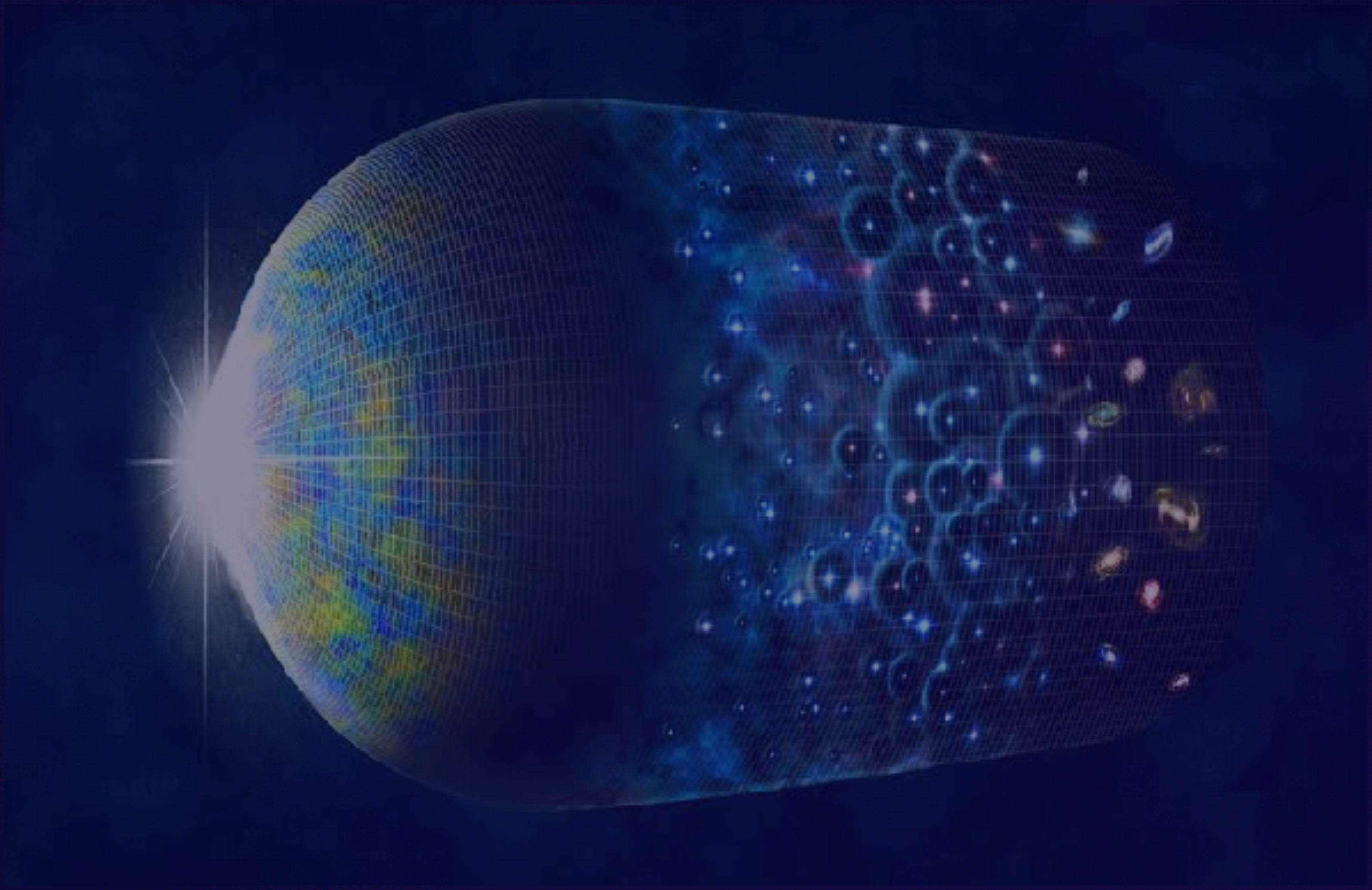
Cosmology:

From the early universe to cosmological observations



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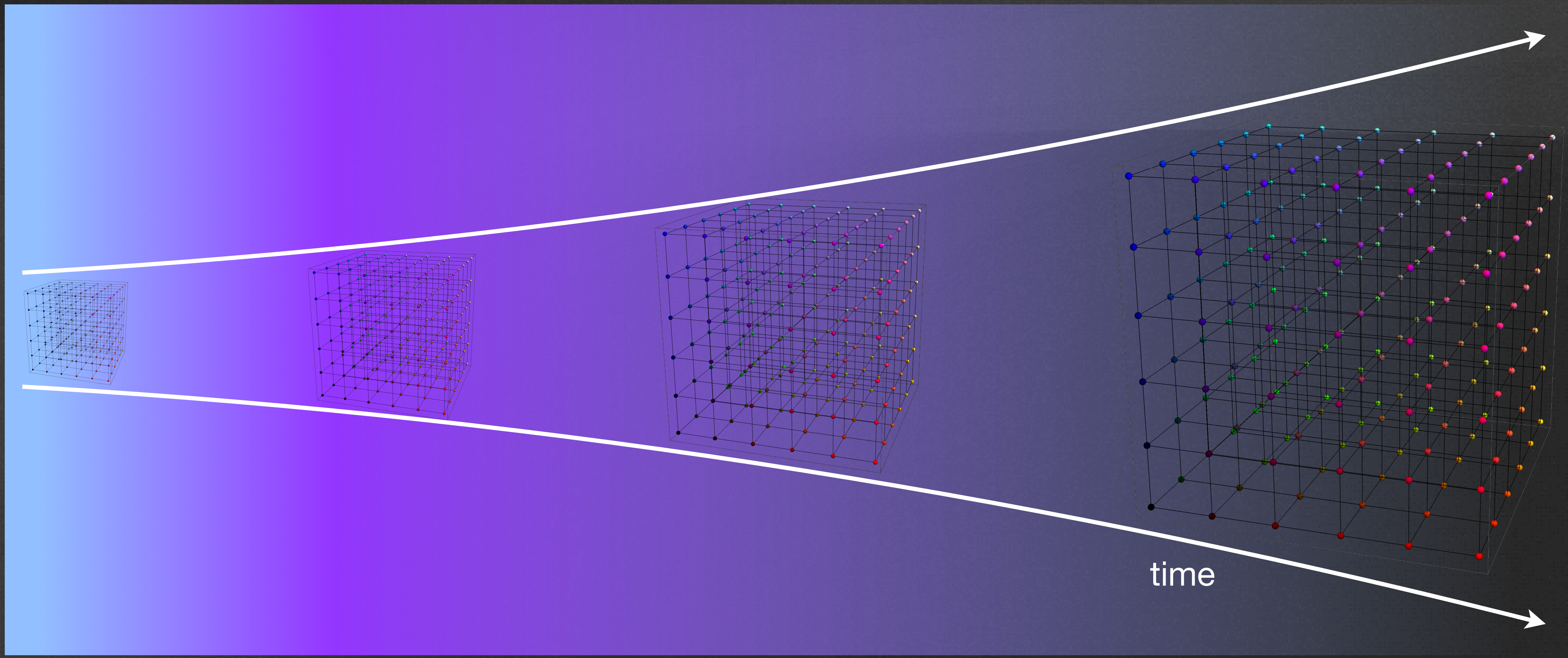
Cosmology in a nutshell

