \sim a^2 P(k, t_i)$ .



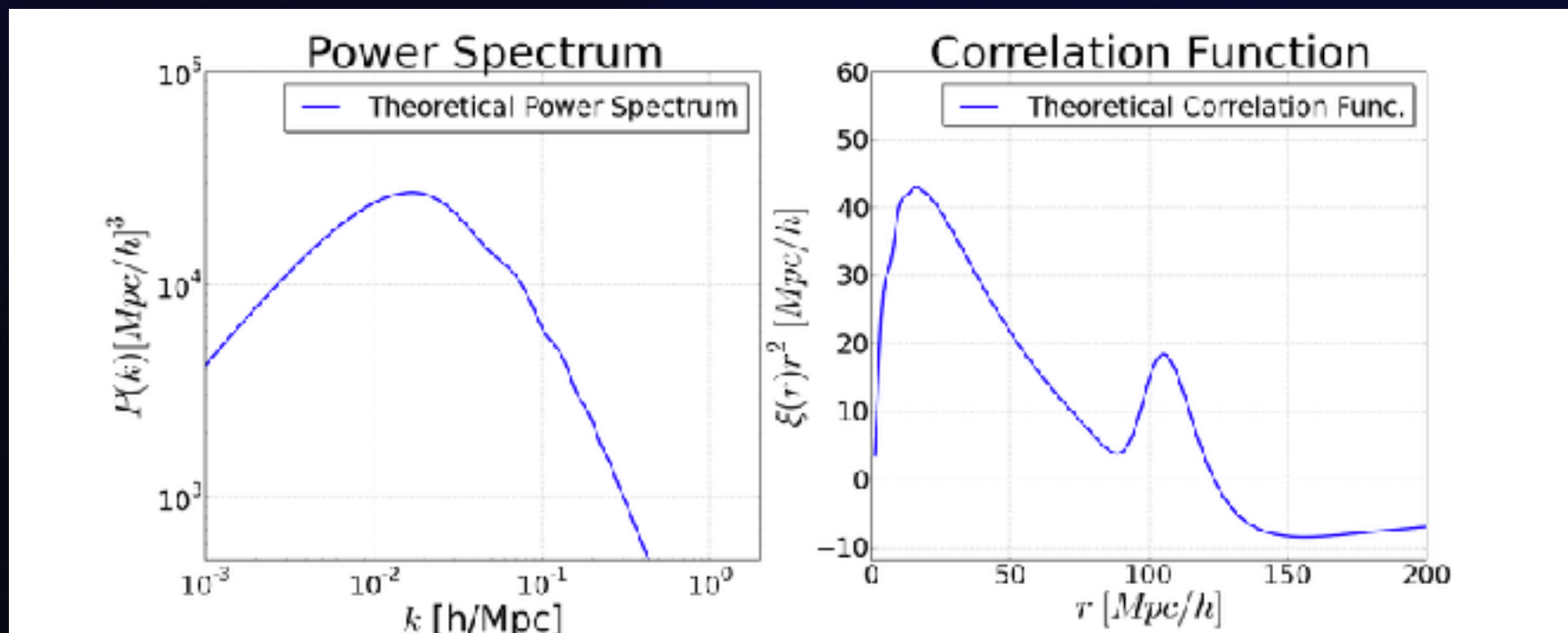
# The perturbed Universe

## Statistical description

- The precise form of the spectrum is of course a bit more complicated: e.g., we still need to account for the pattern of the baryon-photon acoustic oscillations that were imprinted at the time of the formation of the CMB.
- A precise calculation of the power spectrum (today's value) is shown below.
- A complementary way to represent the way that matter clusters in the Universe is through the 2-point correlation function:

$$\xi(R) = \langle \delta(\vec{x}) \delta(\vec{x} + \vec{R}) \rangle = \frac{1}{2\pi^2} \int dk k^2 \frac{\sin kR}{kR} P(k)$$

👉 **Question:** Show that  $\xi(R)$  and  $P(k)$  are in fact related by this equation.



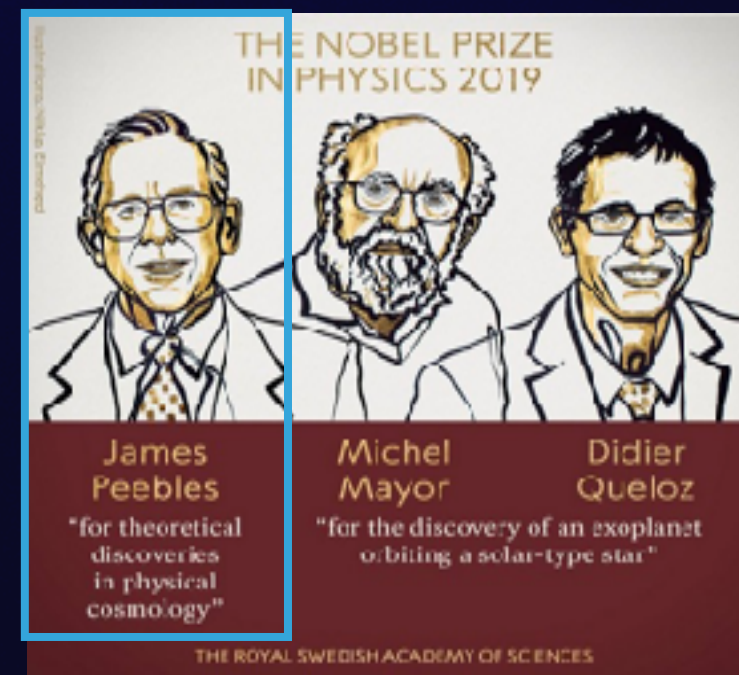
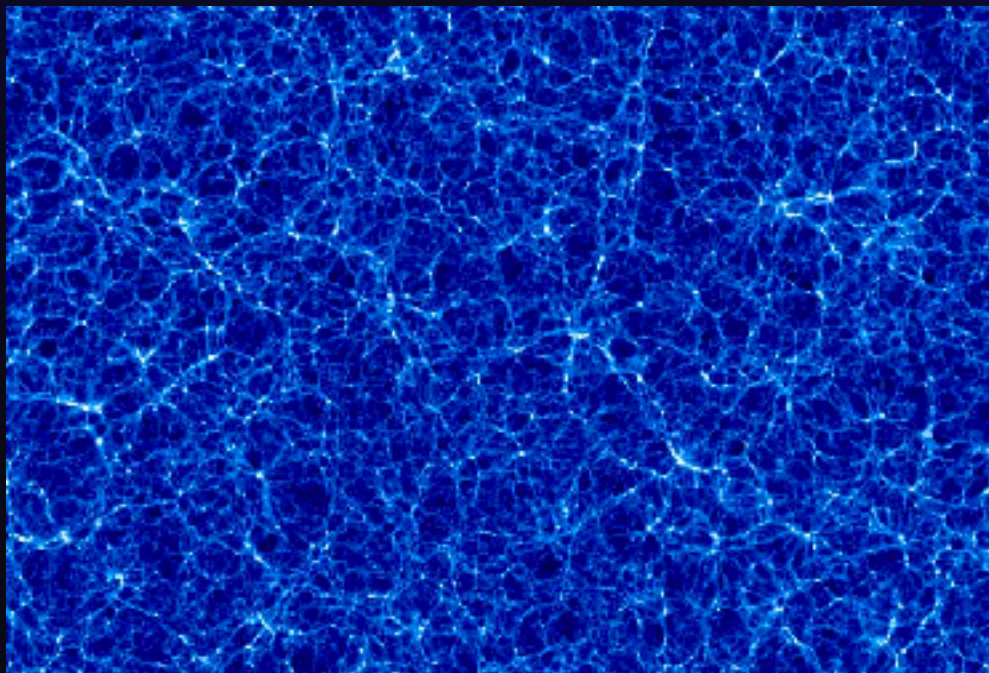
Loureiro 2015



# The perturbed Universe

## Simulating the cosmos

- One of the first people to realize this close link between the primordial Universe and structure formation was James (“Jim”) Peebles.
- From the initial conditions written in the CMB we can use mathematical calculations and numerical simulations that allow us to predict how the Universe should look like, after billions of years of evolution.
- When we look at the matter distribution, we see a web of structures such as walls, filaments, nodes, halos etc.





# The perturbed Universe

## Simulating the cosmos

- These simulations are evolving very fast: in the beginning we had dark matter-only simulations: baryons and radiation were not included (hydrodynamics and radiation feedback are *hard problems to solve!*)
- Recently, greater computing power and new numerical techniques paved the way to better, more accurate simulations that also include, to some extent, the atoms and radiation that we actually observe.

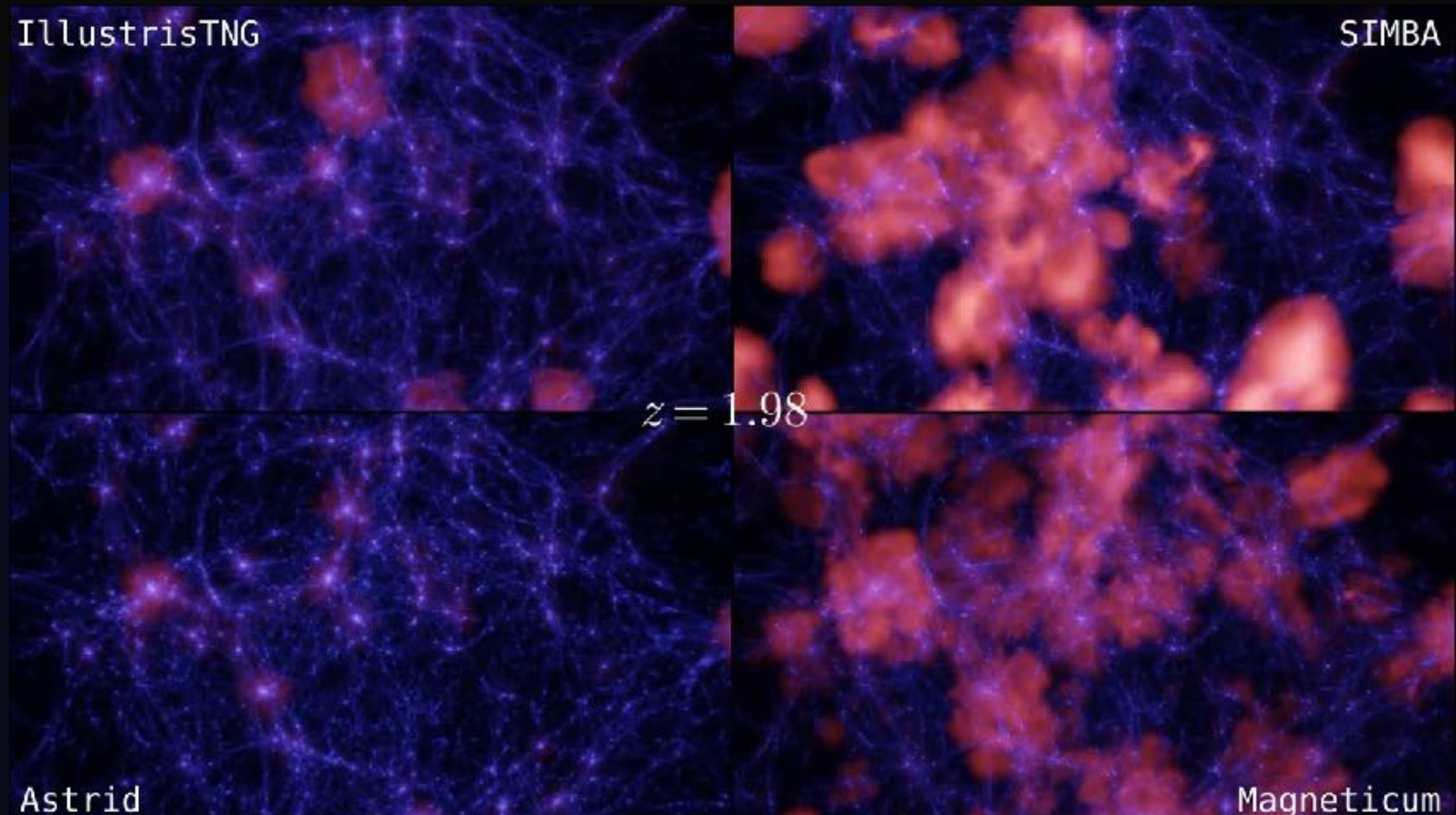




# The perturbed Universe

## Simulating the cosmos

- Baryonic physics can be important — and very complicated...





# Observational cosmology

Now you see me, now you don't

- The basic problem of cosmology is summarized by this figure from the James Webb Space Telescope:



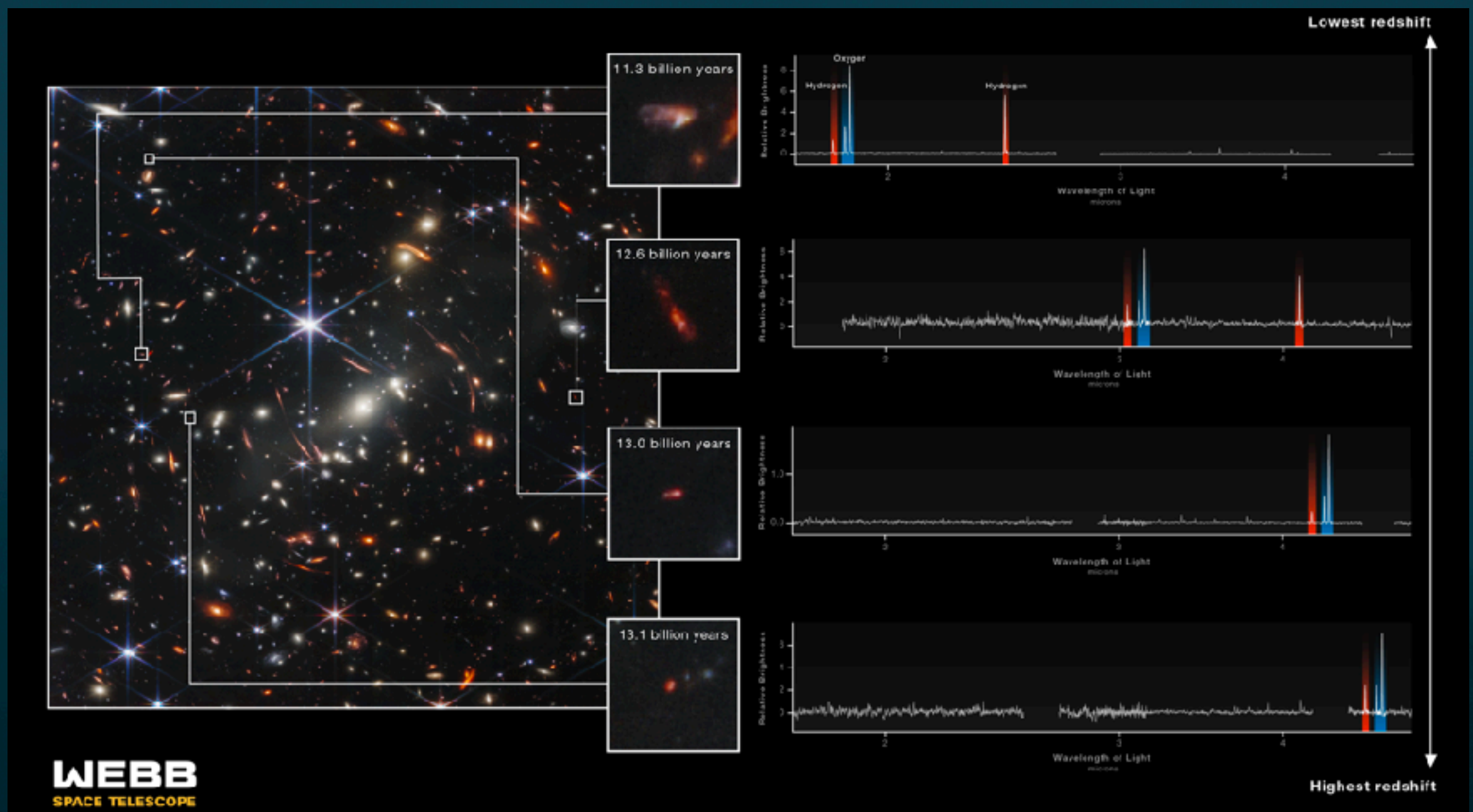
SMACS 0723 (NIRCAM)



# Observational cosmology

Now you see me, now you don't

- It can be very hard to figure out what is what... you need spectra for the objects in this image.

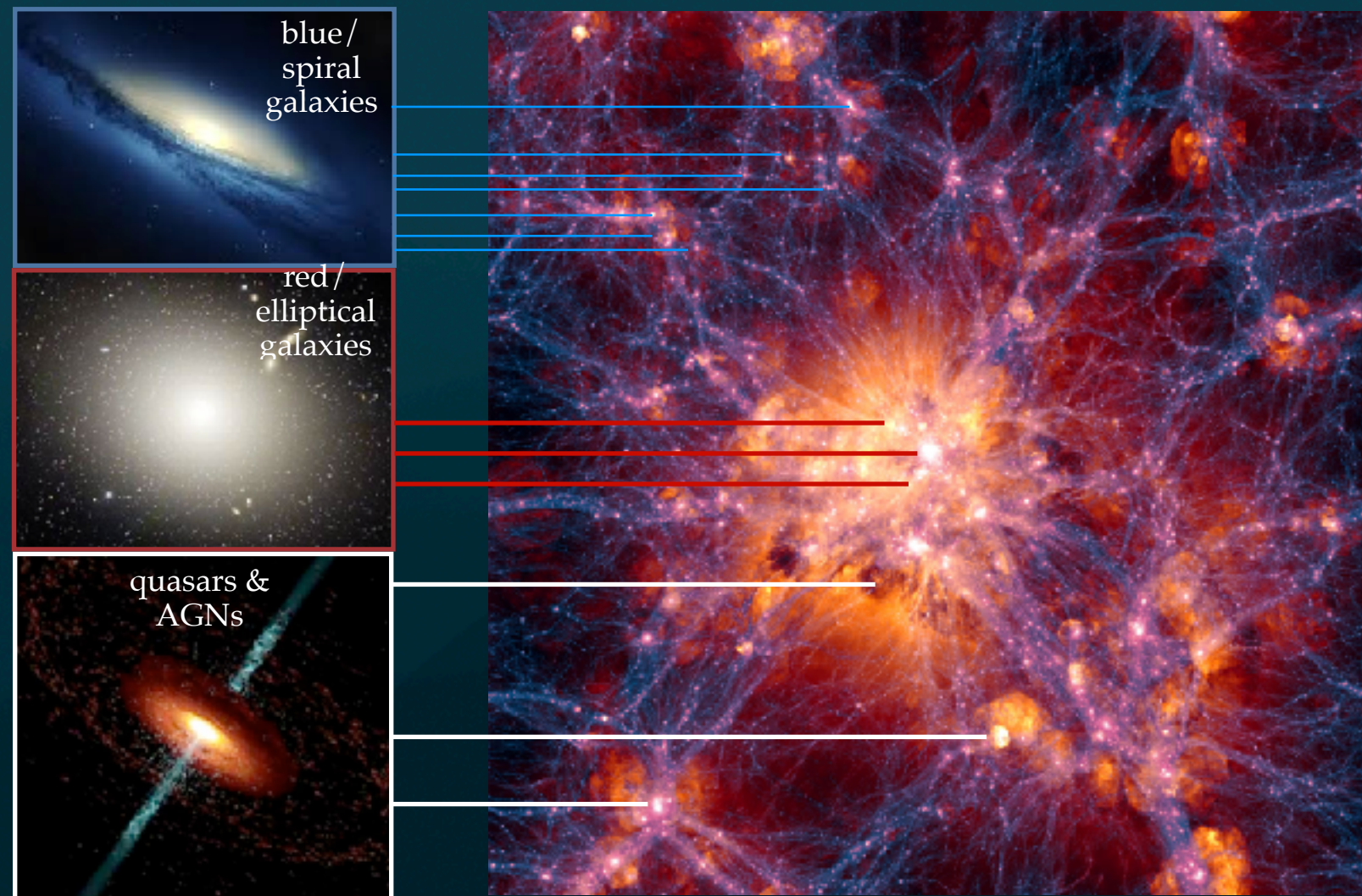




# Tracers of large-scale structures

## And how to observe them

- And reality is yet harsher: remember that **visible matter** is a small fraction ( $\lesssim 20\%$ ) of the total mass.
- In any large concentration of matter we can find a dazzling variety of astronomical objects: red elliptical galaxies, blue spiral galaxies, active galactic nuclei (AGNs), supernovas, neutral gas, ionized gas...

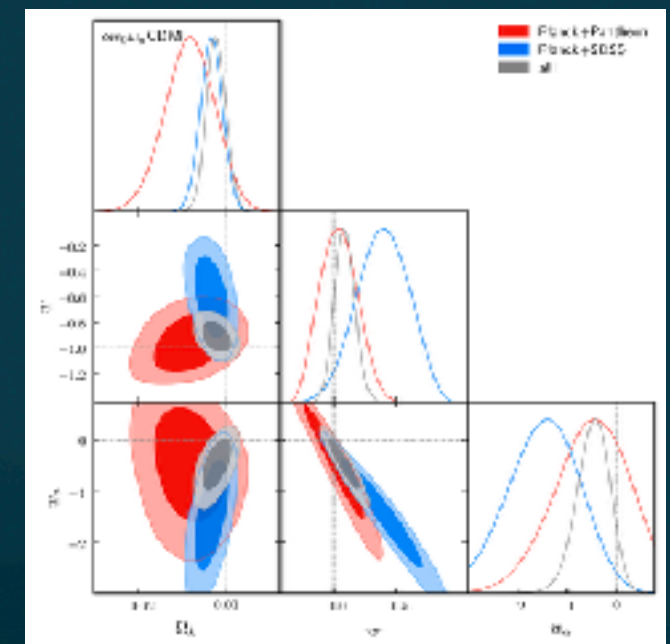
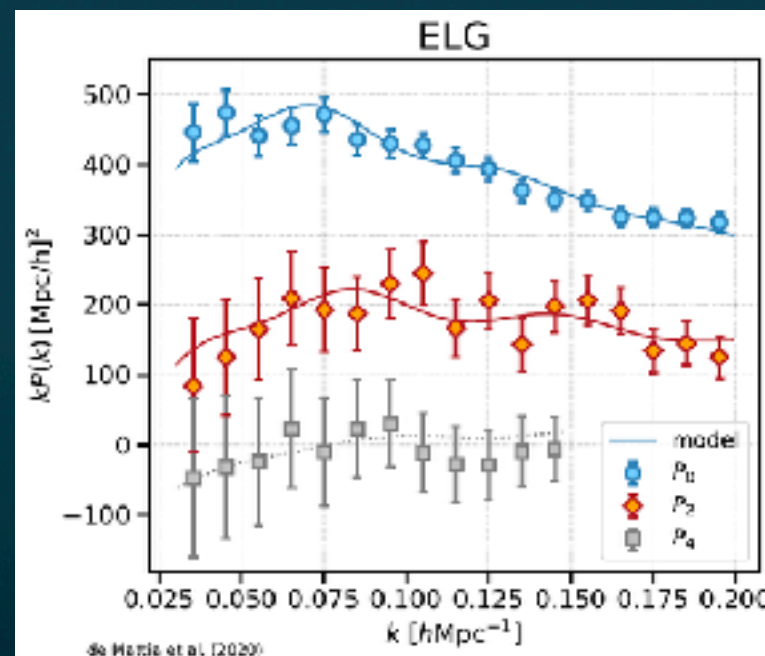




# Tracers of large-scale structures

## And how to observe them

- And the problem, my friends, is really this:



How to get from **here** .....to **here**.....to **there**.

How **observations** impact our **constraints**?

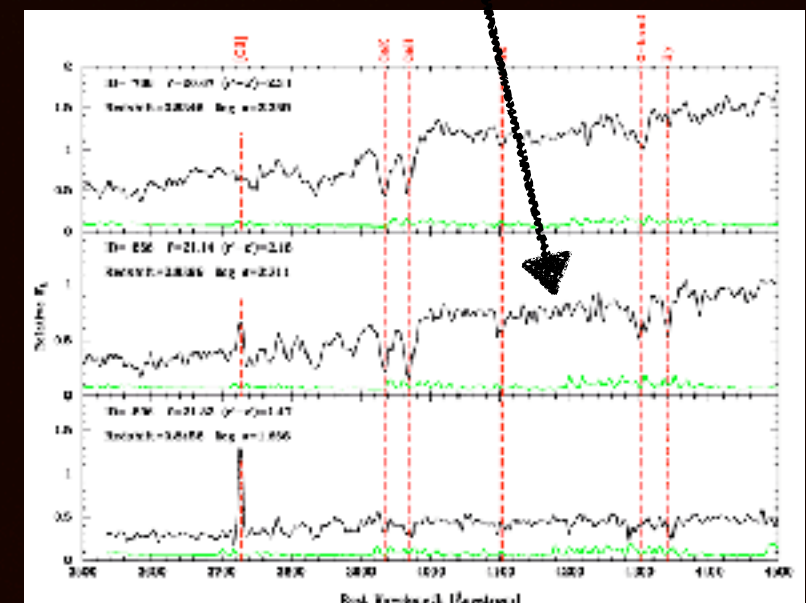
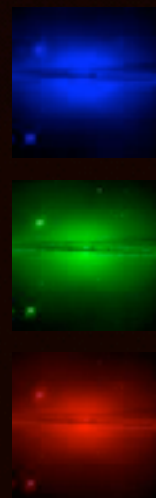
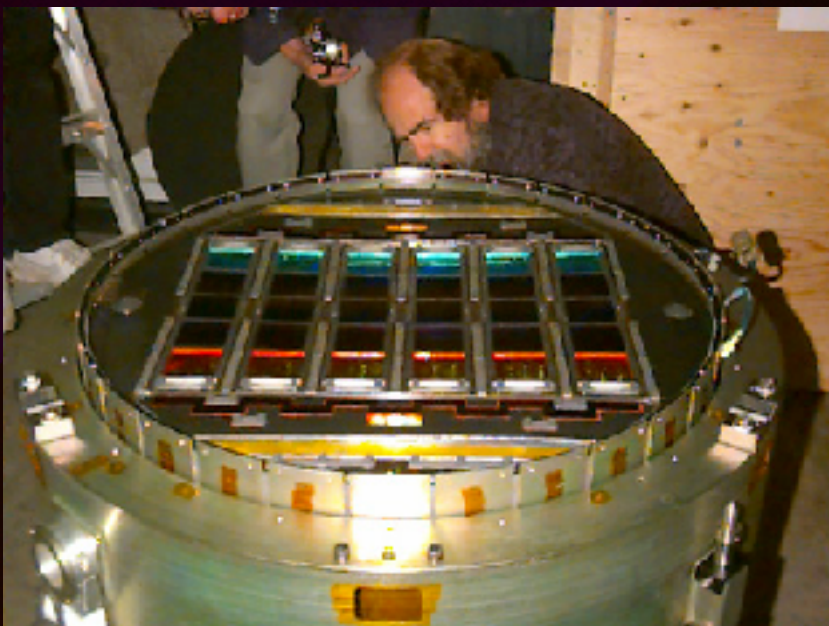
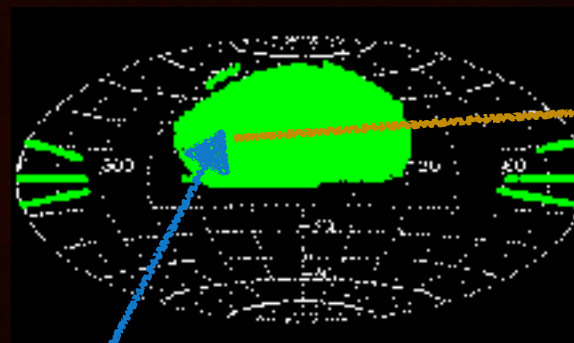
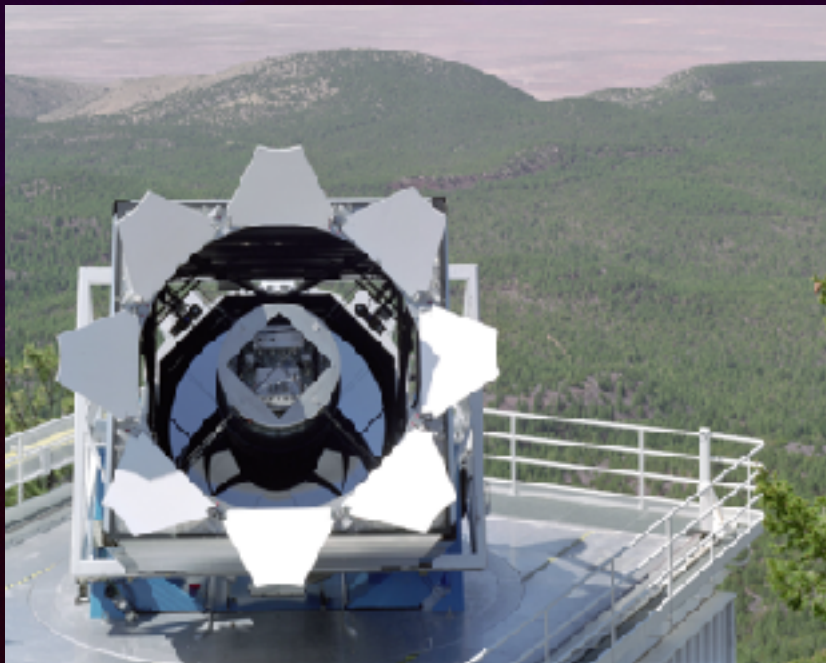
How our theoretical “desires” can inform the way that we plan for **new surveys**?



# A survey of surveys

## Spectroscopy

- The Sloan Digital Sky Survey/BOSS/eBOSS

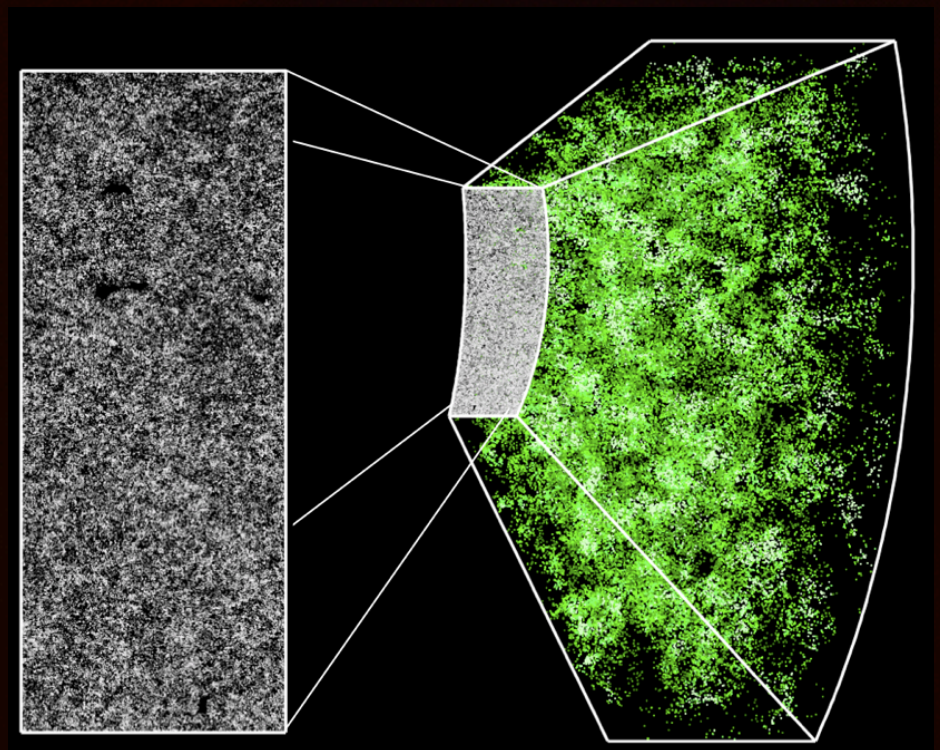
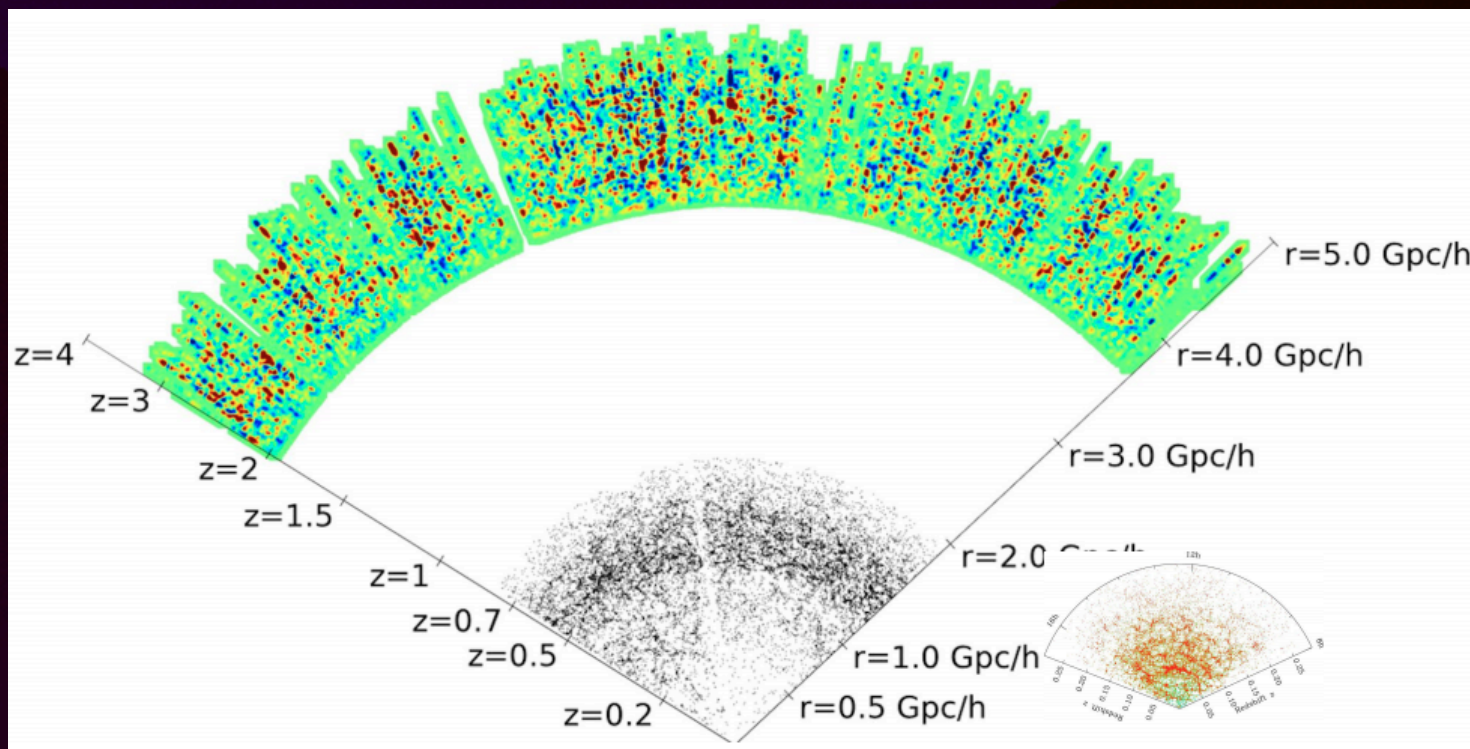




# A survey of surveys

## Spectroscopy

- The Sloan Digital Sky Survey/BOSS/eBOSS
  - Total area:  $\sim 10,000 \text{ deg}^2$
  - Redshifts:  $z < 0.7$  (galaxies),  $2.1 < z < 3.2$  (Ly- $\alpha$  “forest”)

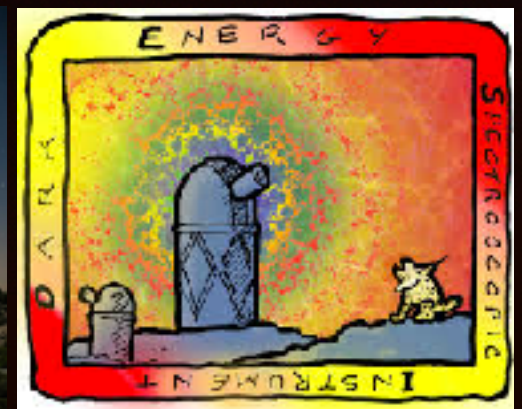
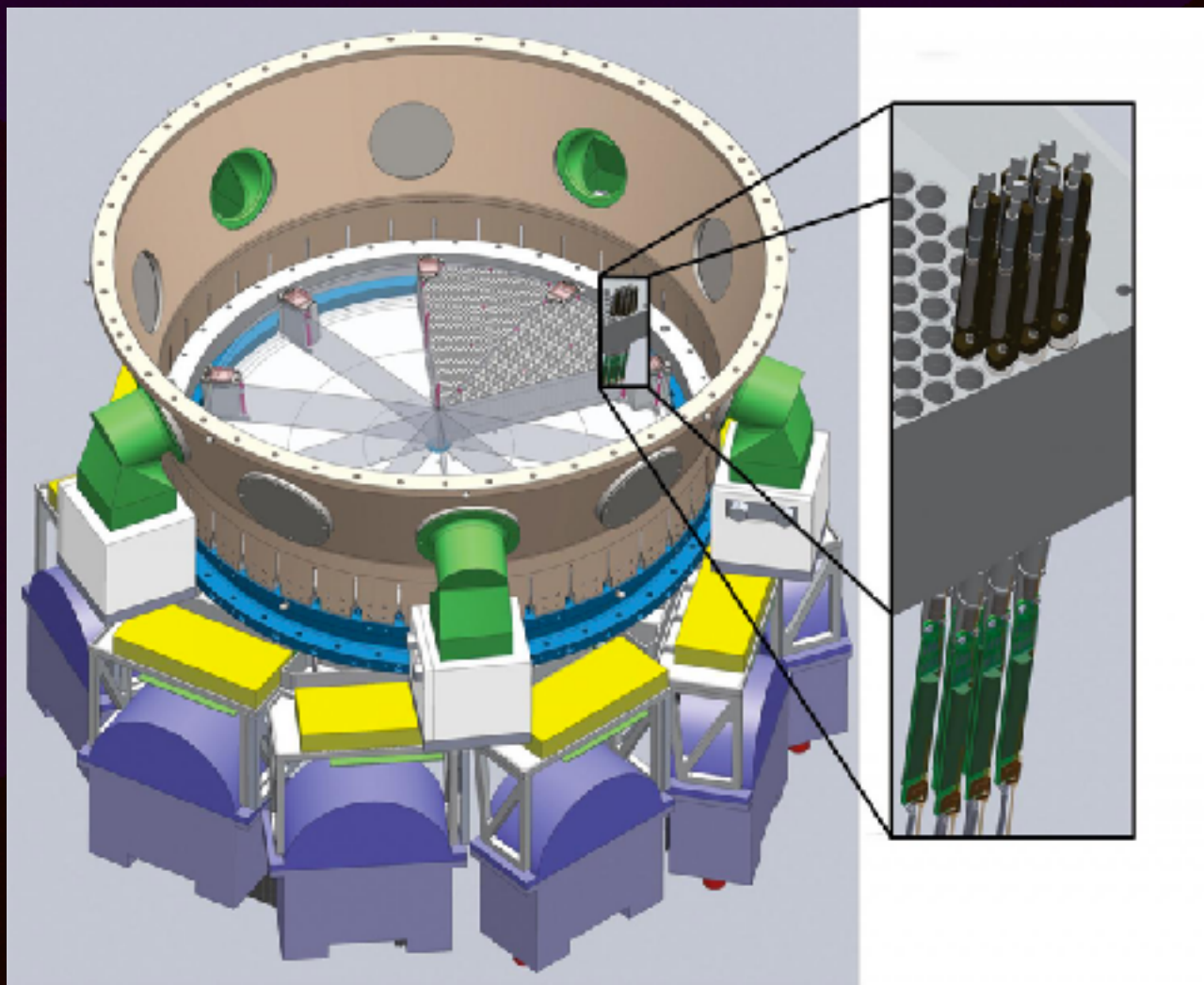