The Friedmann-Lemaître-Robertson-Walker (FLRW) metric

- The simplest cosmology we could possibly conceive is an extension of Newton's "absolute" space, where, in a certain coordinate frame, the Universe is basically the same at every spatial point — i.e., is **homogeneous at each instant in time.**
- But we **could** allow this space to be **different** at each $t = const.$ hypersurface, so this would look something like this:

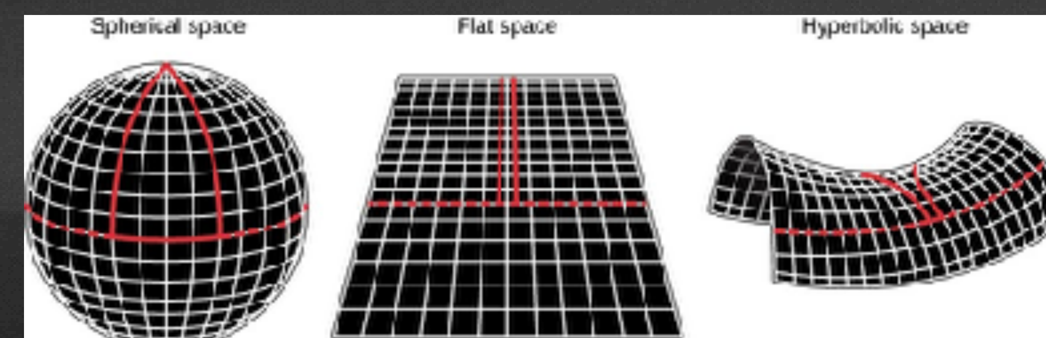


Cosmology in a nutshell

The FLRW metric

- It turns out that **Euclidean space** is not the only *reasonable* **3D space**: more than 100 years ago (before Relativity!), Gauss, Lobachevsky and Bolyai discovered that Euclidean space can be **generalized**!

- There is a **single parameter** that spans the variety of homogeneous 3D spaces: **spatial curvature,** K — a physical quantity with dimensions of (1/distance)^2 . In fact, $R_K = |K|^{-1/2}$ is the **curvature radius**!



$K > 0$ Closed | $K \to 0$ Flat | $K < 0$ Open

- The metric of these generalized "spatial sections" is given by:

$$dl^2 = d\chi^2 + \frac{1}{K}\text{sen}^2\left(\sqrt{K}\chi\right) d\Omega^2$$

which can also be written, in different coordinates, as:

$$dl^2 = \frac{dR^2}{1 - K R^2} + R^2\, d\Omega^2 = \frac{dr^2 + r^2 d\Omega^2}{\left[1 + \frac{K}{4} r^2\right]^2} = \frac{dx^2 + dy^2 + dz^2}{\left[1 + \frac{K}{4}(x^2 + y^2 + z^2)\right]^2}$$

If you are familiar with General Relativity or Differential Geometry, you can show that:

$$dl^2 = \gamma_{ij}\, dx^i dx^j$$

$$\Rightarrow \; R_{ijkl} = K\left(\gamma_{ik}\gamma_{jl} - \gamma_{jk}\gamma_{il}\right)$$

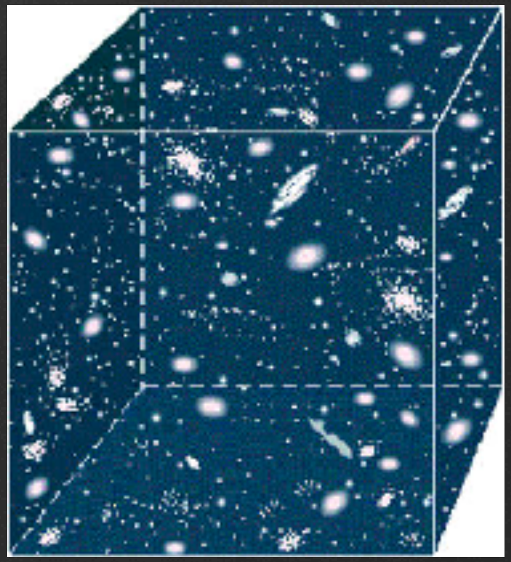
Cosmology in a nutshell

The FLRW metric

- These spatial volumes have a fundamental property: all points are equivalent. In fact, this is exactly what we need, because the **Cosmological Principle** states that, on sufficiently large scales, the Universe is **homogeneous** and **isotropic:**

Homogeneity without isotropy

Isotropy without homogeneity



Our Universe appears to be **both** homogeneous and isotropic!

Cosmology in a nutshell

The FLRW metric

- The **only** metric which encodes a **homogeneous and isotropic spatial section** is the FLRW, given by:

$$ds^2 = -c^2dt^2 + a^2(t)\,dl^2 \quad ,$$

where $a(t)$ is a function of time (the ***scale factor***), and the spatial section is given by our former acquaintance:

$$dl^2 = \gamma_{ij}\,dx^i dx^j = d\chi^2 + \frac{1}{K}\mathrm{sen}^2\left(\sqrt{K}\chi\right)d\Omega^2$$

Alexander **Friedmann**

- Consider now what happens for two points in the same $t = const$. hypersurface. The **distance** between those two positions is:

$$\Delta s = a(t)\,\Delta l \quad ,$$ where we call Δl the **comoving distance**

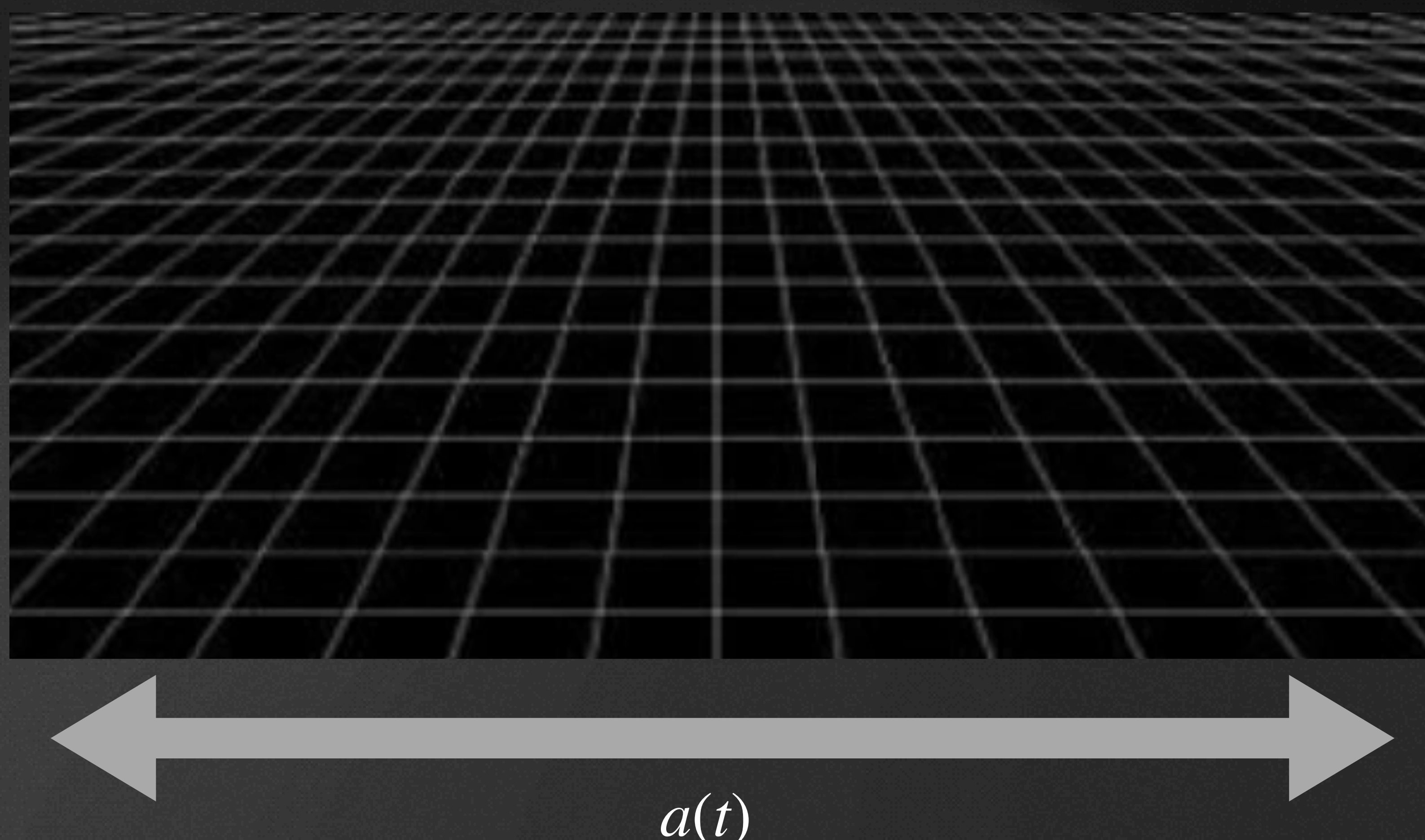
This means that the ***physical distance*** between any two points is **changing with time** according to the scale factor $a(t)$

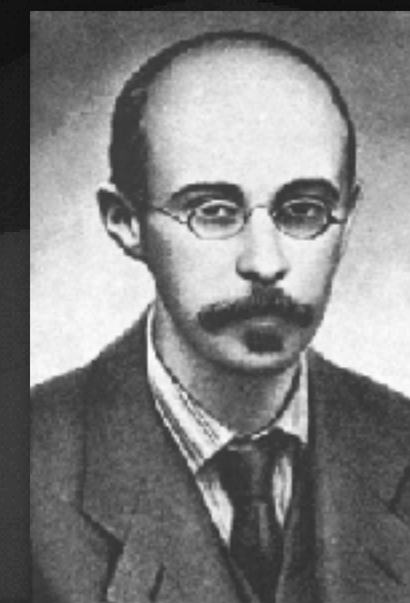
Georges **Lemaître**

Cosmology in a nutshell

The FLRW metric

- We can think of the FLRW metric in terms of a spatial section that changes dynamically with time according to the scale factor $a(t)$





Alexander **Friedmann**

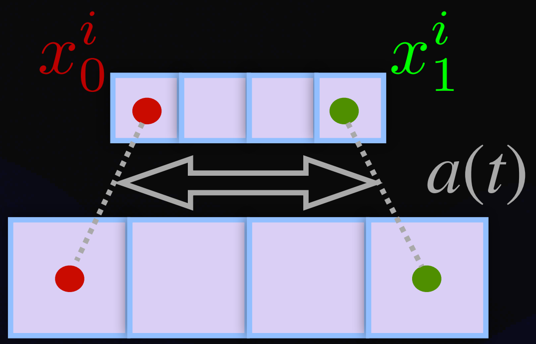
Georges **Lemaître**

By convention, we anchor the distance scale to today (t_0) so $a(t_0) = 1$

Cosmic expansion

and redshift!