# A survey of surveys

## Spectroscopy

- **DESI - Dark Energy Spectroscopy Instrument (~2022-2026)**
  - Total area:  $\sim 14,000 \text{ deg}^2$
  - Redshifts:  $z < 1.3$  (galaxies),  $2.1 < z < 3.5$  (Ly- $\alpha$  "forest")

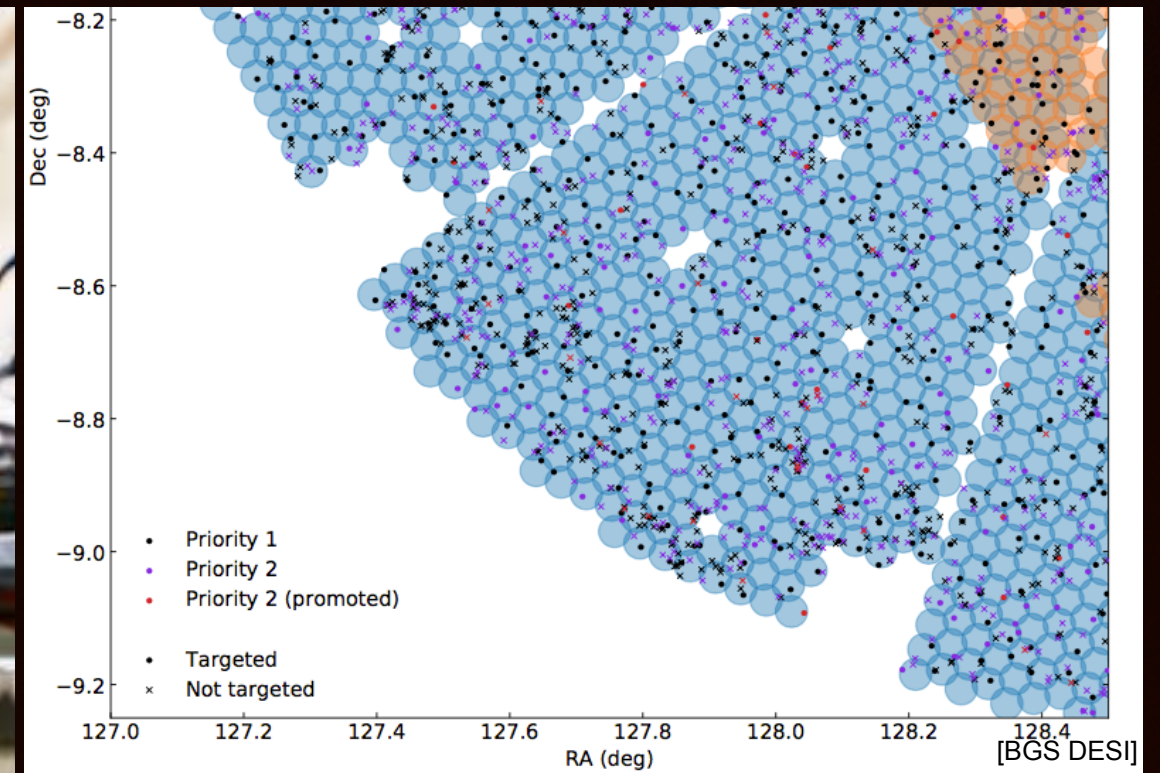
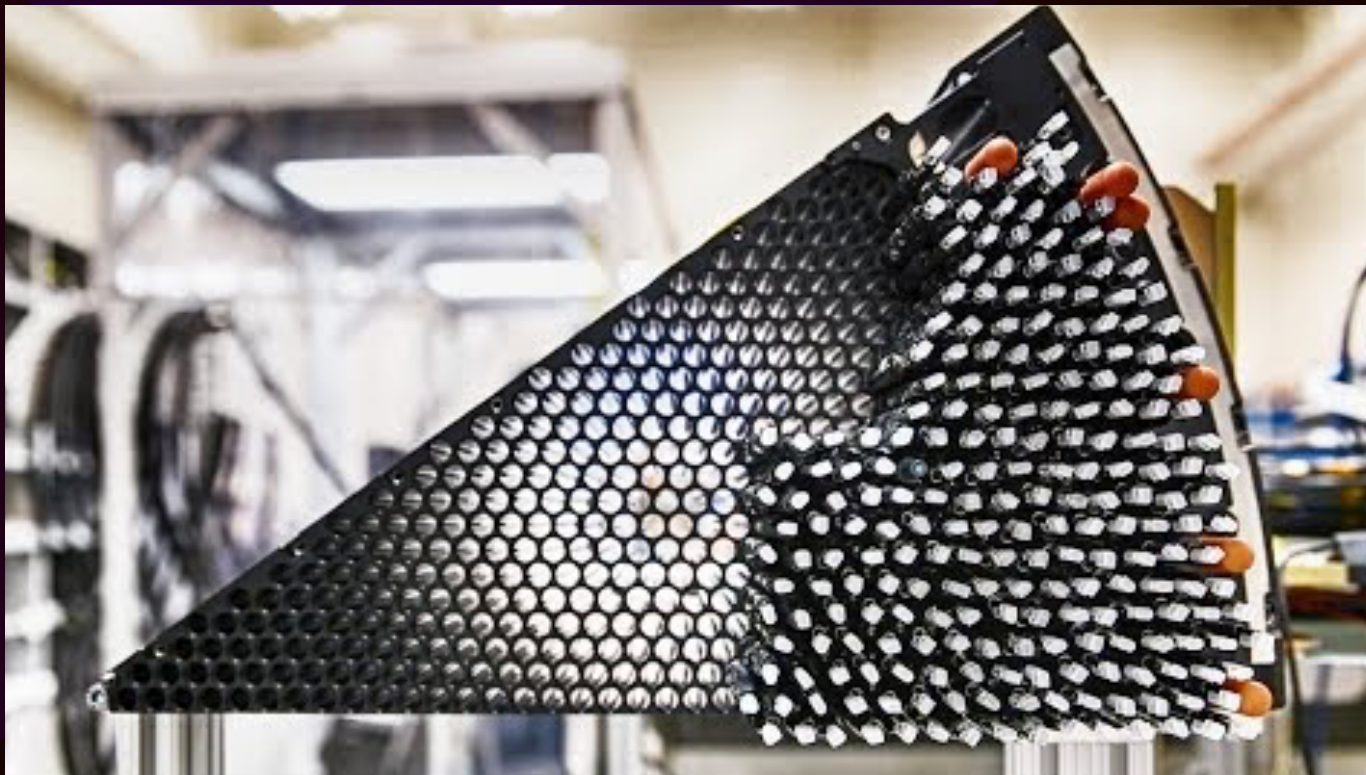




# A survey of surveys

## Spectroscopy

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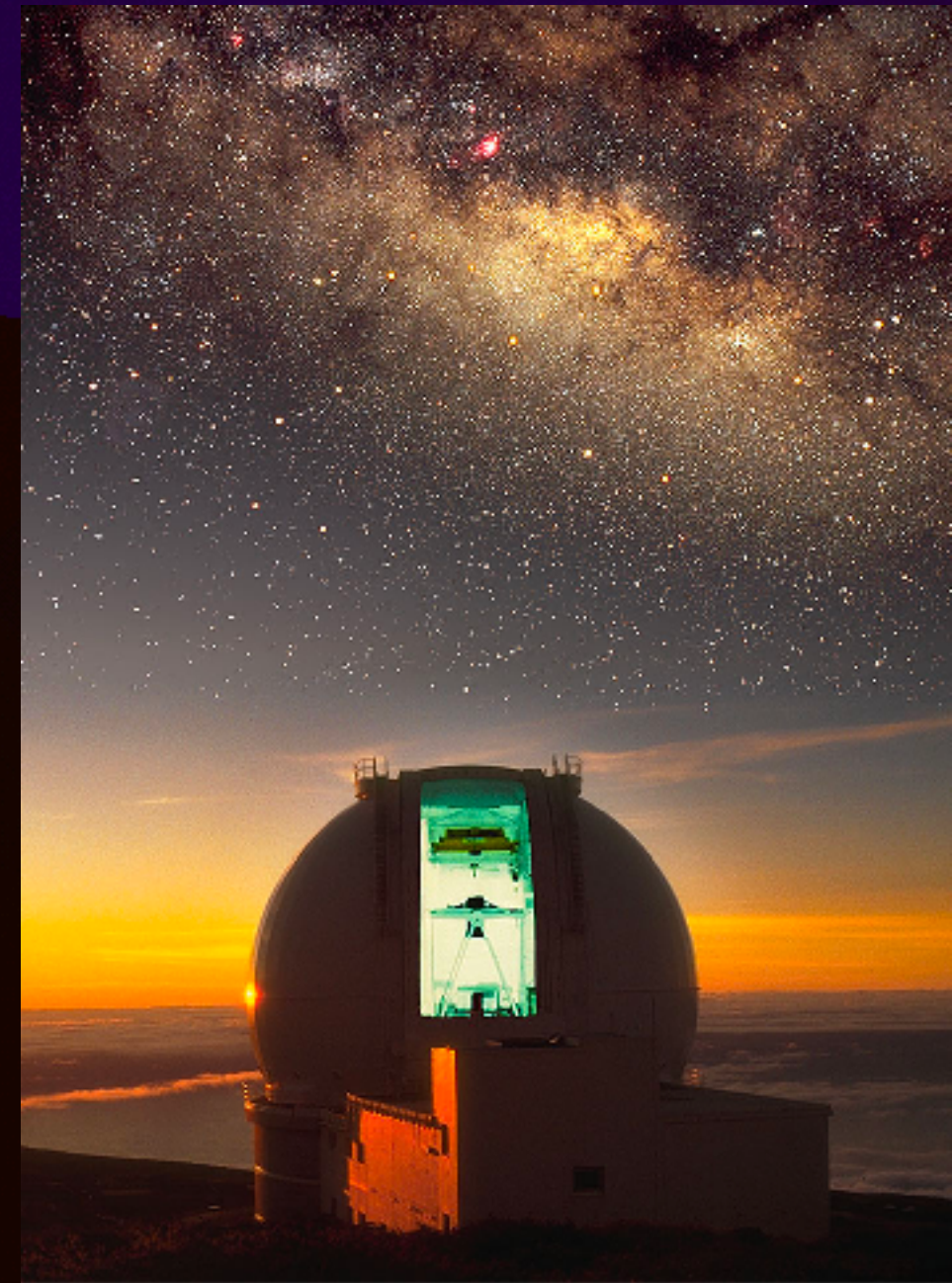
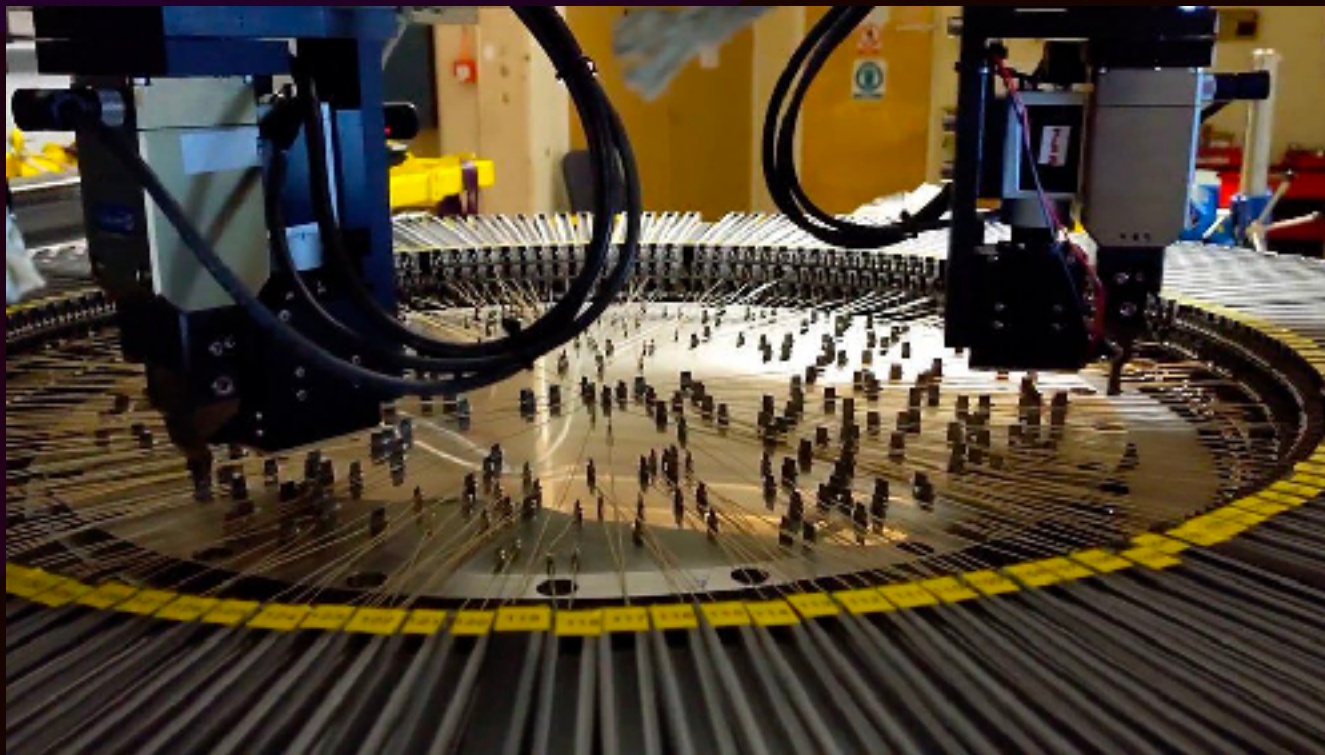




# A survey of surveys

## Spectroscopy

- **WEAVE at the William Herschel Telescope (~2023-2029)**
  - Total area:  $\sim 10,000 \text{ deg}^2$
  - Targets: LoFar galaxies + Ly- $\alpha$  “forest” ( $2.3 < z < 4.0$ ) + some “galactic archeology”

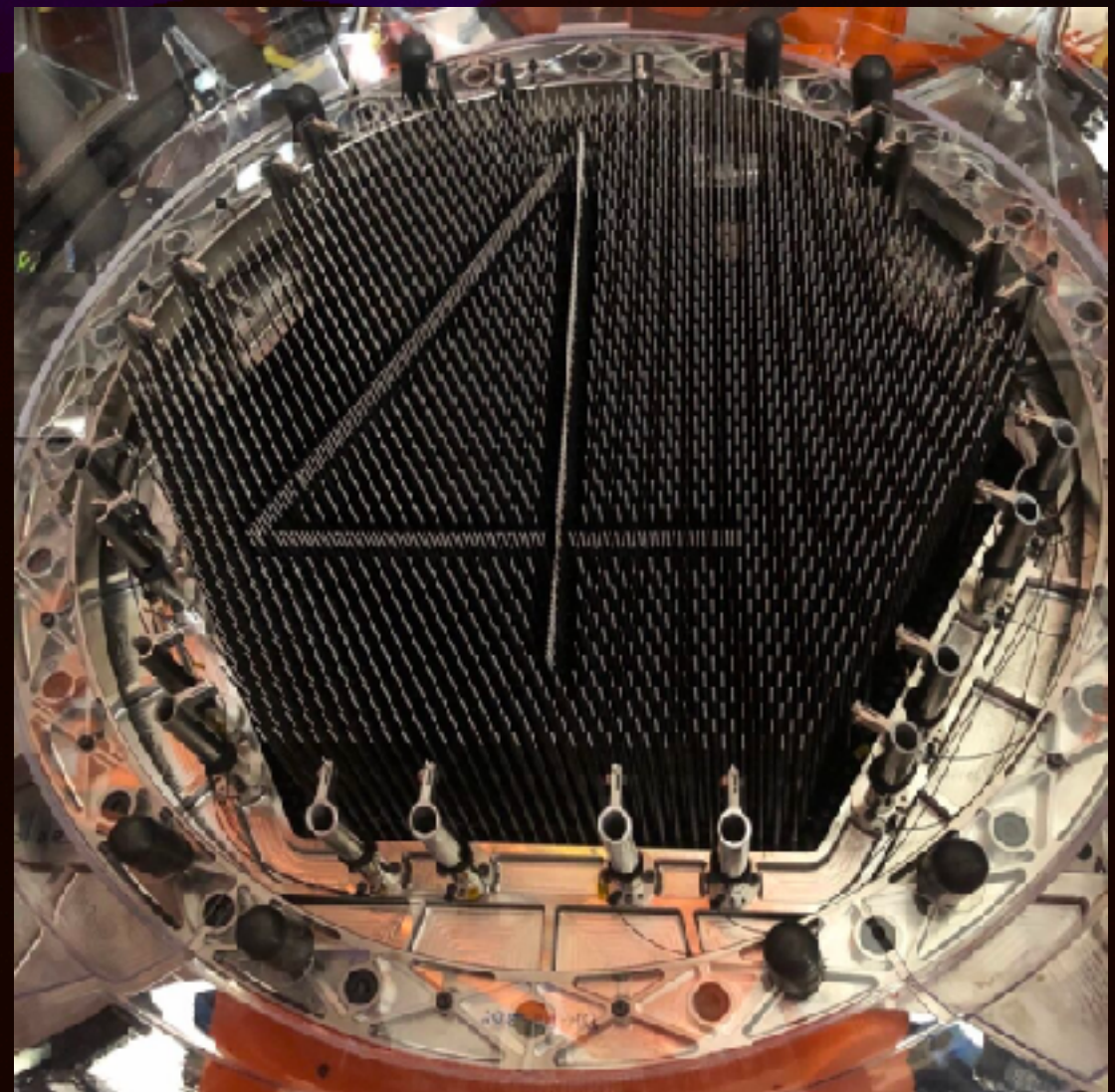




# A survey of surveys

## Spectroscopy

- **4MOST @ ESO's VISTA telescope (~2023-2029)**
  - Mostly Galactic Archeology (Milky Way structure)
  - 13,000,000 spectra of galaxies, AGNs and transient sources