- Let's consider a pair of points in space. Using our FLRW metric we have that the distance between those two points is, at each $t = const$. hypersurface:



$$ds^2 = -c^2dt^2 + a^2(t)\, dl^2 \quad \Rightarrow \quad \Delta s^2 = a^2(t)\, \Delta l^2$$

- The speed with which these two points are separated over time can be cast in terms of the distance itself:

$$v = \frac{d}{dt}\Delta s = \frac{d}{dt}\left[a(t)\, \Delta l\right] = \dot{a}\Delta l \quad \Rightarrow \quad v = \frac{\dot{a}}{a}\, a\, \Delta l = \frac{\dot{a}}{a}\Delta s = H\, \Delta s \qquad ,$$

where $H(t) = \dot{a}/a$ is the **Hubble-Lemaître function**.

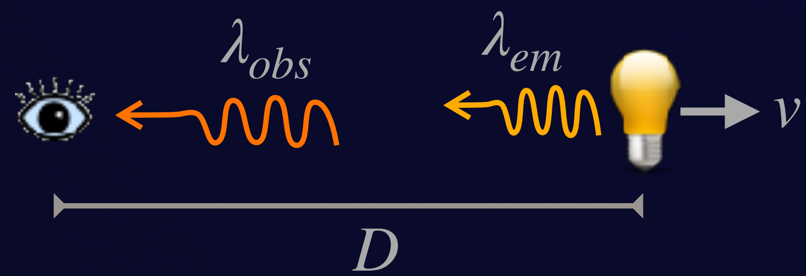
Alexander Friedmann 1922

Georges Lemaître 1927

- But this speed also affects the light signals that propagate between those two points due to the Doppler effect:

$$\frac{\lambda_{obs}}{\lambda_{em}} = 1 + \frac{v}{c} \quad \rightarrow \quad 1 + \frac{H\,D}{c} \quad ,$$ where here we take $D = \Delta s$ for simplicity.

This is the ***redshift***: $z = \dfrac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} \rightarrow \dfrac{HD}{c}$



Cosmic expansion

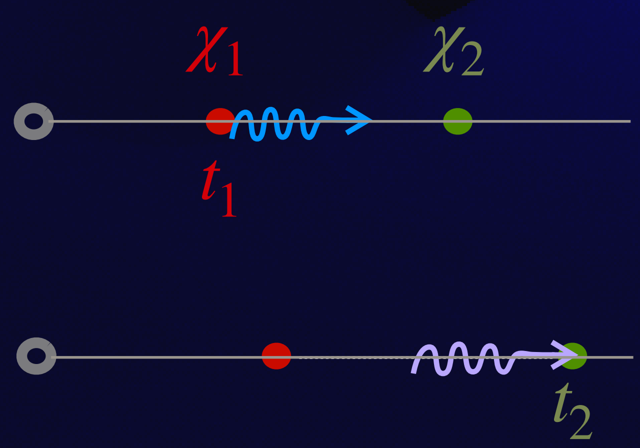
and redshift!

- We can establish a direct relationship between redshift and distance by considering a light signal emitted from one point to the other. Since photons travel along the light cone we have:

$$ds^2 = -c^2dt^2 + a^2(t)\left[d\chi^2 + \frac{1}{K}\text{sen}^2\left(\sqrt{K}\chi\right)d\Omega^2\right] \to 0$$

- For simplicity, let's assume that the light signal travels from a distant point (a galaxy) to the origin along a **radial trajectory**, so $d\Omega = 0$ and therefore:

$$cdt = a(t)\,d\chi \quad \Rightarrow \quad \boxed{\chi = \int \frac{cdt}{a(t)}}$$

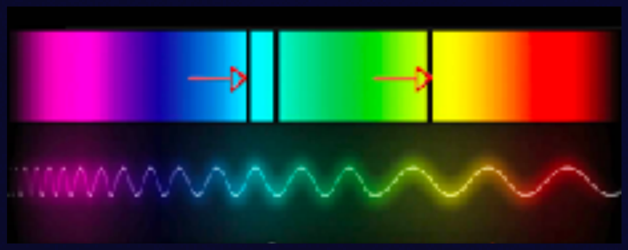


- We can also relate **redshift** to the **scale factor** if we consider the fact that the wavelength of the light is a distance like any other, so:

$$\lambda \sim a(t)\lambda_0 \quad \Rightarrow \quad \lambda_{obs} = \frac{a(t_{obs})}{a(t_{em})}\lambda_{em} \quad , \quad \text{hence}$$

$$\Rightarrow \quad 1 + z = \frac{\lambda_{obs}}{\lambda_{em}} = \frac{a(t_{obs})}{a(t_{em})} \quad , \quad \text{and “obs" is now } (t_0)\text{, so}$$

$$\boxed{1 + z = \frac{1}{a(t)}}$$

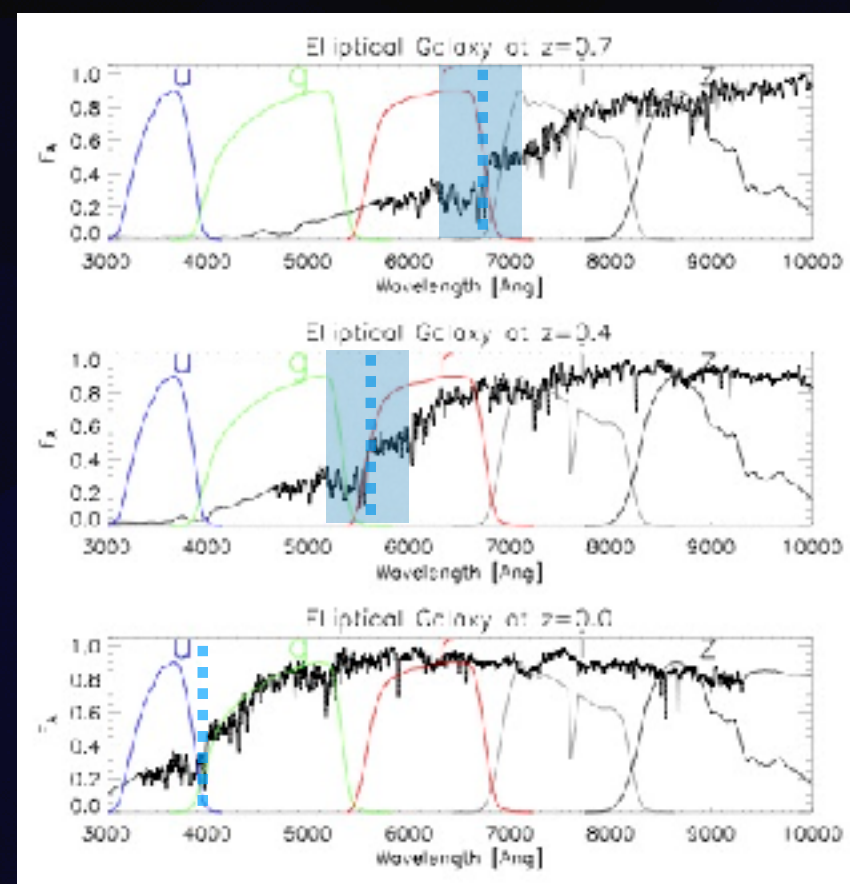


Distances and redshifts

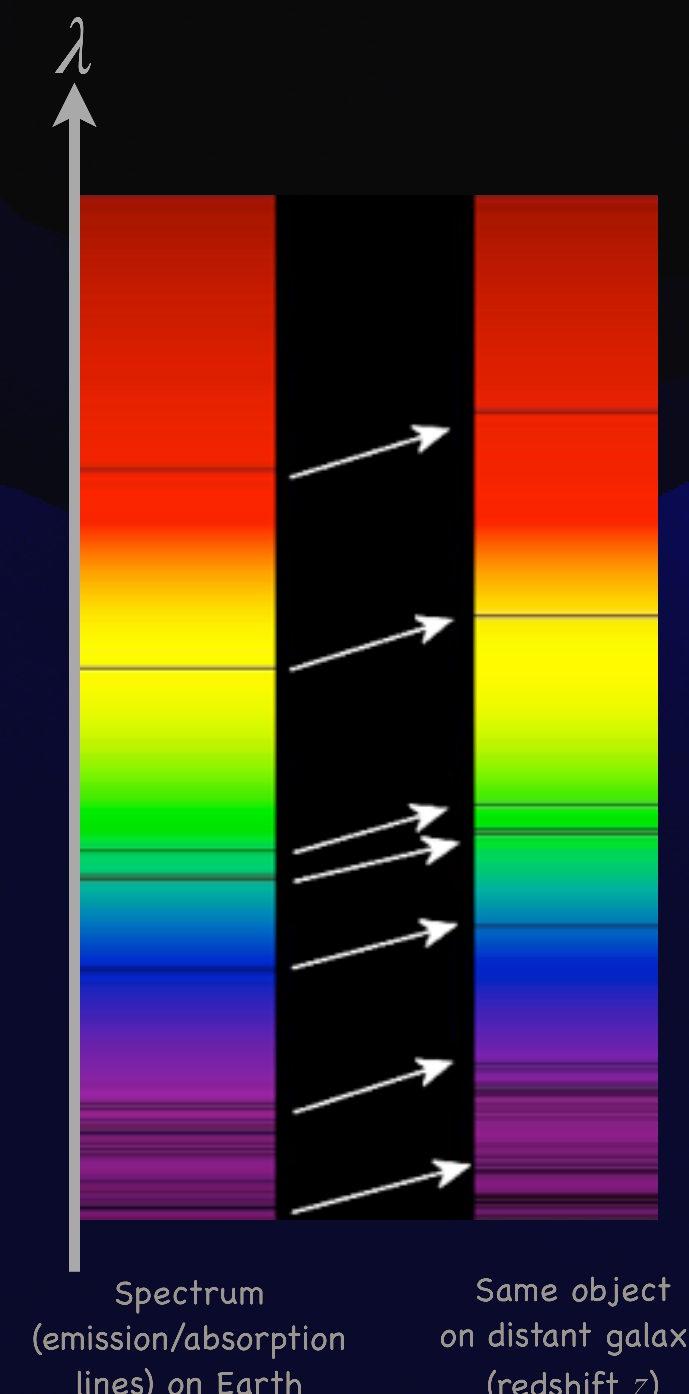
and how to measure them

- Redshift is quite easy to measure: if we can observe the light from a distant star or galaxy, we can measure the same spectral features of the same elements or molecules that we have here on Earth, and then of course:

$$z = \frac{\lambda_{obs}}{\lambda_{em}} - 1$$



- However, sometimes we do measure the light from these galaxies, but we can't resolve the spectral features, because, e.g, we use some filters to observe the light using filters
- These "photometric" redshifts are inherently less accurate than spectroscopic redshifts.



Distances and redshifts
and how to measure them

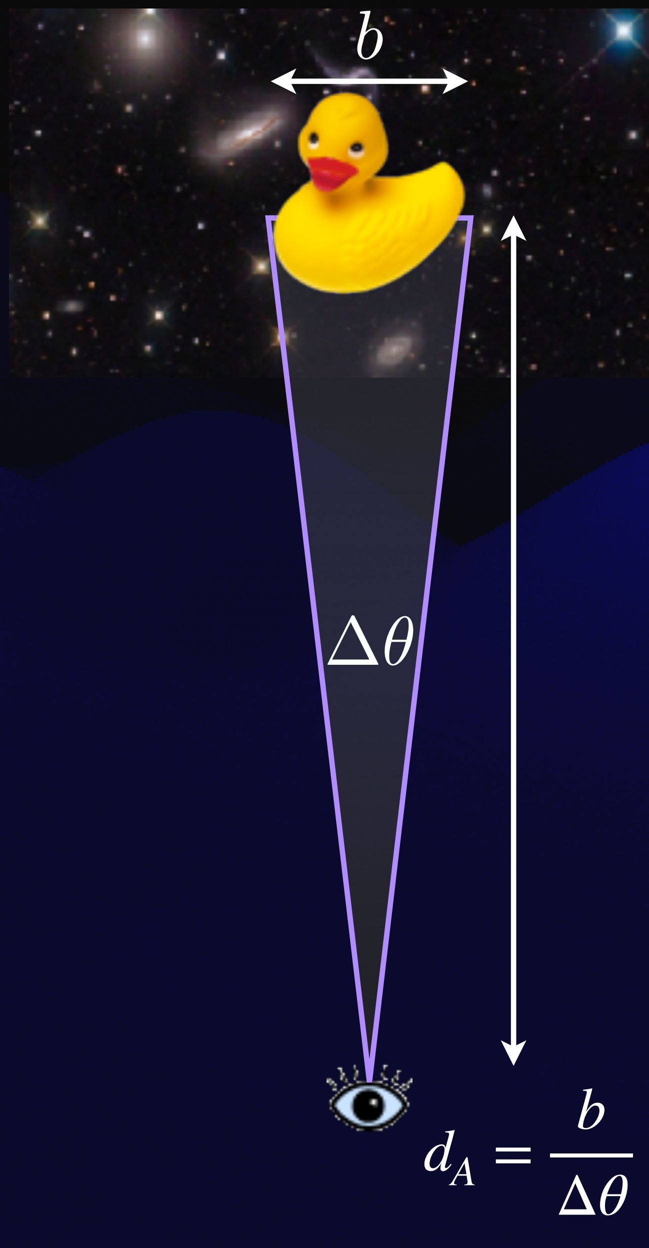
- But what about distances? How on Earth can we measure any length over thousands, millions, billions of light years?
- There are a few ways, but they all involve assuming something about the object that we observe at a distance.
- One way is the following: suppose you know the size (b) of an object that you observe from far away. You could then use the angle subtended by that object to infer the distance.
- Using our FLRW metric and the fact that the object is at basically the same distance from us (χ is constant) we get:

$$\Delta s^2 = a^2 \left[\Delta\chi^2 + \frac{1}{K} \mathrm{sen}^2 \left(\sqrt{K}\chi \right) \Delta\Omega^2 \right]$$

$$\Rightarrow \quad b = a \frac{1}{\sqrt{K}} \mathrm{sen} \left(\sqrt{K}\chi \right) \Delta\theta$$

Hence, if we know b and measure $\Delta\theta$ we can infer that:

$$d_A = a \frac{1}{\sqrt{K}} \mathrm{sen} \left(\sqrt{K}\chi \right) = \frac{b}{\Delta\theta}$$



This is known as the angular diameter distance

Distances and redshifts

and how to measure them

- One of the most widely used method to infer cosmic distances rely on another physical property of the sources: their **luminosities**.

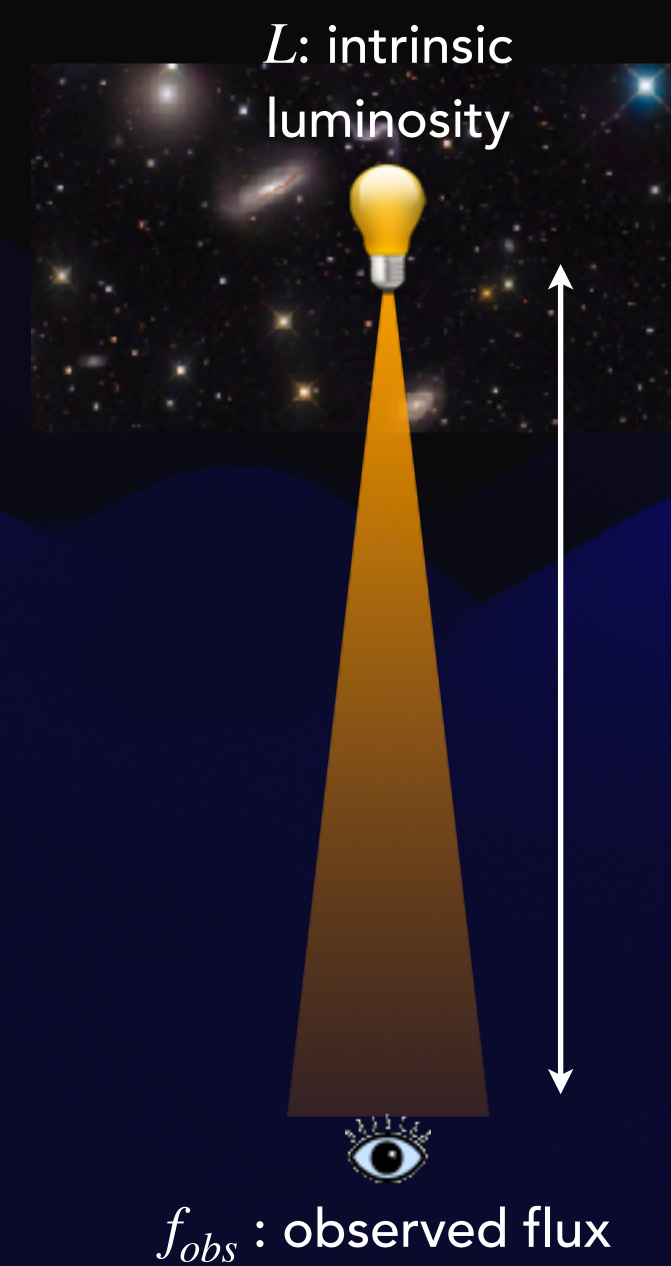
- The **flux** observed here on Earth falls with the **square of the distance** (d_L), hence:

$$f = \frac{L}{4\pi\, d_L^2} \quad \Rightarrow \quad d_L = \sqrt{\frac{L}{4\pi f}}$$

- There is one subtle detail that we need to consider: the **energy** of the light emitted at the source is redshifted as it propagates from the source to our telescopes, $E \sim \nu \sim 1/a$.

- Taking this into account modifies the equation above:

$$f_{obs} = \frac{a_{em}^2}{a_0^2} \frac{L}{4\pi R^2} \quad \Rightarrow \quad d_L = \sqrt{\frac{L}{4\pi f_{obs}}} = \frac{1}{a} \frac{1}{\sqrt{K}} \,\mathrm{sen}\left(\sqrt{K}\chi\right)$$

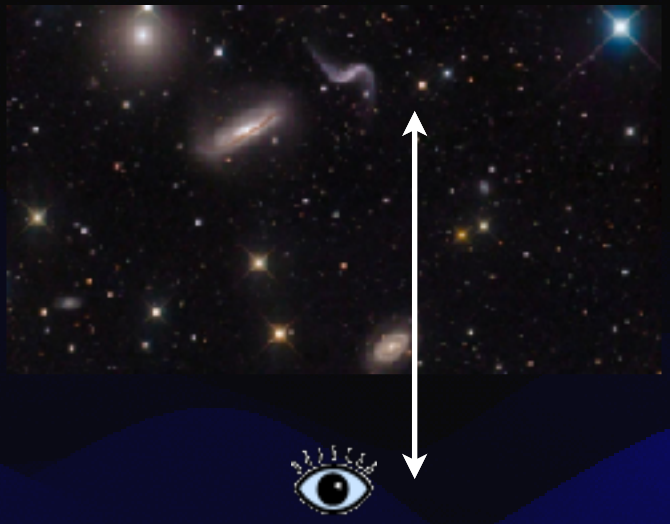


This is known as the **luminosity distance**

Distances and redshifts

and how to measure them

- Notice that for **small distances** all these definitions are equivalent — and they converge to the usual, Euclidean notion of distance!