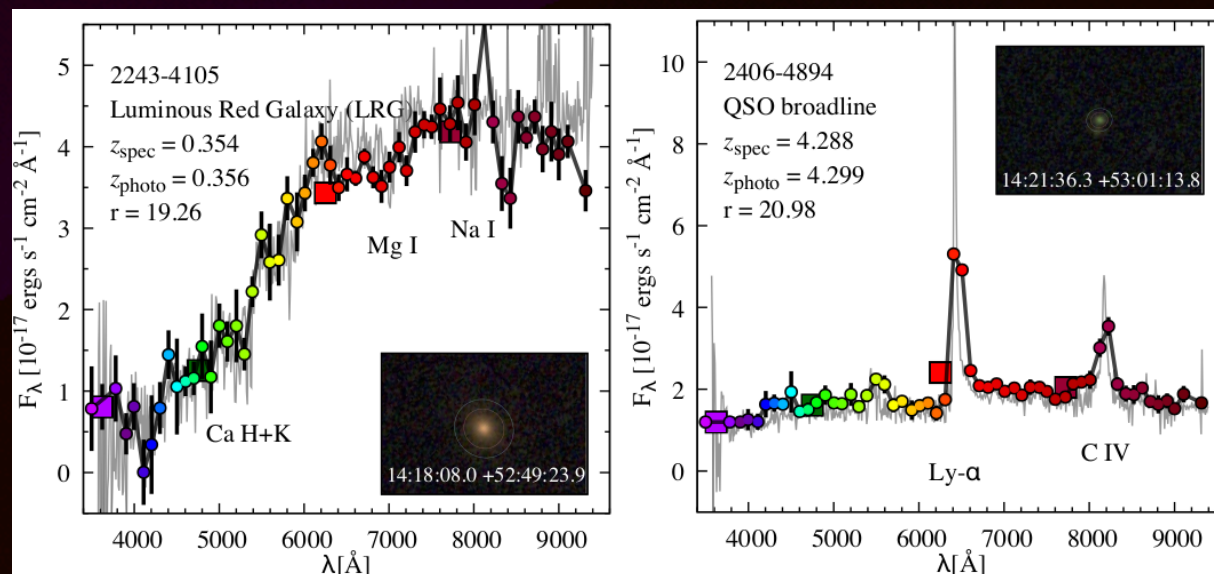




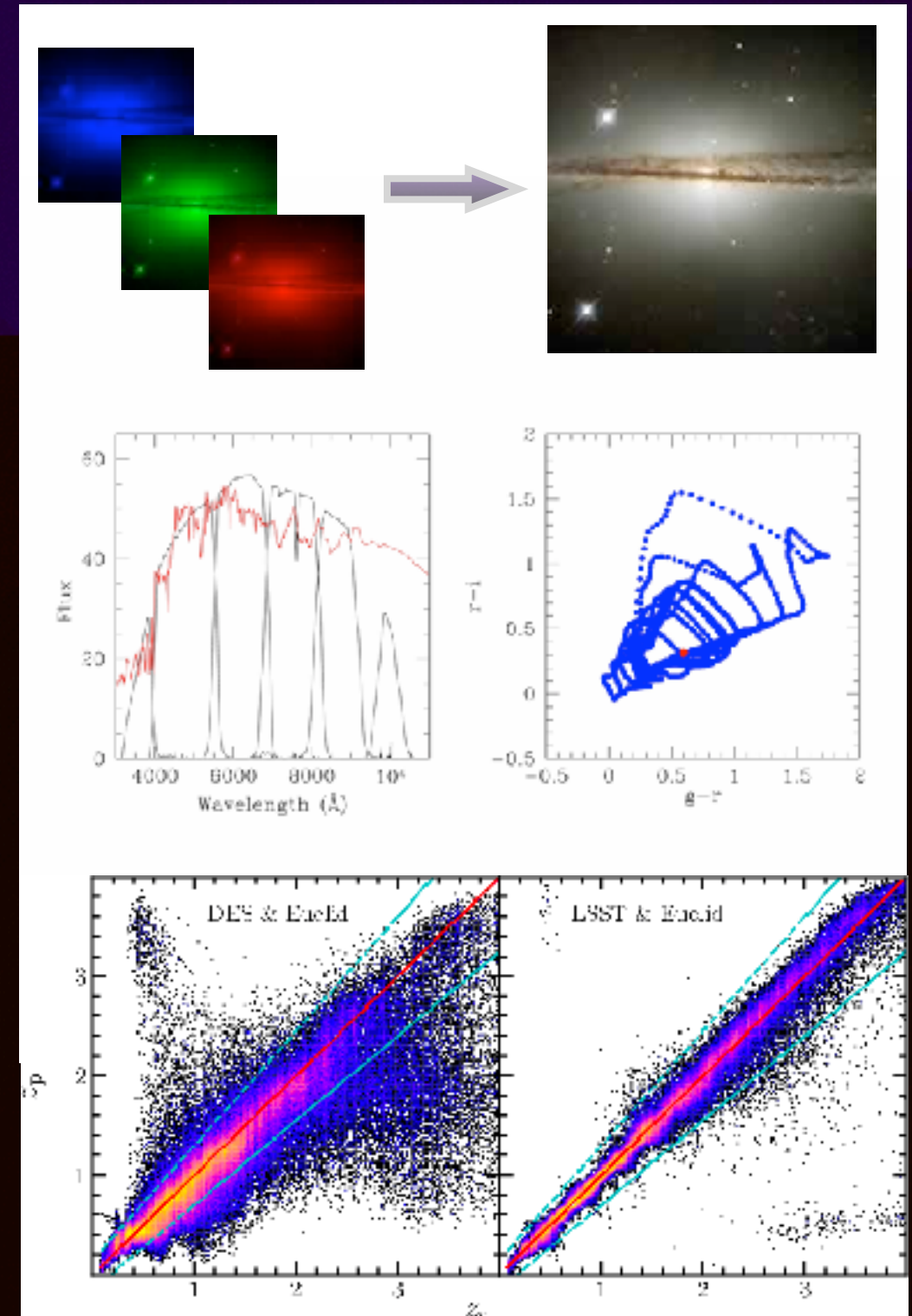
# The galaxy hunter's guide

## Imaging (photometric)

- **All of the objects, but less of the light**
  - Images collect light from all the objects in the field of view
  - Filters select some range of wavelength
  - It's now possible to produce tailor-made, superb filters of almost any kind
  - Narrow filters select specific regions of the spectrum, but admit few photons
- **Pros:** many **more objects** compared with spectroscopy:  $\sim 10^5 \text{ deg}^{-2}$  for  $r < 23$  depth
- **Cons:** cannot resolve individual atomic/molecular lines, so the “**photometric redshifts**” have **larger errors** compared with spectroscopic redshifts



J-PAS survey, Bonoli et al. 2022

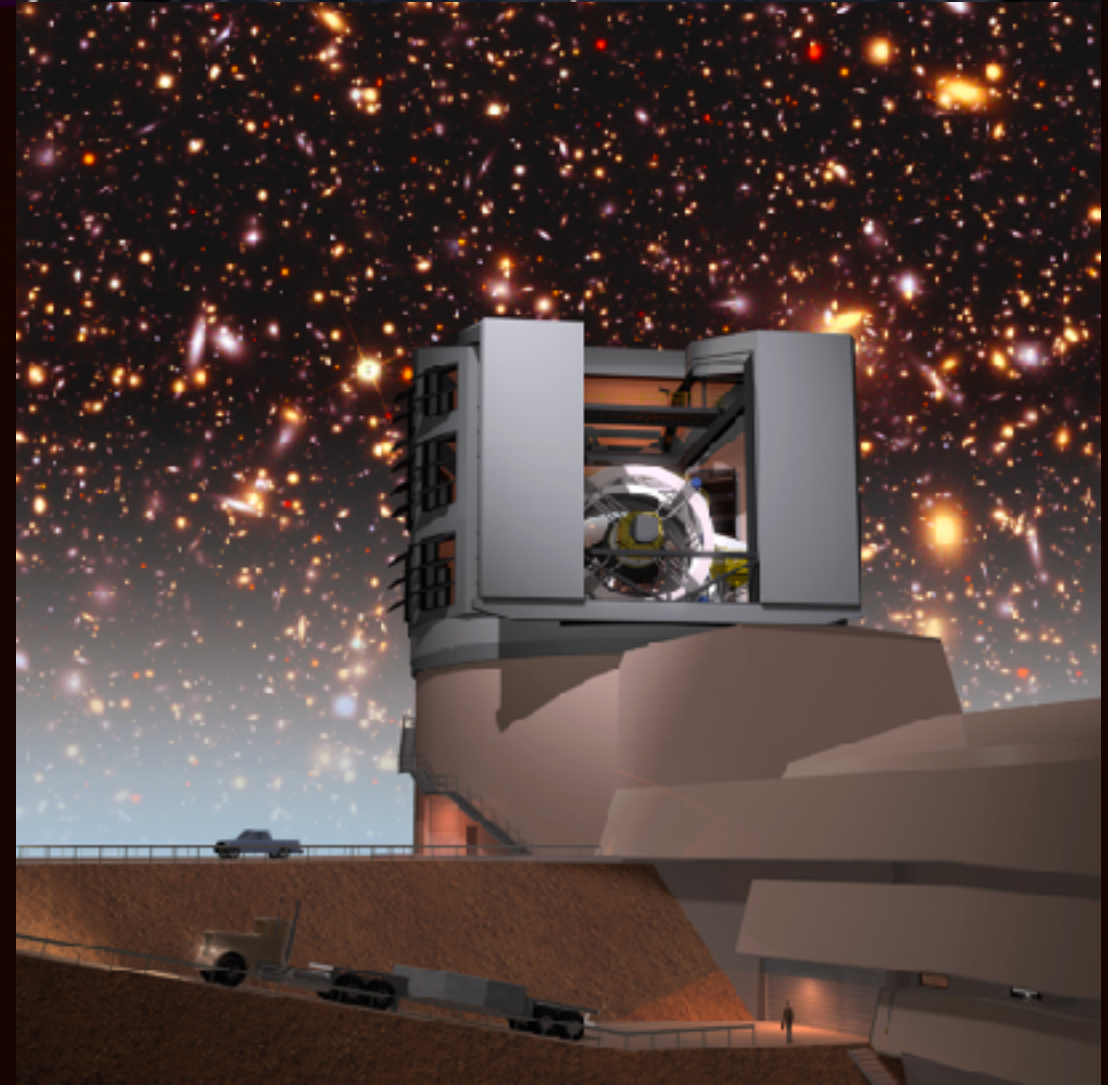




# A survey of surveys

## Imaging (photometric)

- **Some photometric surveys**
  - CFHT
  - Pan-STARRS
  - Vera Rubin LSST





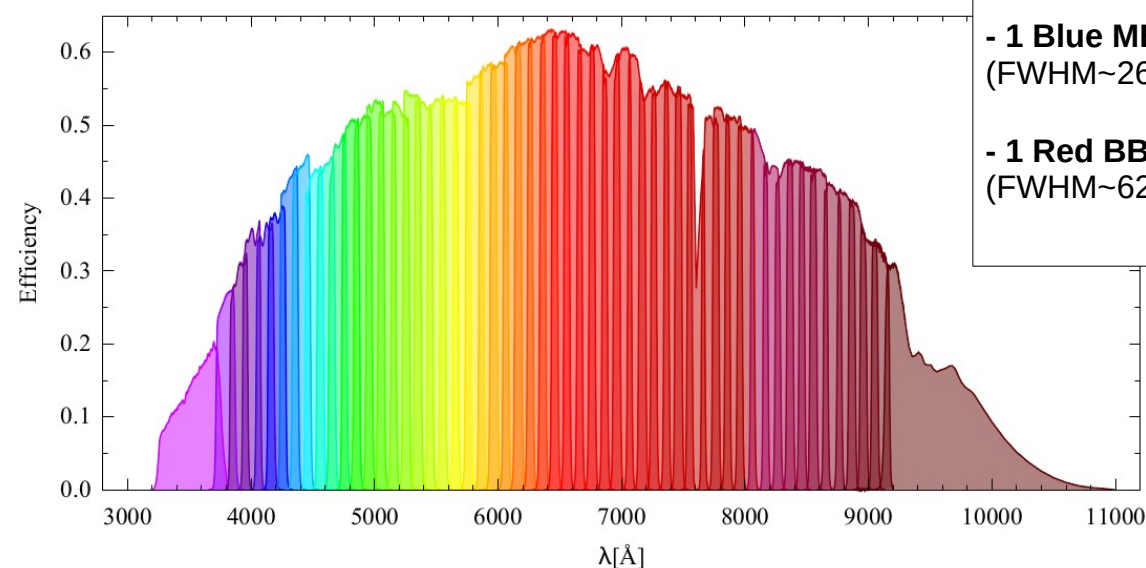
# A survey of surveys

In-between imaging and spectroscopy

- ▶ J-PAS
- ▶ HETDEX
- ▶ Spharx



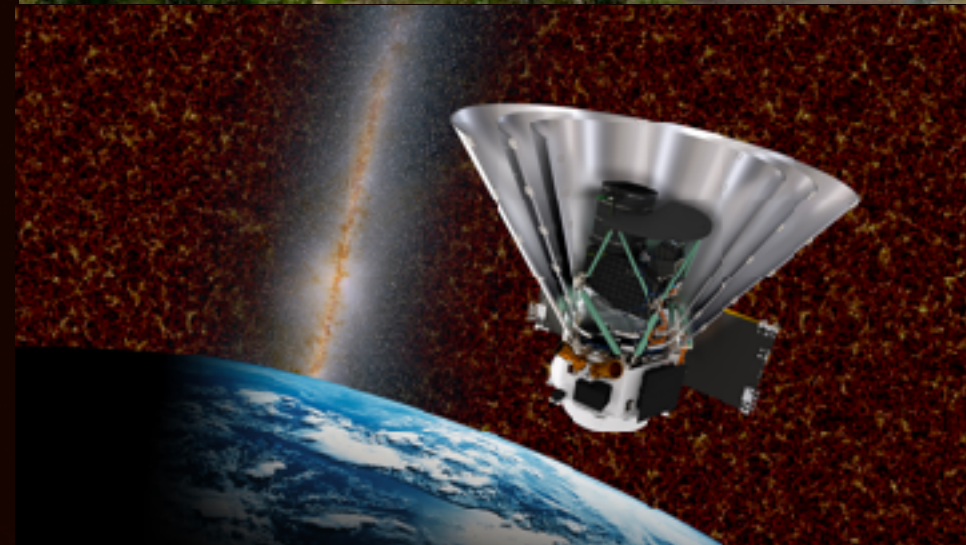
The J-PAS filter system



- **54 NB filters**  
(FWHM~145Å;  $\Delta\lambda$ ~10nm)  
From 3785Å to 9100Å

- **1 Blue MB filter**  
(FWHM~260Å;  $\lambda_c$ ~3600Å)

- **1 Red BB filter**  
(FWHM~620Å;  $\lambda_c$ ~9500Å)





# What do we measure in the LSS?

## From density fluctuations to tracers

- The matter distribution is dominated by dark matter. The fluctuations are best described by the Fourier power spectrum:

$$\delta(\vec{x}, t) = \frac{\delta\rho(\vec{x}, t)}{\bar{\rho}(t)} \leftrightarrow \tilde{\delta}(\vec{k}, t) \leftrightarrow \langle \tilde{\delta}(\vec{k}, t) \tilde{\delta}^*(\vec{k}', t) \rangle = (2\pi)^3 \delta_D(\vec{k} - \vec{k}') P(k; t)$$

- But we don't observe dark matter: what we see are galaxies, quasars, Hydrogen clouds (Ly- $\alpha$ , 21cm), etc.





# What do we measure in the LSS?

## From density fluctuations to tracers

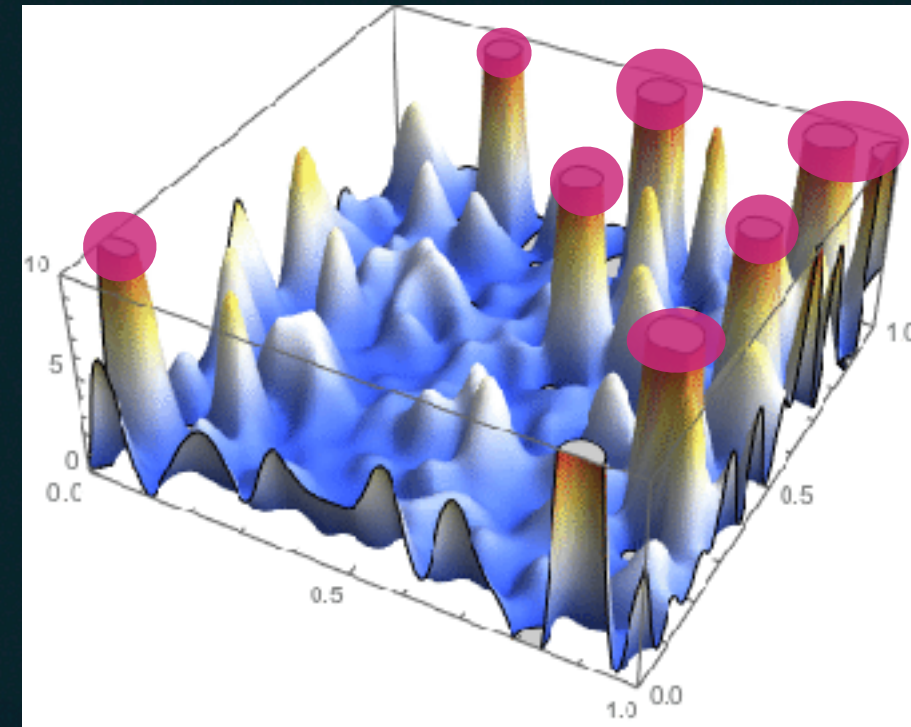
- OK, but how do we go from the fluctuations in a fluid (DM) to the fluctuations in tracers (halos or galaxies)?

$$\delta_{DM}(\vec{x}) = \frac{\delta\rho_{DM}(\vec{x})}{\rho_{DM}}$$

The only way to estimate these fluctuations is to **count the galaxies**, and then measure how these counts deviate from the mean. Something like this:

$$\delta_g(\vec{x}) = \frac{\delta N_g(\vec{x})}{N_g}$$

- This is trickier than it looks. Just think about it: where exactly are you counting those galaxies? We need to define a **volume** for this counting — something which is quite arbitrary.
- There are two ways to go forward:
  - If we want to study the Fourier spectrum of the galaxies, then we almost certainly shall use some numerical implementation of the Fast Fourier Transform (FFT), for which we need to define a **grid** in real space and in Fourier space. The size of the cells in this grid define the volume inside which we count the galaxies
  - OR, we compute the **real-space** 2-point correlation function of the galaxy counts directly.





# What do we measure in the LSS?

## The two-point correlation function

- Let's start with the **2-point correlation function**, which we introduced in a previous lecture:

$$\xi(R) = \langle \delta(\vec{x}) \delta(\vec{x} + \vec{R}) \rangle = \frac{1}{\bar{\rho}^2} \langle \rho(\vec{x}) \rho(\vec{x} + \vec{R}) \rangle - 1$$

where by "expectation value" we mean, in practice, the **mean** over a certain volume:

$$\xi(R) = \frac{1}{\bar{\rho}^2} \frac{1}{V} \int_V d^3x \rho(\vec{x}) \rho(\vec{x} + \vec{R}) - 1$$

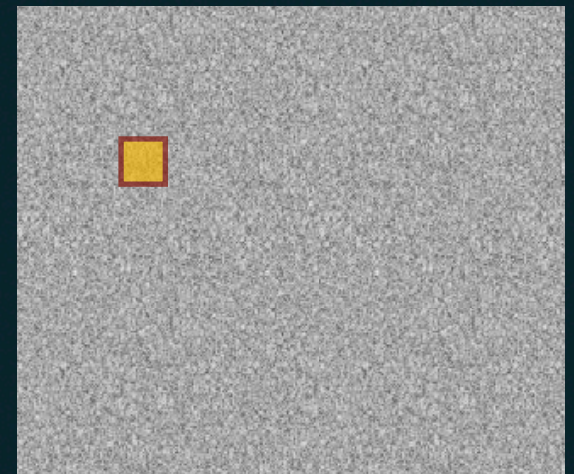
- You can think of the 2-pt CF as the **excess probability** of finding **two objects** (halos, galaxies, etc.) separated by a **distance**  $R$ . Let's explore this probabilistic approach here.
- Consider first a **random**, but **homogeneous** distribution of objects with mean number density  $\bar{n}$ . The probability that we will find some number of objects in a volume  $\delta V$  in that case is simply:

$$dP = \bar{n} \delta V \quad ,$$

which leads to the expectation value  $\langle N \rangle_V = \int_V \bar{n} dV = \bar{n} V = \bar{N}$

- The "random" nature of the distribution needs to be specified: for "point processes", where each instance (or bin, or volume) is independent from each other, the natural statistics to consider is that of the **Poisson distribution**.
- For a Poisson process, the fluctuations in the number counts are described by:

$$\langle NN \rangle_V = \bar{N}(\bar{N} + 1) \quad , \quad \text{i.e.,} \quad \text{var}(N) = \bar{N}$$