Angular-diameter distance:

$$d_A = a\frac{1}{\sqrt{K}}\,\mathrm{sen}\left(\sqrt{K}\chi\right) \to \frac{1}{\sqrt{K}}\left(\sqrt{K}\chi - \frac{1}{3!}K^{3/2}\chi^3 + \ldots\right)$$

$$\Rightarrow d_A \simeq \chi + \ldots$$

And similarly for the luminosity distance:

$$\Rightarrow d_L \simeq \chi + \ldots$$

Typically, within a few hundred million light years, all these distances are nearly the same.

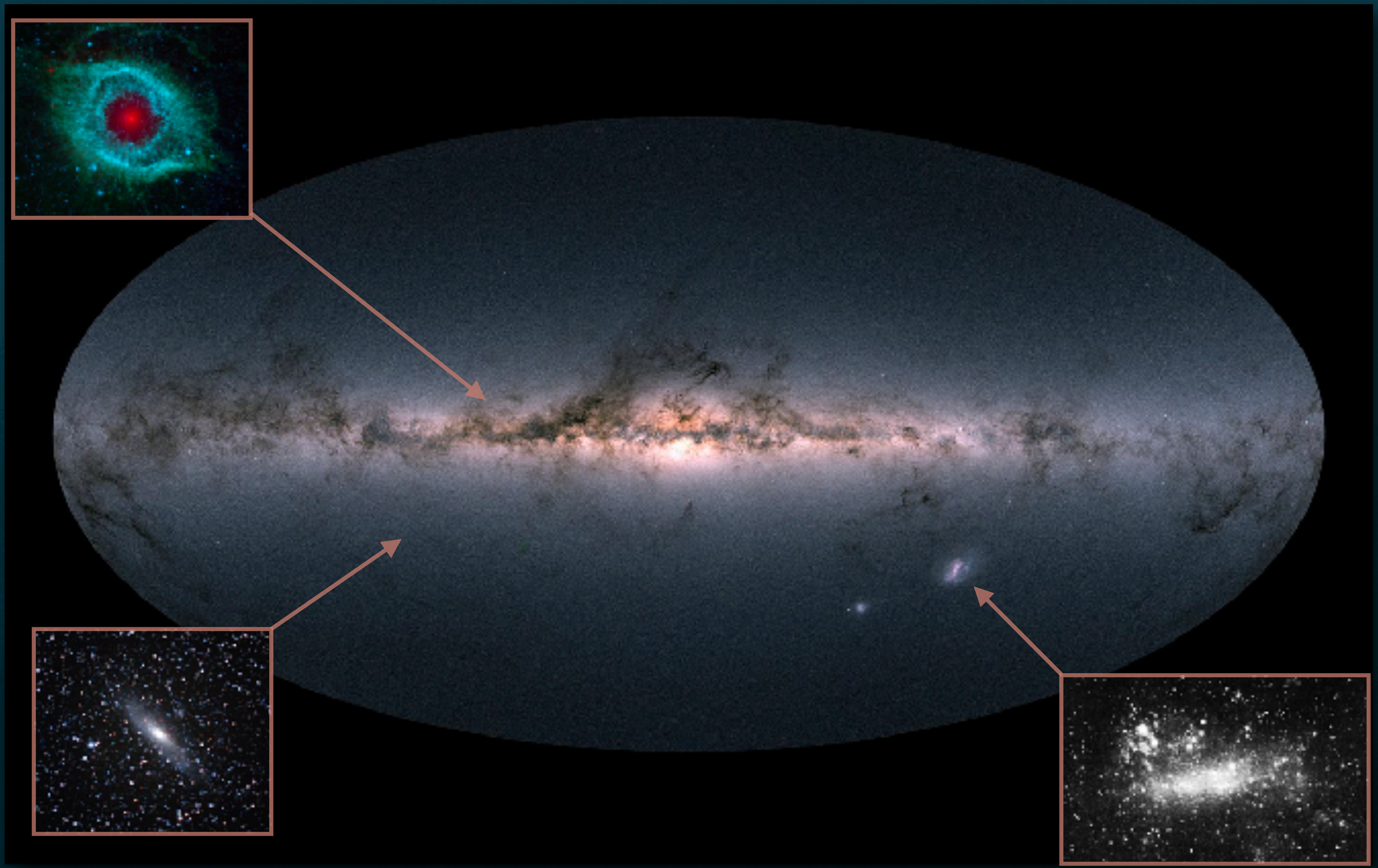
- Cosmological unit of distance: Mpc = 3,26 light-years