# What do we measure in the LSS?

## The two-point correlation function

- Now consider the probability that we will find galaxies in **two different volumes**  $\delta V_1$  and  $\delta V_2$ . If the distribution is **homogeneous**, this is given by:

$$dP_{12} = \bar{n}\delta V_1 \bar{n}\delta V_2$$

- However, if the objects have any tendency to **cluster as a function of the distance**, then this becomes:

$$dP_{12} = \bar{n}\delta V_1 \bar{n}\delta V_2 (1 + \xi_{12}) \quad ,$$

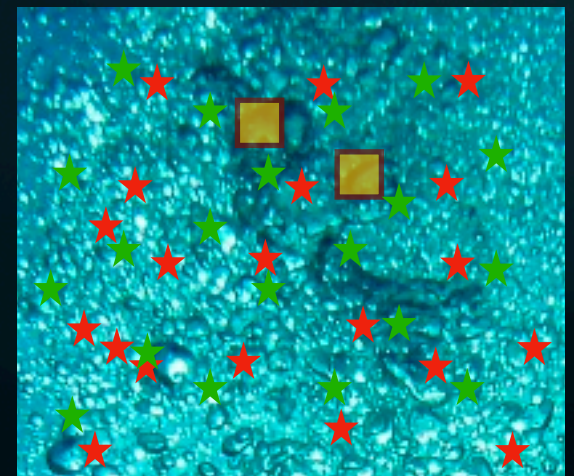
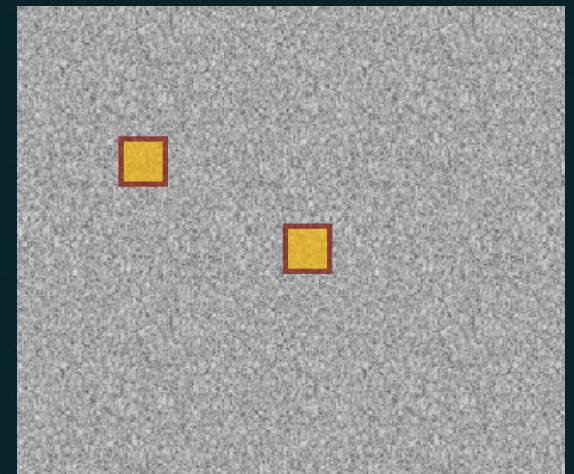
where  $\xi_{12} = \xi(R_{12})$  is the **correlation function** at the distance  $R_{12}$

- In the presence of **both a point process (Poissonian) and clustering**, we get that the expectation value of the counts of tracers has two contributions:
  - For tracers in two **different positions**, the point processes are uncorrelated, and we get only the **signal of the clustering**,  $\xi(R)$ ;
  - But if we take the distance to zero, we are probing the point process in the **same region**, which has a variance as determined by the Poisson statistics.
- The result is that any estimation of the correlation function using discrete tracers results in:

$$\langle n(\vec{r})n(\vec{r}') \rangle = \xi(\vec{r} - \vec{r}') + \frac{1}{\bar{n}}\delta_D(\vec{r} - \vec{r}')$$



**Question: show this identity.**  
[This extra term is called shot noise — here, assumed Poissonian]





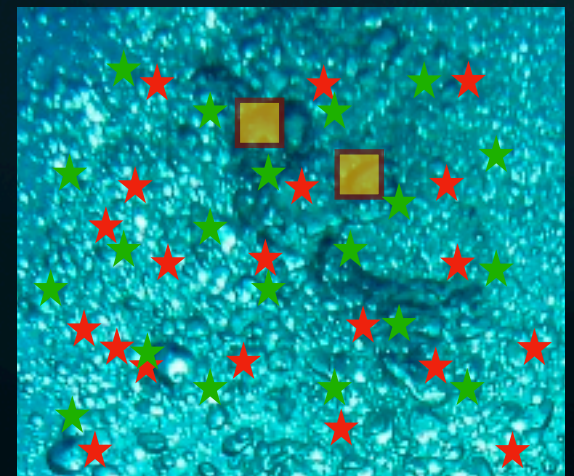
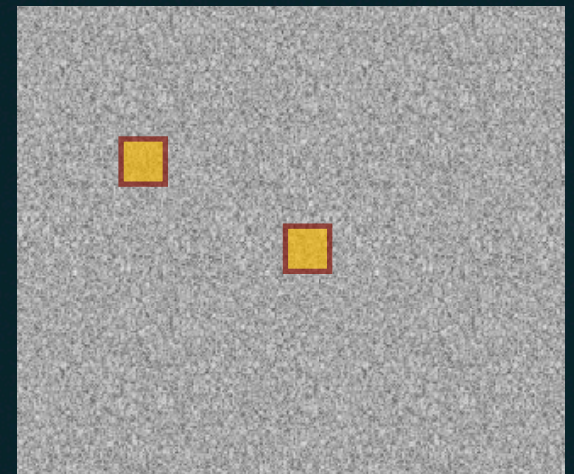
# What do we measure in the LSS?

## The two-point correlation function

- But with halos, or galaxies, we don't really have anything like " $\rho_g(\vec{x})$ ", so how do we compute in practice the correlation function?
- For that purpose there are various **estimators**. All of them use both:
  - the **real data set** ( $D$ ) — the list of positions of the tracers, and
  - a **random data set** ( $R$ ) — with the same mean distribution, but **no clustering**.
- Here are some of the main estimators of the correlation function:

- Peebles & Hauser (1974): 
$$\xi = \frac{\langle DD^{tr} \rangle}{\langle RR^{tr} \rangle} - 1$$
- Hamilton (1993): 
$$\xi = \frac{\langle DD^{tr} \rangle \langle RR^{tr} \rangle}{\langle DR^{tr} \rangle^2} - 1$$
- Landy & Szalay (1993): 
$$\xi = \frac{\langle (D - R)(D - R)^{tr} \rangle}{\langle RR^{tr} \rangle}$$

The LS estimator has been proven to be the "optimal" (least covariance) estimator, under some assumptions





# What do we measure in the LSS?

## The power spectrum

- The other “summary statistics” we can use to measure clustering is the Fourier-space **power spectrum**.

- Mathematically, the power spectrum is straightforward:

$$\delta(\vec{x}) \rightarrow \tilde{\delta}(\vec{k}) \quad , \quad \text{and the 2-point correlation is} \quad \langle \tilde{\delta}(\vec{k}) \tilde{\delta}^*(\vec{k}') \rangle = (2\pi)^3 \delta_D(\vec{k} - \vec{k}') P(\vec{k})$$

- The main advantage of the power spectrum is the **simplicity** of the statistics: because the Gaussian random fields used in cosmology are defined in Fourier space, and there is an underlying fundamental theory behind this statistics, we can assume Gaussianity as an excellent first approximation.
- This means that the higher-order momenta of the PDF are defined by **Gaussian statistics**. Consider a set of Gaussian random variables  $X_i$ , all with zero mean for simplicity. Their “statistical momenta” are:

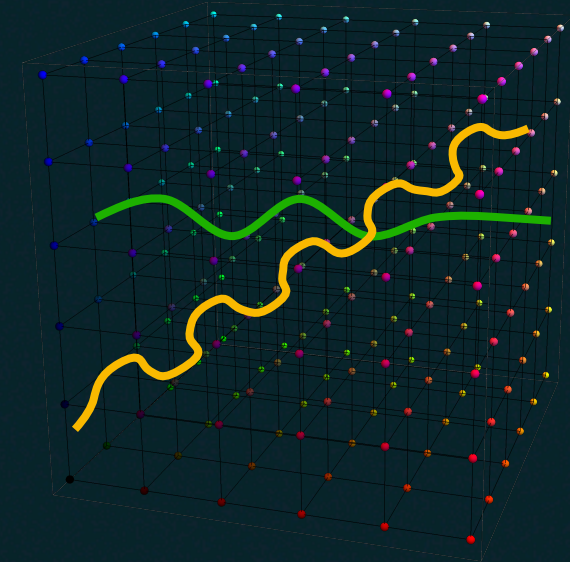
$$\langle X_i \rangle = 0$$

$$\langle X_i X_j \rangle = \delta_{ij} \sigma^2$$

$$\langle X_i X_j X_m \rangle = 0$$

$$\langle X_i X_j X_m X_n \rangle = \sigma^4 \left( \delta_{ij} \delta_{mn} + \delta_{im} \delta_{jn} + \delta_{in} \delta_{jm} \right) \quad , \quad \text{etc.}$$

- The 4-point function allows us to compute the **covariance of the power spectrum**
- **In practice we often rely on simulations to determine the statistics of our data!**

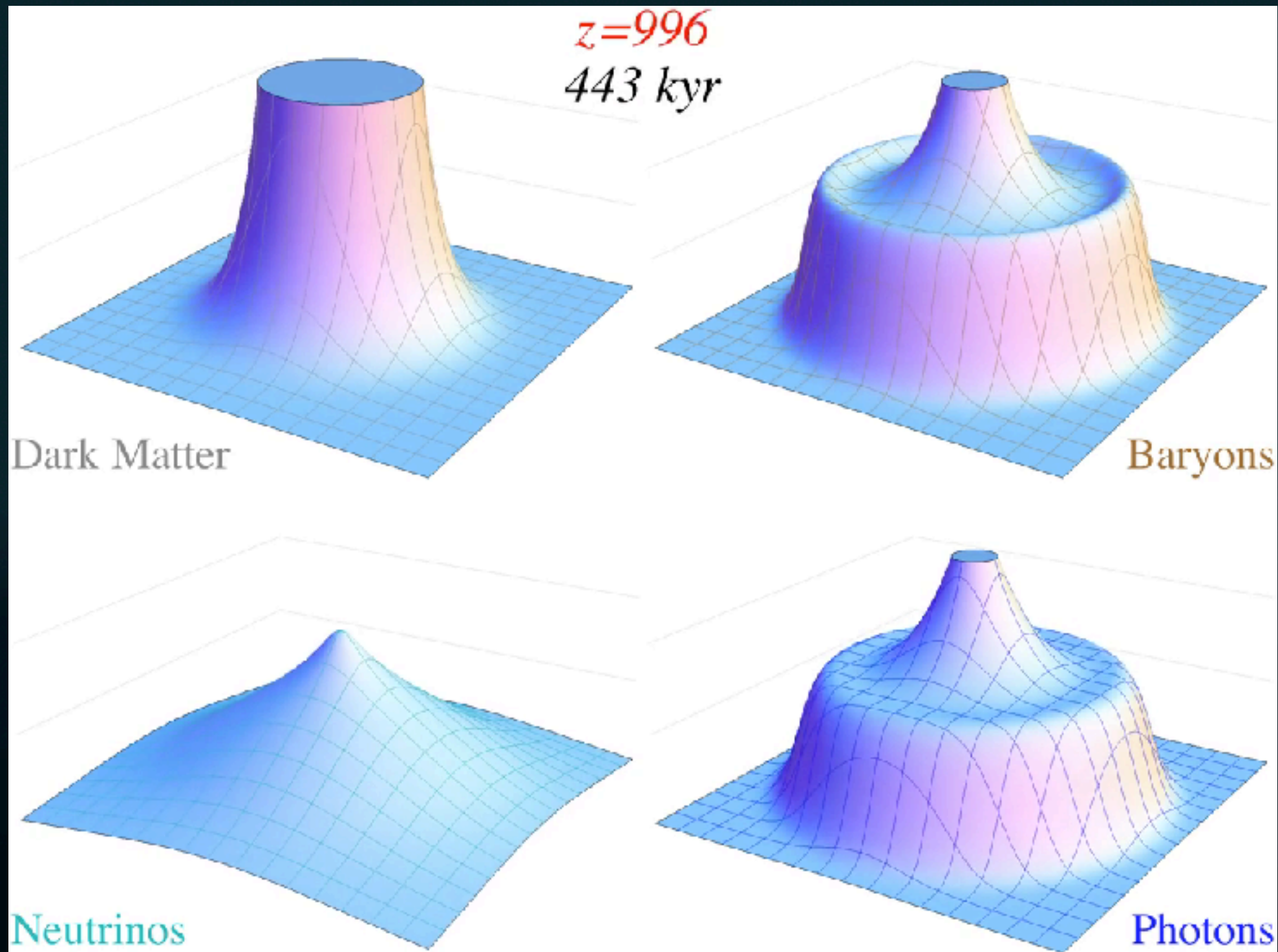


👉 Wick's theorem in QFT; Isserli theorem for stochastic variables



# What do we measure really well in the LSS?

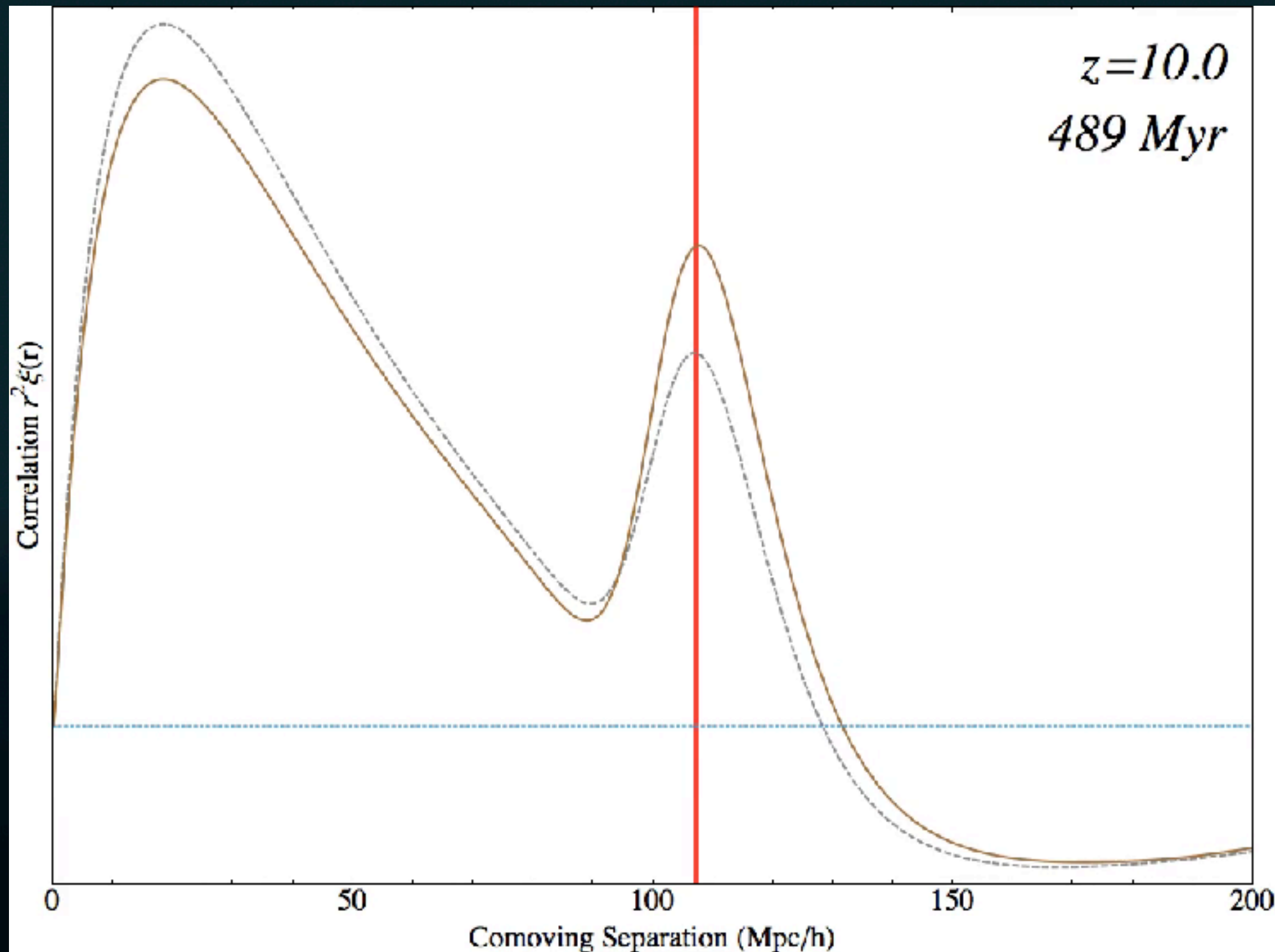
## Baryon acoustic oscillations (BAOs)





# What do we measure really well in the LSS?

Baryon acoustic oscillations (BAOs)

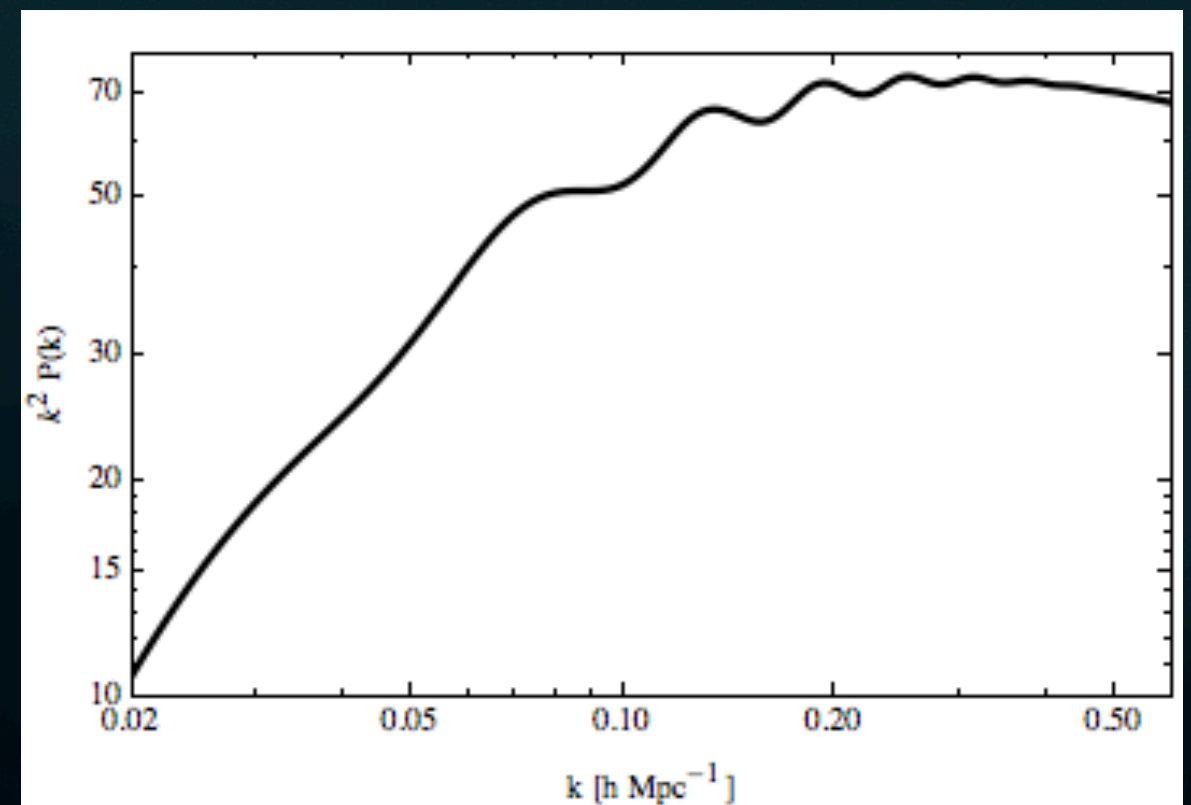
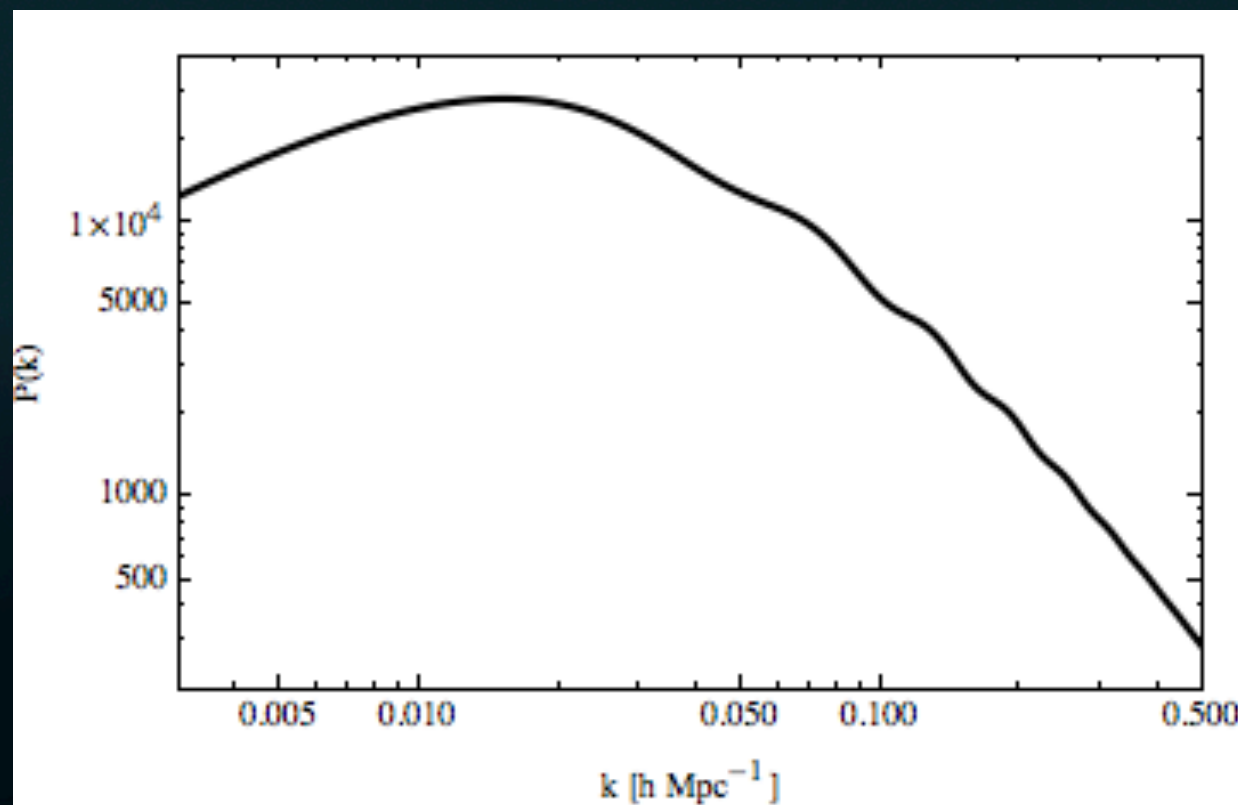




# What do we measure really well in the LSS?

## Baryon acoustic oscillations (BAOs)

- In real space we have a radial feature in the correlation function
- In the power spectrum, this feature becomes **oscillations**
- Never lose sight of the fact that by measuring these features we are measuring **distances**!
- And we can measure these distances **along** and **across** the line of sight to the galaxies



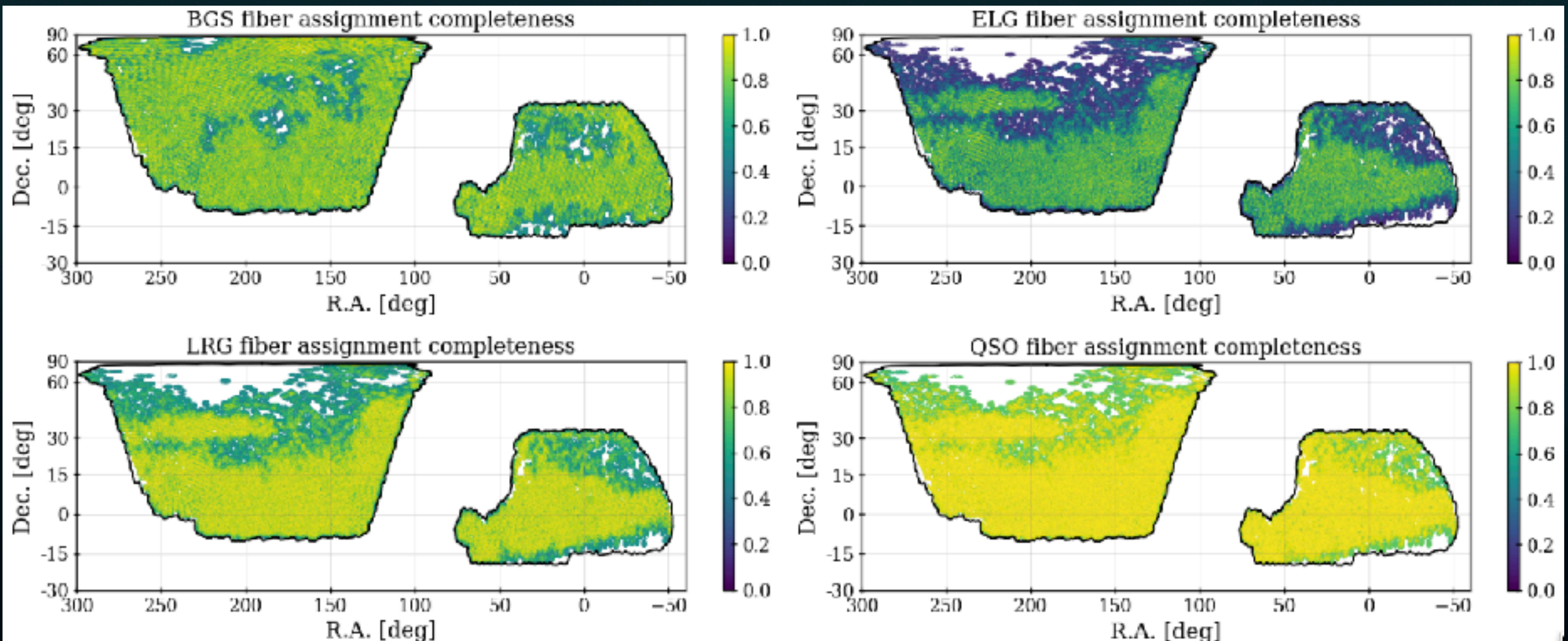


# Latest observations: DESI DR2

arxiv:2503.14738

- BGS: Bright Galaxy Sample
- LRG: Luminous Red Galaxies
- ELG: Emission Line Galaxies
- QSO: Quasars

| Tracer | # of good $z$ | $z$ range | Area [deg <sup>2</sup> ] | $C_{\text{assign}}$ | $z$ succ. |
|--------|---------------|-----------|--------------------------|---------------------|-----------|
| BGS    | 1,188,526     | 0.1 – 0.4 | 12,355                   | 75.5%               | 98.8%     |
| LRG    | 4,468,483     | 0.4 – 1.1 | 10,031                   | 82.6%               | 99.0%     |
| ELG    | 6,534,844     | 0.8 – 1.6 | 10,352                   | 53.7%               | 73.9%     |
| QSO    | 2,062,839     | 0.8 – 3.5 | 11,181                   | 93.6%               | 68.0%     |

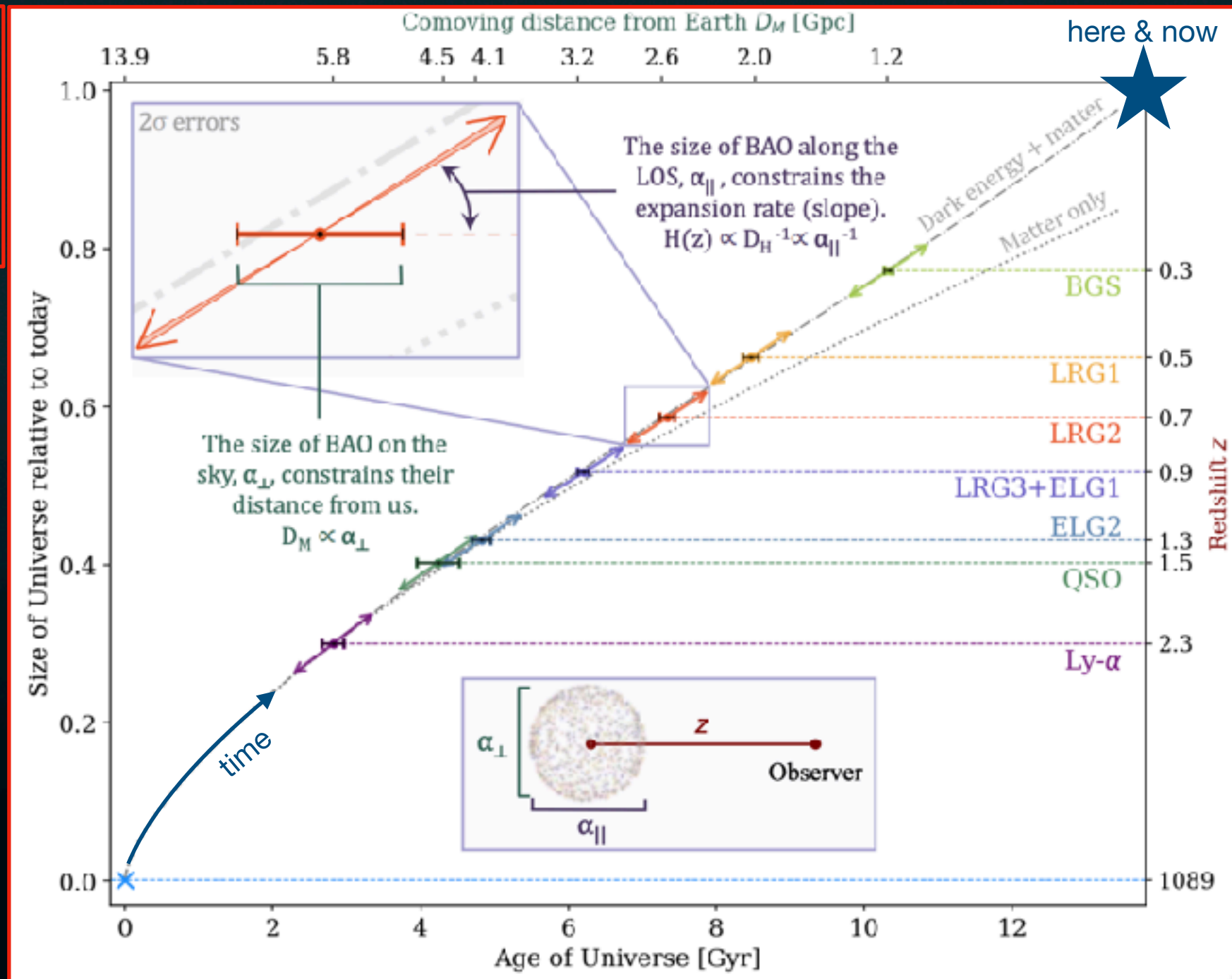




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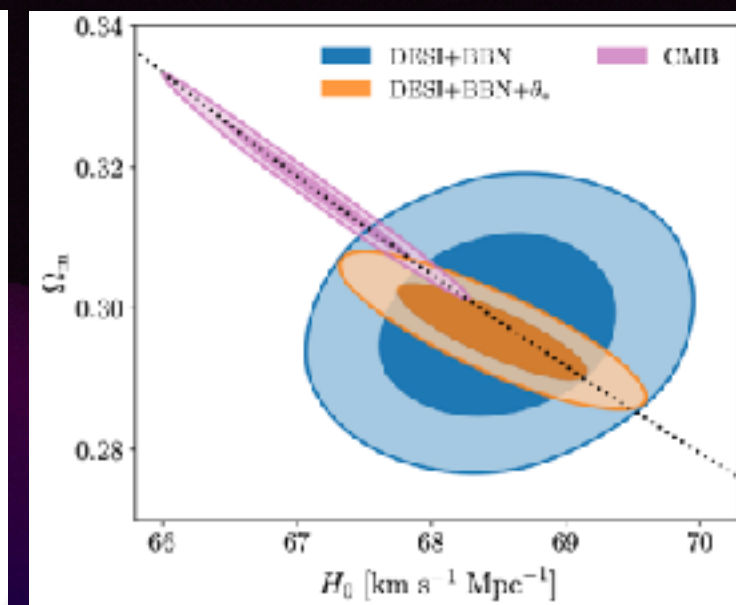
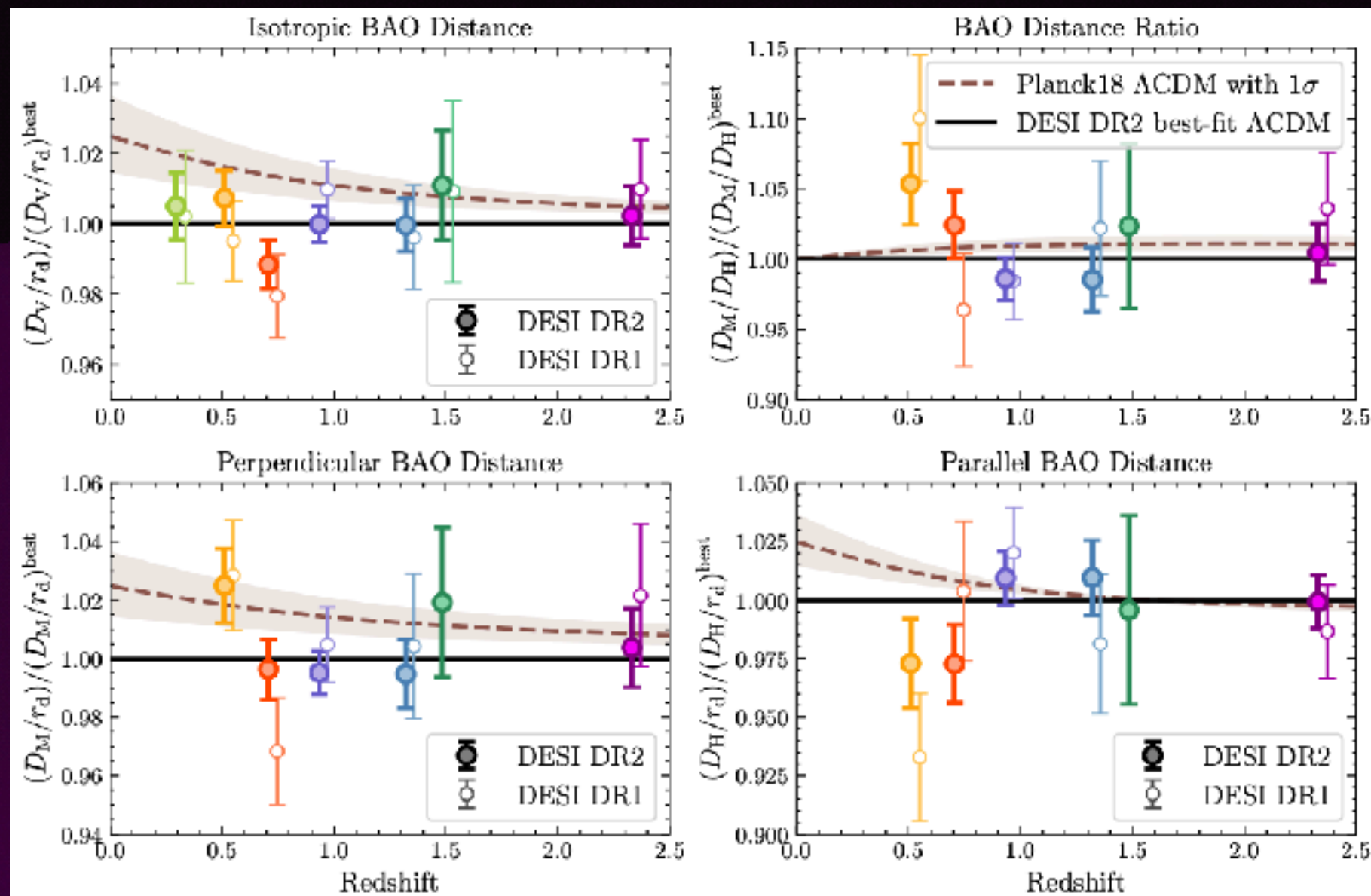




# Latest observations: DESI DR2

arxiv:2503.14738

- First question:** let's assume that the cosmological model is  $\Lambda$ CDM . How does galaxy clustering data compares with CMB (Planck)?



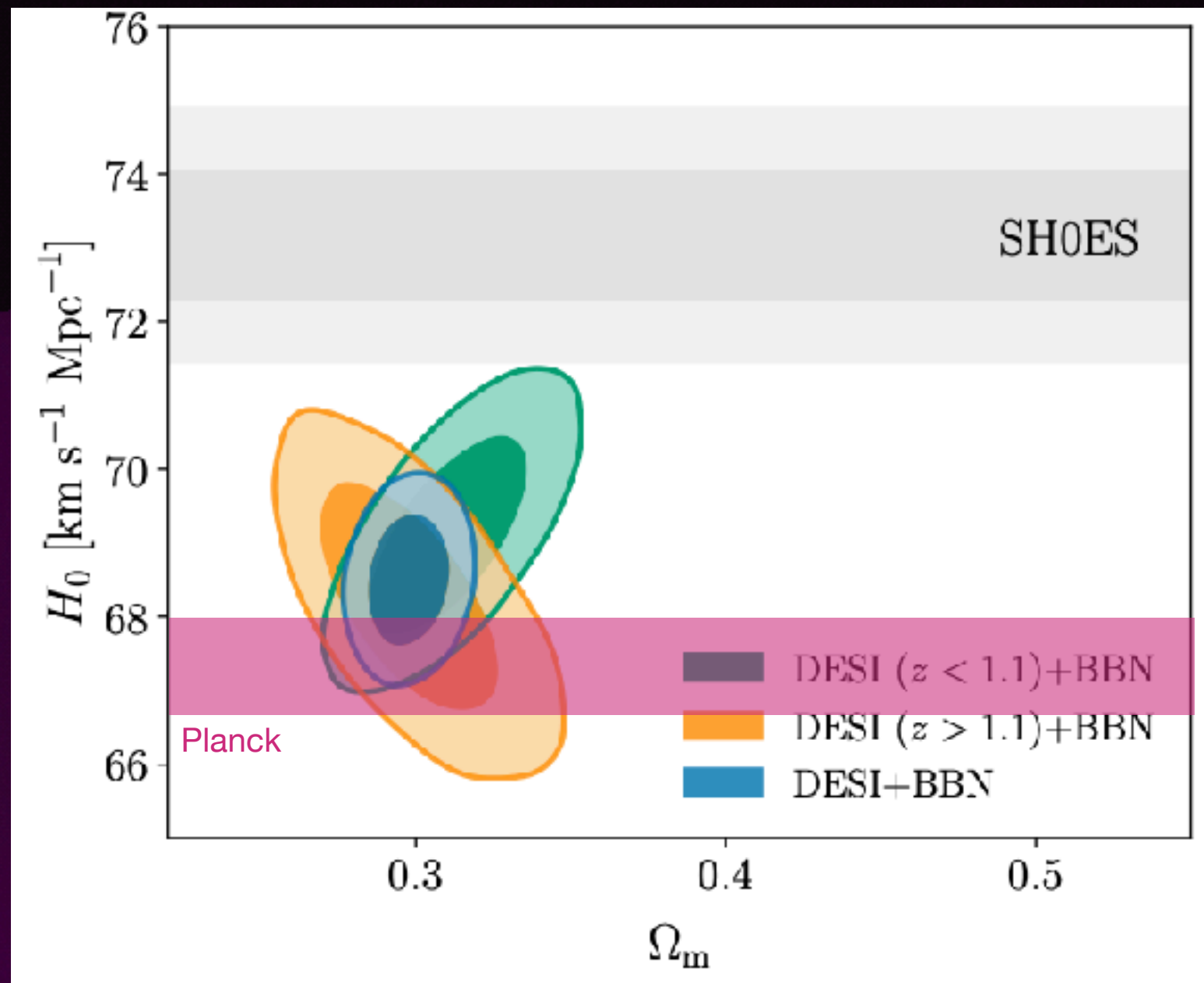
Planck “likes”  
slightly higher  $\Omega_m$   
and slightly lower  
 $H_0$  than DESI



# Latest observations: DESI DR2

arxiv:2503.14738

- **Second question:** How about  $H_0$  and the Cepheids/Supernova data (SH0ES)?



DESI **confirms** the CMB's preference for a low  $H_0$  value

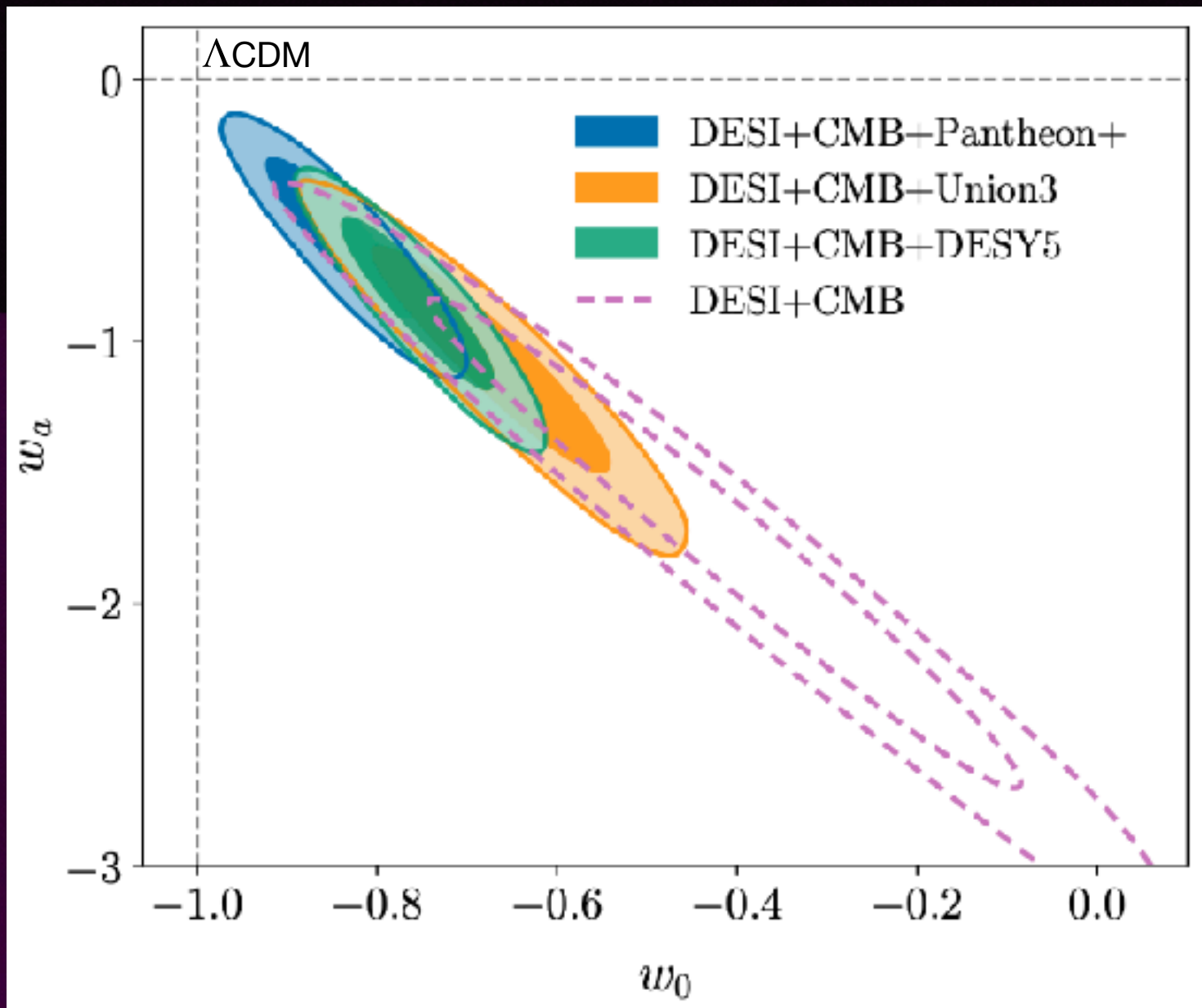
(BUT, notice that this is not totally independent from the CMB — it's based upon the prediction of the BAO scale, which comes from the CMB.)



# Latest observations: DESI DR2

arxiv:2503.14738

- **Third question:** what is dark energy?



Equation of state:

$$w(a) = w_0 + (1 - a) w_a$$

Cosmological Constant ( $\Lambda$ ):

$$w_0 = -1, \quad w_a = 0$$

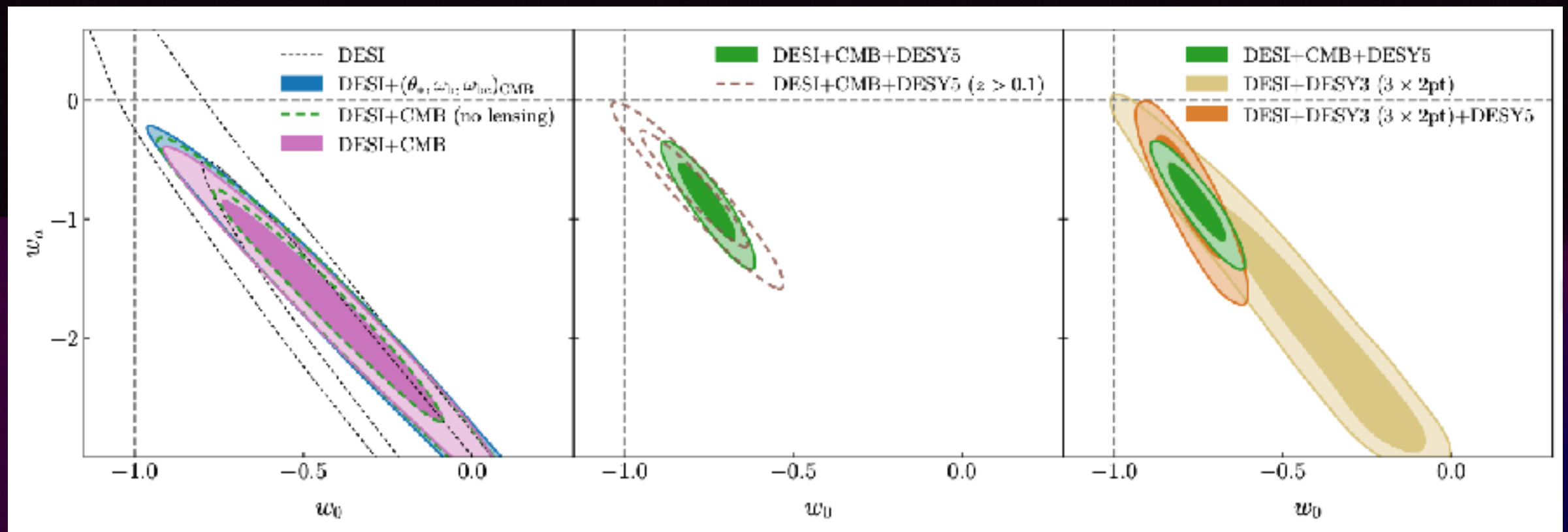
$\gtrsim 2.8\sigma$  evidence for  
**dynamical dark energy!**



# Latest observations: DESI DR2

arxiv:2503.14738

- **Third question:** dynamical dark energy? Is it Supernova's fault?



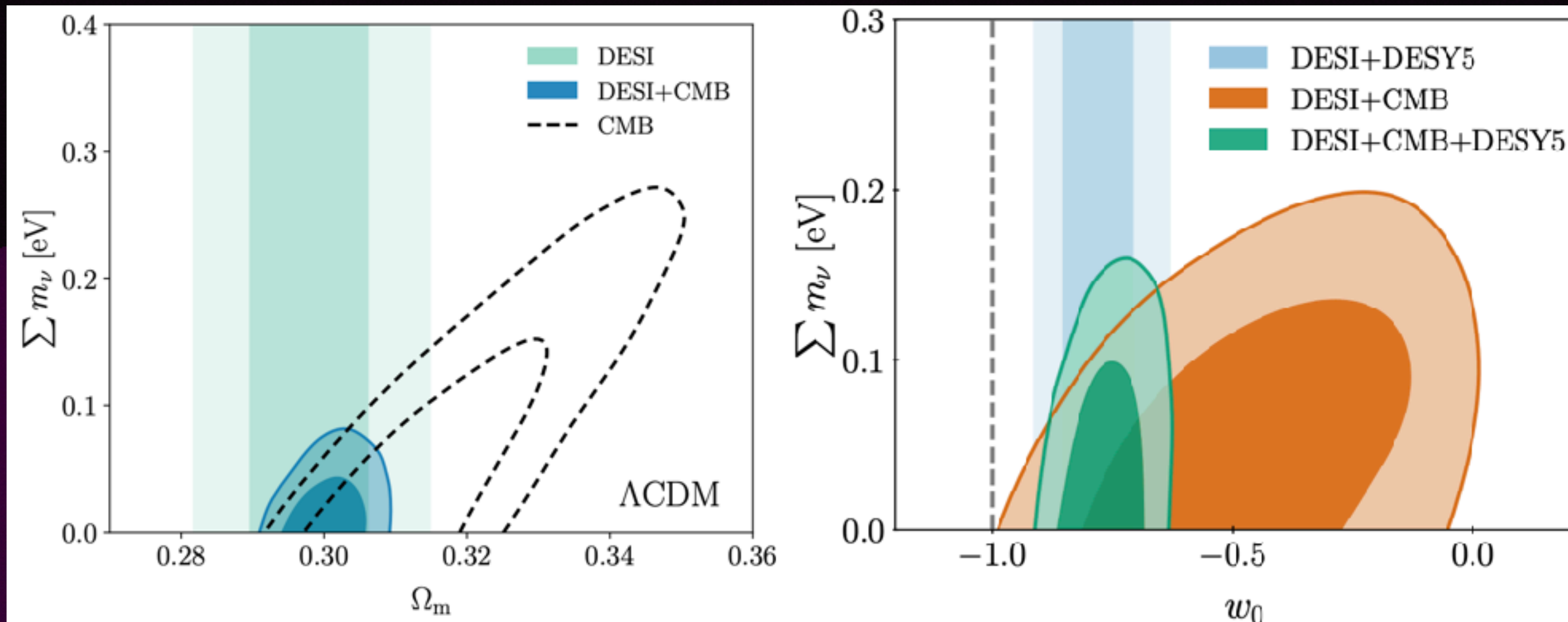
No!



# Latest observations: DESI DR2

arxiv:2503.14738

- **Fourth question:** have we detected neutrino masses?

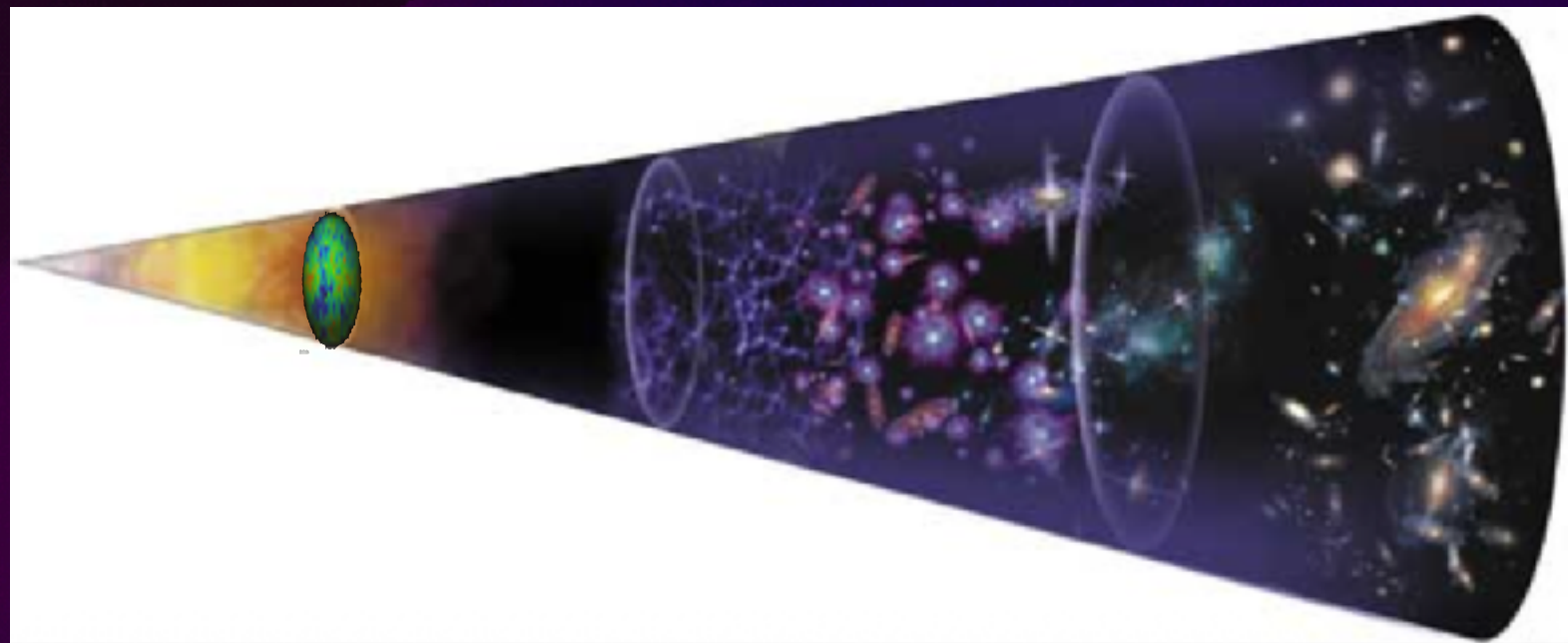


Not yet... but it's getting harder to accommodate the inverted mass ordering scenario.



# What have we learned?

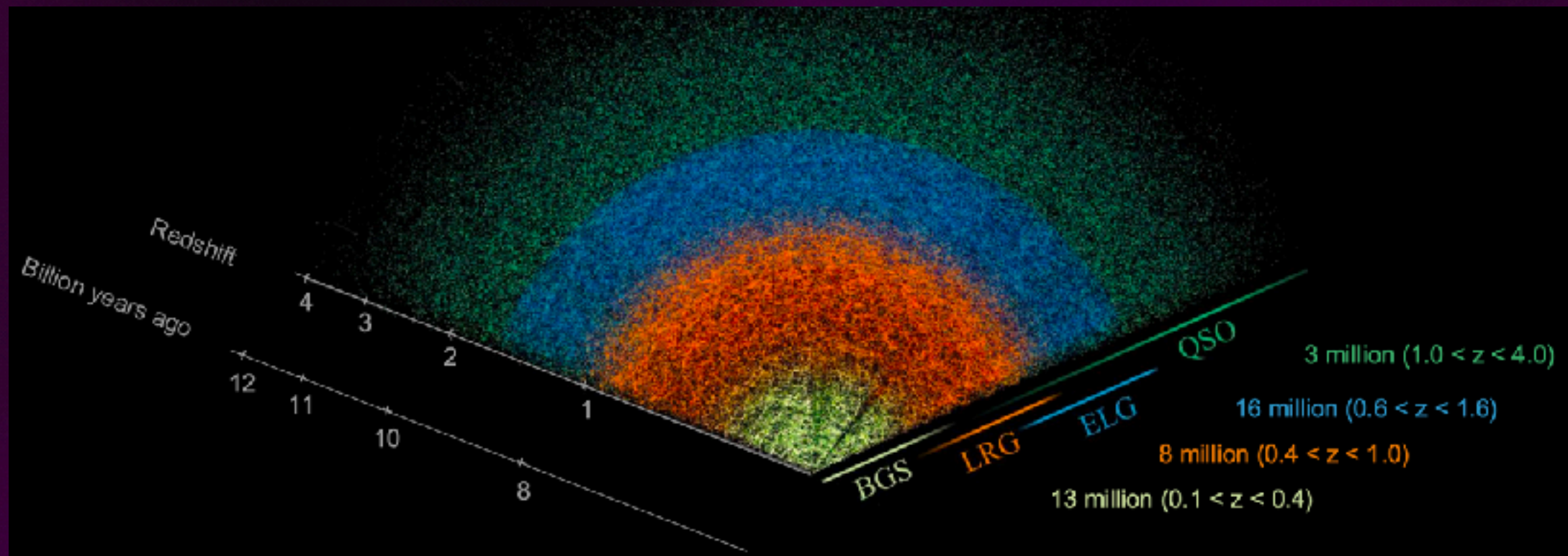
- The notion of a **Big Bang** has been effectively **replaced** by **inflation**, which is the standard theory for the **initial conditions** for the Universe.
- Our **earliest physical observable**, the **CMB**, which tells us that:
  - a. We live in a nearly **flat Universe** ( $K \rightarrow 0$ );
  - b. Of **all** the (cold) **matter**, **only about 20% is made of baryons**; the rest is **dark matter**
  - c. The Universe indeed **started very homogeneous**: at  $z \sim 10^3$  ( $t \lesssim 4 \cdot 10^5$  yrs), the density fluctuations were  $\delta\rho/\rho \lesssim 10^{-4}$ ; in **agreement** with the predictions of **inflation**!
  - d. There is a **characteristic BAO scale** (i.e., the baryon-photon acoustic horizon) imprinted on radiation and matter, that persists until today in the galaxy distribution





# What have we learned?

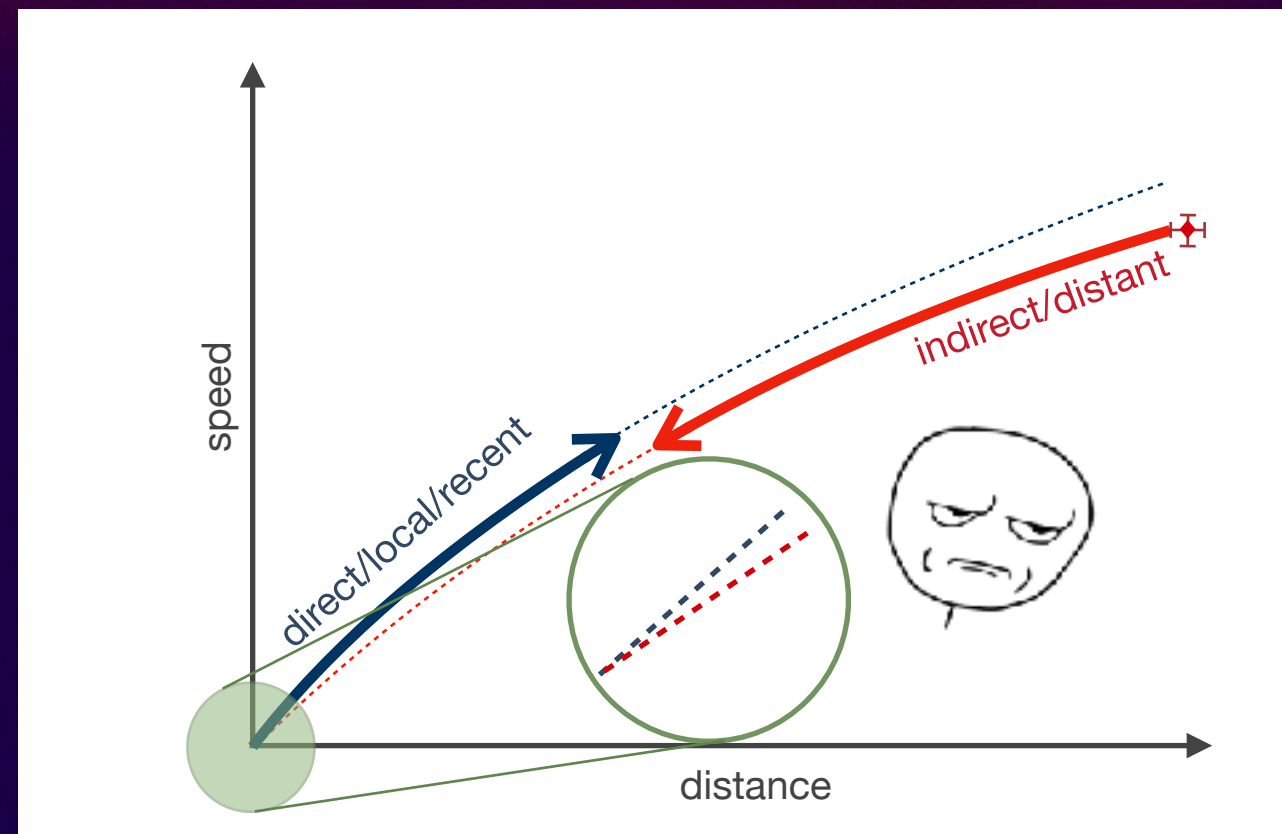
- **Galaxy surveys** are able to use the BAO scale and the general “shape” of the power spectrum/correlation function to **measure distances**, and these observations say that:
  - a. The Universe’s **expansion** is indeed **accelerating**
  - b. The **source** for this seems to be some kind of **dark energy**; but could still be Modified Gravity
  - c. In the case of dark energy, observations seem to indicate that it is not a cosmological constant ( $\Lambda$ ), but some kind of **dynamical dark energy**. **This means: a new field.**





# What to watch for

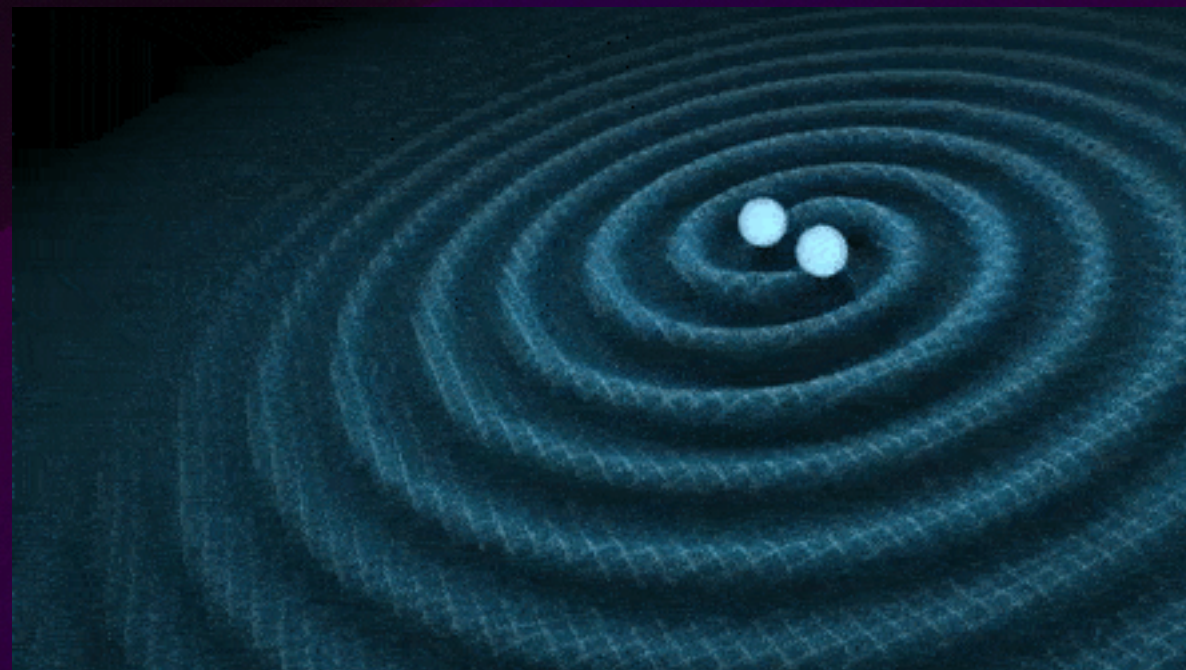
- We didn't have time to discuss this, but the **greatest "crisis" in cosmology** is the  $H_0$  tension:
  - Local/late (SH0ES):  
 $H_0 = (72.9 \pm 0.75) \text{ km s}^{-1} \text{ Mpc}^{-1}$  (di Valentino et al. 2021)
  - Early/far (Planck, ACT):  
 $H_0 = (67.4 \pm 0.5) \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Aghanim et al. 2020)
- These teams have reached the **limits** of what they can do — we **need external data sets**
  - Galaxy surveys are still improving, and they will continue contributing to our understanding of dark energy, but they are limited in what they can do for  $H_0$
- This **discrepancy of  $\gtrsim 5.5 \sigma$**  could be pointing to some **deep, basic issues** in our standard cosmological models





# What to watch for

- The **most exciting** new data set to appear in cosmology in years is now coming from the LIGO+Virgo+Kagra observations of **gravitational waves** from **binary black hole (BBH)** mergers.
  - The fact that we can directly measure the distance to these events (Schutz 1967) is now becoming of consequence: by combining the distances from BBHs with redshifts from galaxies **we can measure  $H_0$  and the expansion history of the Universe** in a completely **independent** way from the other observables.
  - In my opinion, in the near future (less than 10 years) **these observations will solve this crisis. What will come out of it? Stay tuned!**





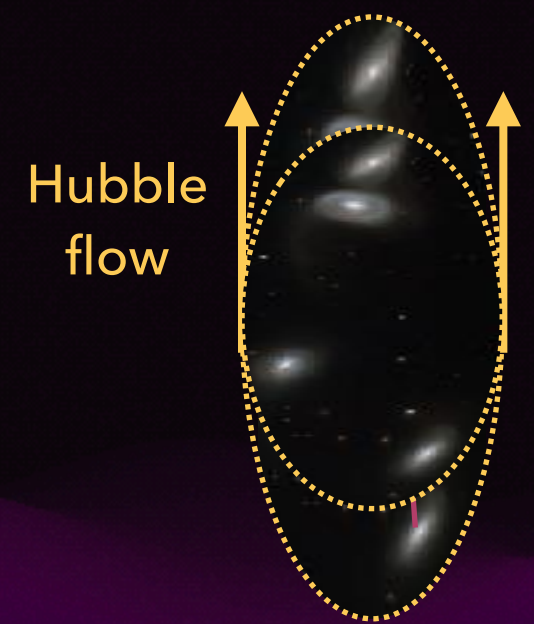
Extra slides: Redshift-Space distortions



# Redshift-space distortions

## In through the looking-glass

- We measure the angular positions of the galaxies, but we don't really measure their radial distances. All that we have is their redshifts — and we infer the radial distance using the Hubble flow, i.e.,  $z \leftrightarrow \chi(z)$ .
- However, the cosmological redshift appears to us mixed with the **peculiar velocities** of the galaxies (their motions with respect to the comoving coordinate frame). This affects, to a greater or lesser extent, all galaxies.
- Think, in particular, of a galaxy group that is starting to form: the members have a tendency to **fall towards the center** of that structure.
- That individual structure will appear to us “squashed” along the radial direction.
- The global effect of this degeneracy between the Hubble flow and the peculiar velocities on LSS is called redshift-space distortions.

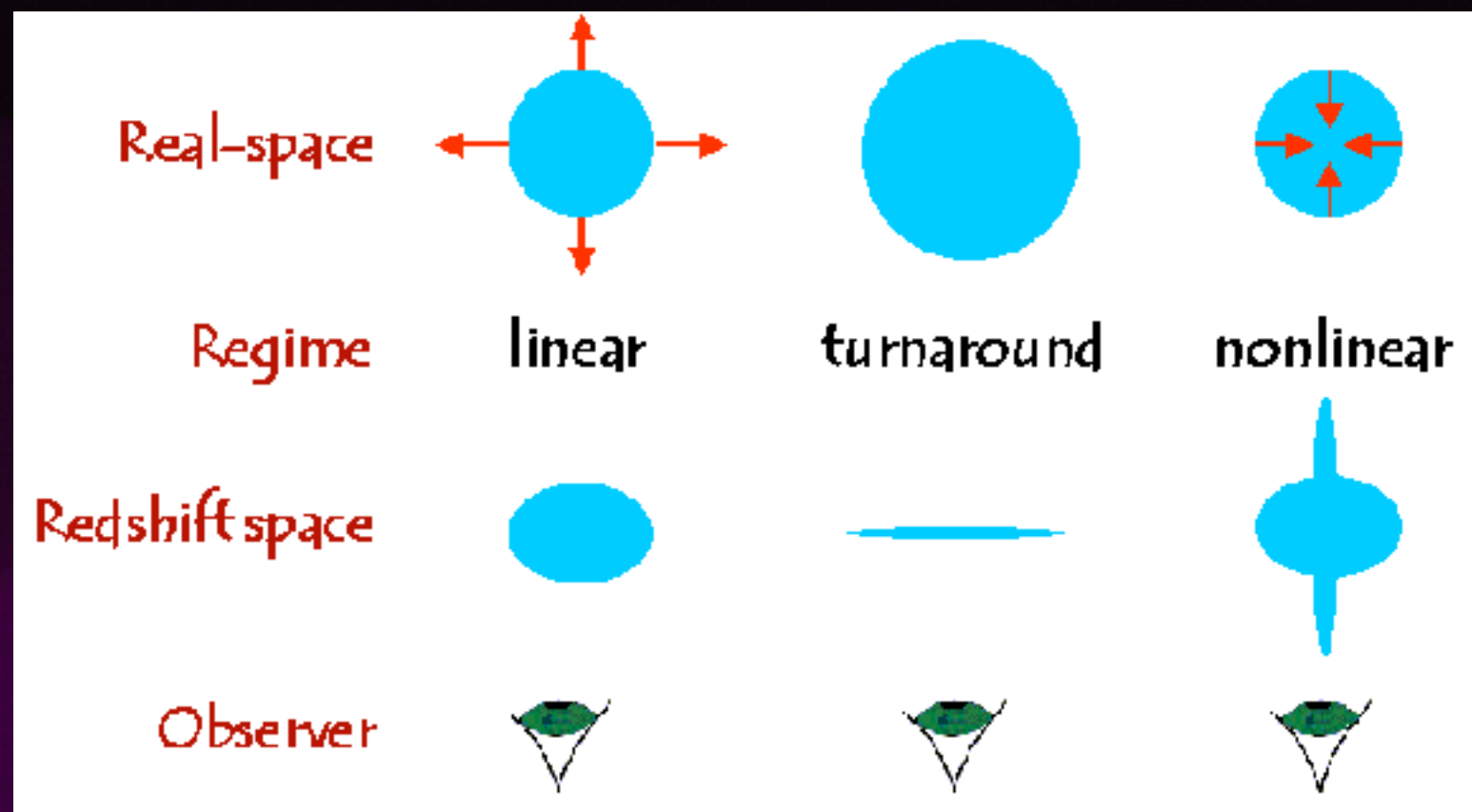




# Redshift-space distortions

## In through the looking-glass

- RSDs distort the measurements of LSS along the line of sight — not along the angular directions.
- Hence, it introduces a kind of anisotropy in the clustering of matter.
- The strength of the RSDs is also something we can measure — and use to extract information.



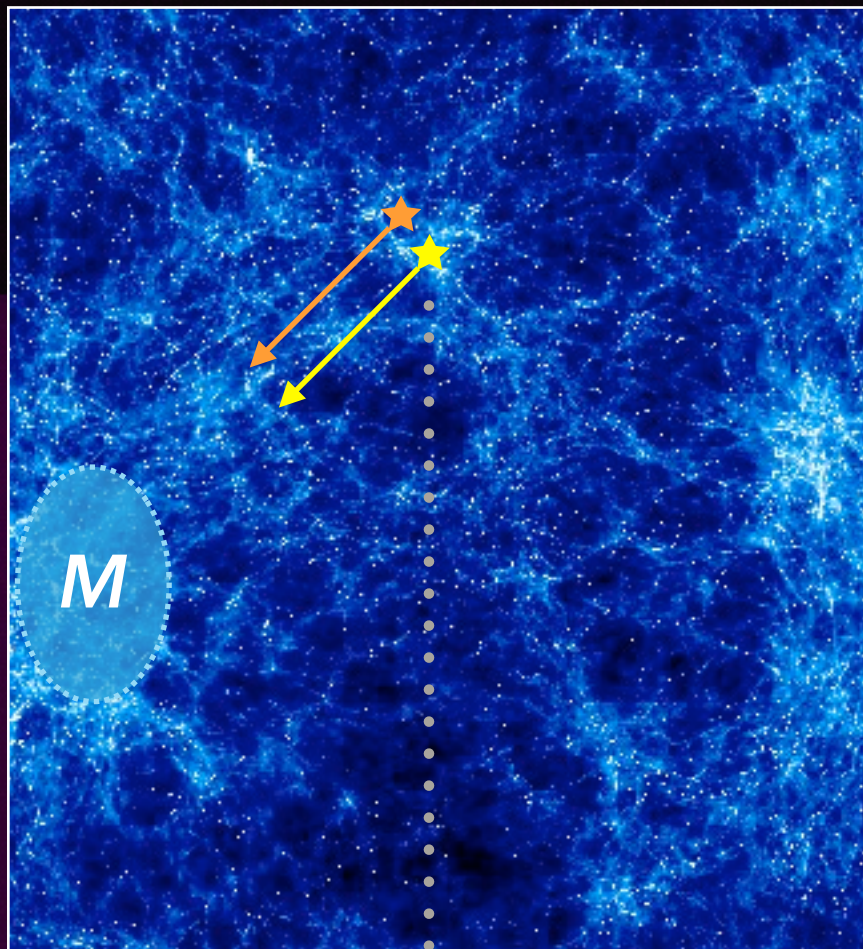
Kaiser 1984



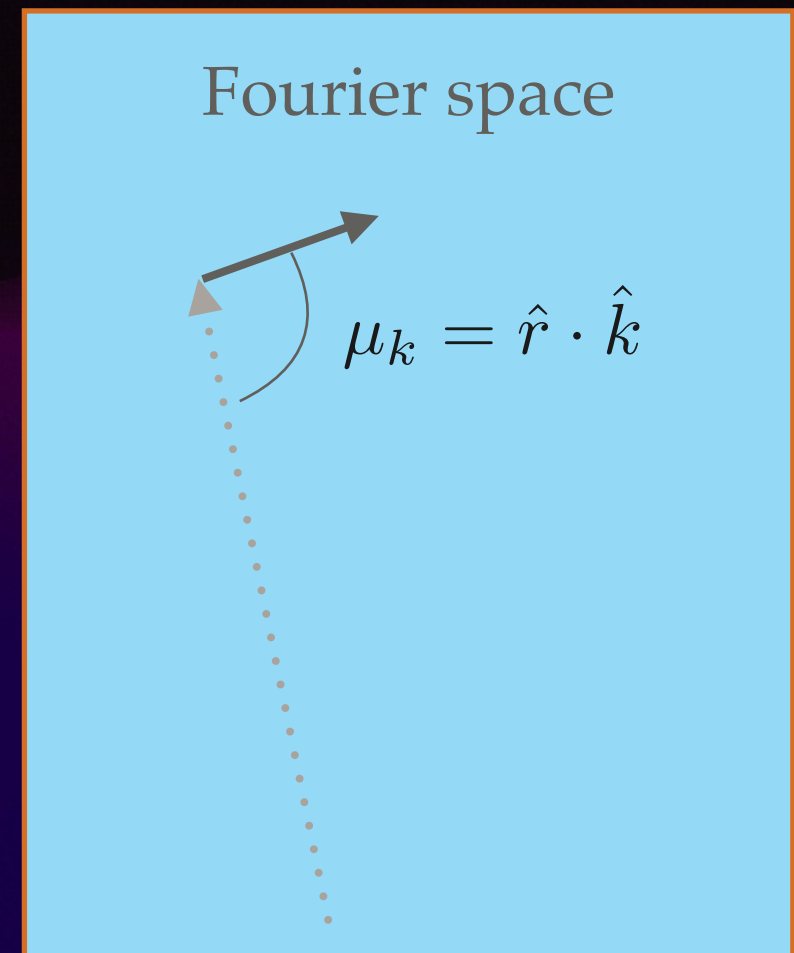
# Redshift-space distortions

## and the equivalence principle

- In fact, there is some very **interesting information** on RSDs, considering the fact that the motions of galaxies around some central mass do not depend on the masses of the galaxies (that's the equivalence principle).



The velocity field reflects the gravitational force in an **unbiased** way



$$P_g(k) \simeq (b_g + f\mu_k^2)^2 P_m(k)$$

Kaiser 87  
Guzzo et al. 04  
Percival & White 09  
Raccanelli et al. 13  
+ ...



# Redshift-space distortions

## and the equivalence principle

- Both dark energy and modified gravity can describe the **same expansion history**, but changing gravity's laws also affects the **Poisson equation** and the **geodesic equation**

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + 8\pi G T_{\mu\nu}^E$$

$$\Delta G_{\mu\nu} + G_{\mu\nu} = 8\pi G T_{\mu\nu}$$



SAME  
MATTER,  
DIFFERENT GRAVITY

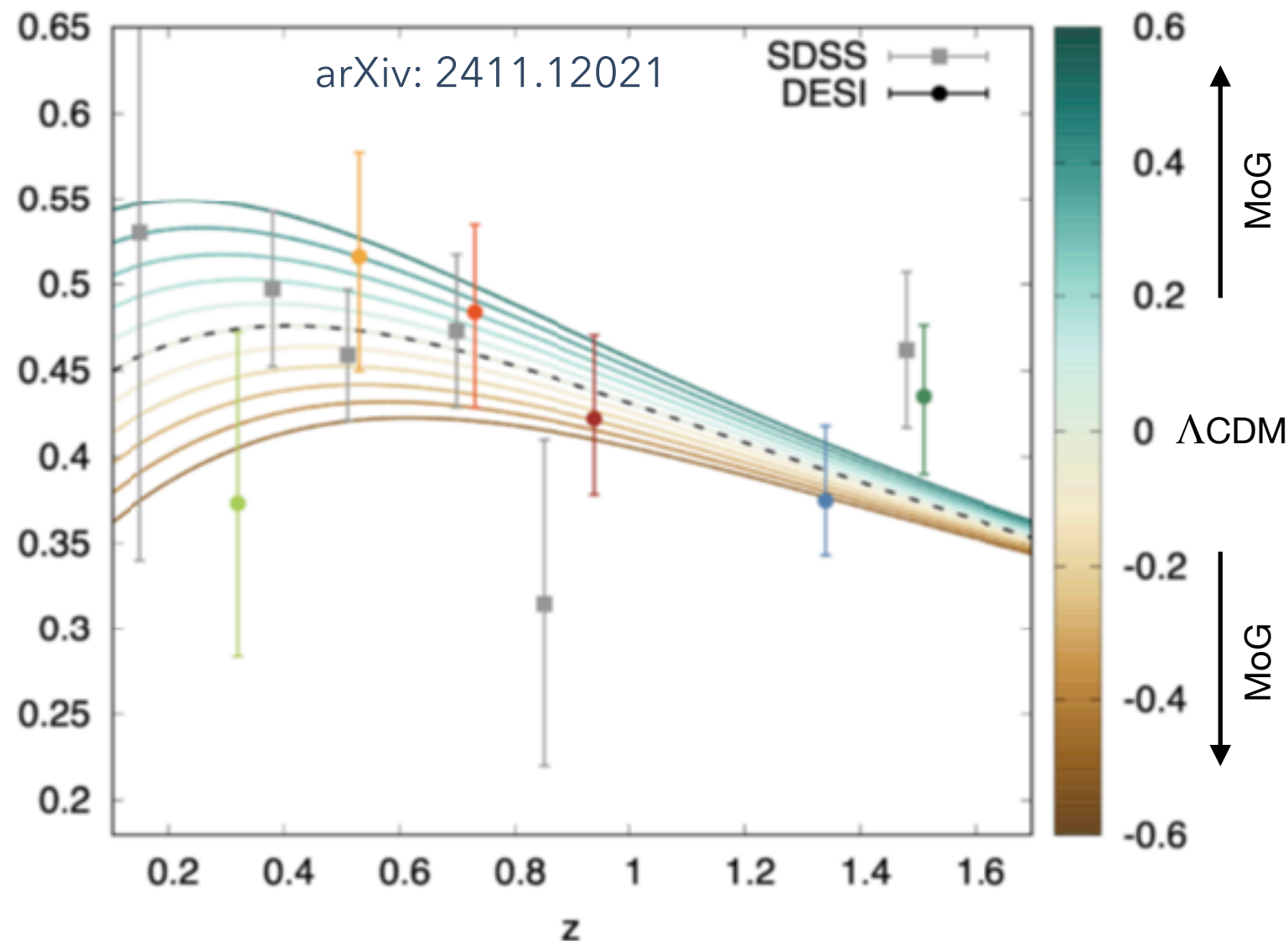
$$\nabla^2 \Phi = \frac{16\pi G}{3} \delta\rho - \frac{1}{6} \delta R(f_R)$$



# Redshift-space distortions

## and the equivalence principle

- The measure of these flows of LSS is given by the **matter growth rate** ( $f$ )
- The growth rate is partly degenerate with galaxy bias ( $b_g$ ), and both are degenerate with the **amplitude of the fluctuations** measured on a “standard” scale of  $8 h^{-1} \text{ Mpc}$  ( $\sigma_8$ )



Peculiar velocities, RSDs  
 ⇒ matter growth rate  $f$

$$f = \frac{d \ln G}{d \ln a} \simeq \Omega_m^\gamma$$

$$P_g(k) \simeq (b_g + f\mu_k^2)^2 P_m(k)$$

$$= (b_g\sigma_8 + f\sigma_8\mu_k^2)^2 \frac{P_m(k)}{\sigma_8^2}$$



# Redshift-space distortions

## and the equivalence principle

- We project the angular dependence (along v. across the line of sight) into a monopole and a quadrupole