1 Mpc =
3.26 million light years =
30.856.775.800.000.000.000.000 m

Cosmic expansion

The Hubble-Lemaître law

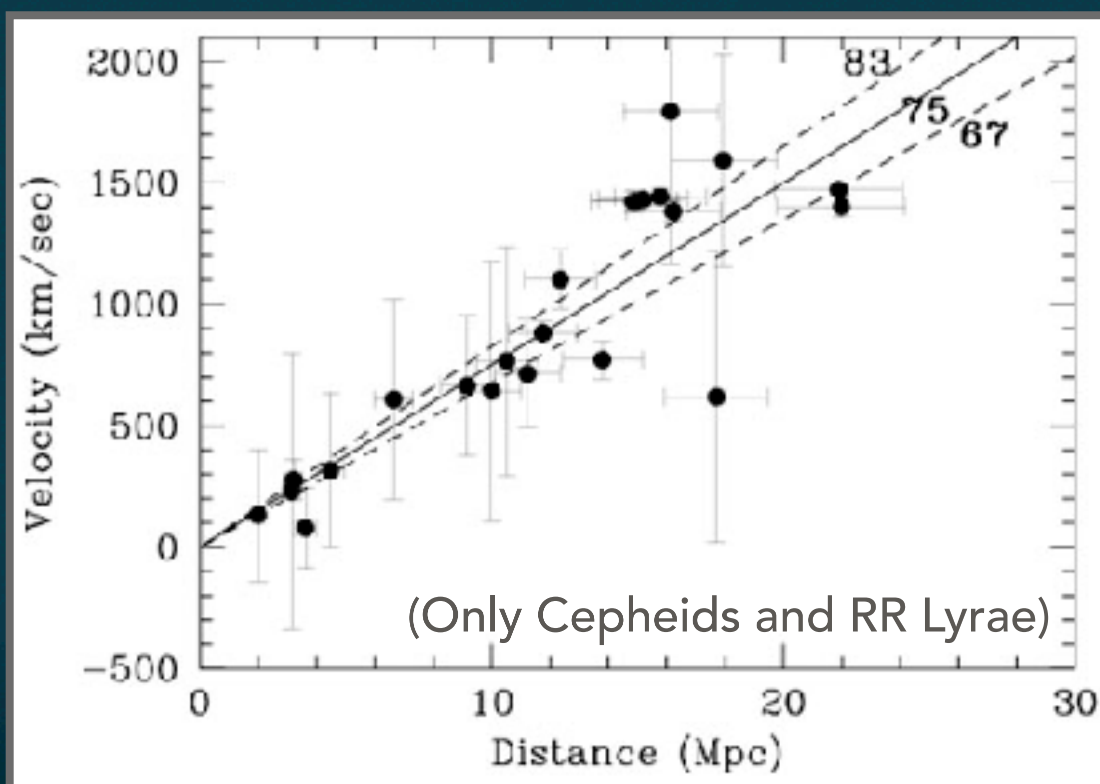
- That story starts with one of the great questions of the beginning of the 20th century: are the “nebulas" **inside** our galaxy, or are they **outside**? This meant **measuring distances to objects that are incredibly far away!**

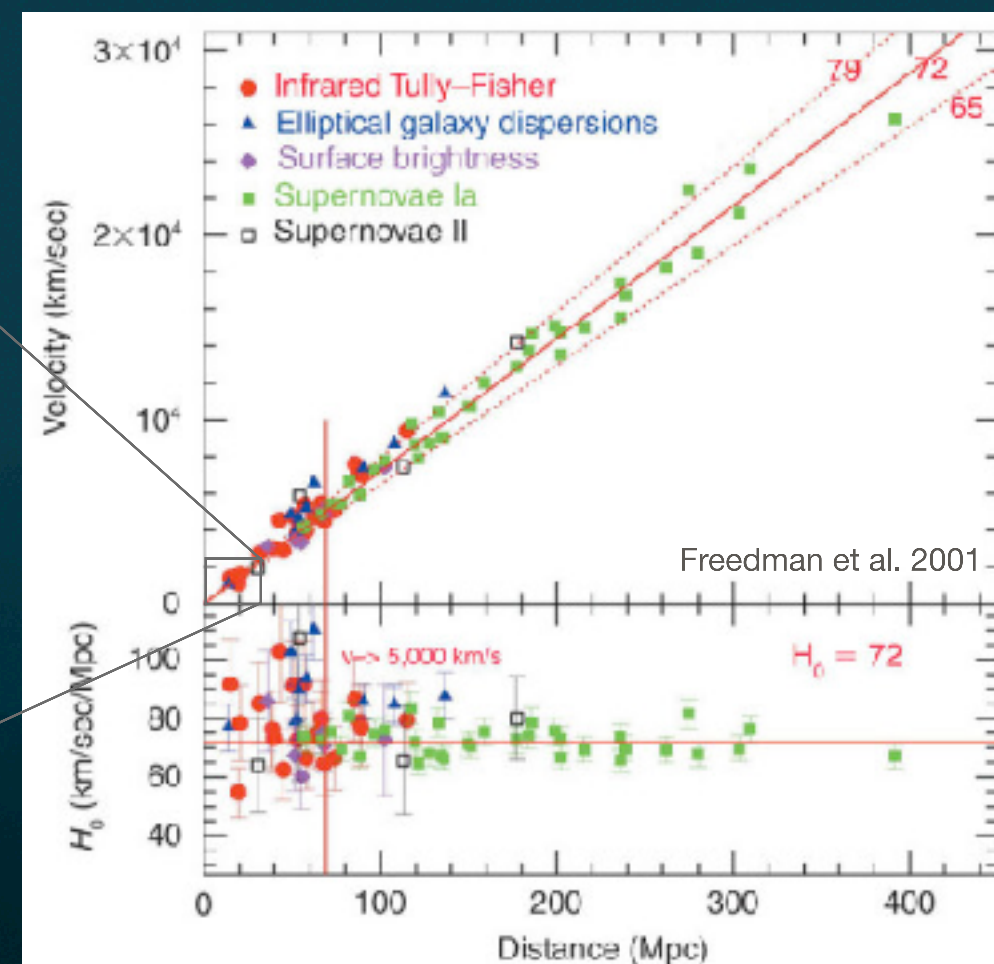


Cosmic expansion

The Hubble-Lemaître law

- With time (and better instruments) this diagram was extended for Cepheids in more distant galaxies, to other types of variable stars, and other types of "standard candles"





$$v = \frac{\dot a}{a}\, a\, \Delta l = \frac{\dot a}{a} \Delta s = H\, \Delta s \quad , \quad \frac{\lambda_{obs}}{\lambda_{em}} = 1 + \frac{v}{c} \;\; \to \;\; 1 + \frac{H\,D}{c} \quad , \quad z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} \to \frac{HD}{c}$$

The hot Big Bang

Going backwards in time…

- Hubble and Lemaître showed that the Universe is **expanding**. *[This is in fact "built in" General Relativity, and could have been predicted by Einstein or by Friedmann… but were not!]*
- This means that galaxies are becoming **more distant with time**, and the Universe is getting **colder**.
- All relativistic (covariant) theories of gravity relate this **dynamics** (the expansion) with the **matter sources.** In Einstein's theory of **General Relativity** (GR), this is given by the **Friedmann equations.**
- Starting from the FLRW metric:

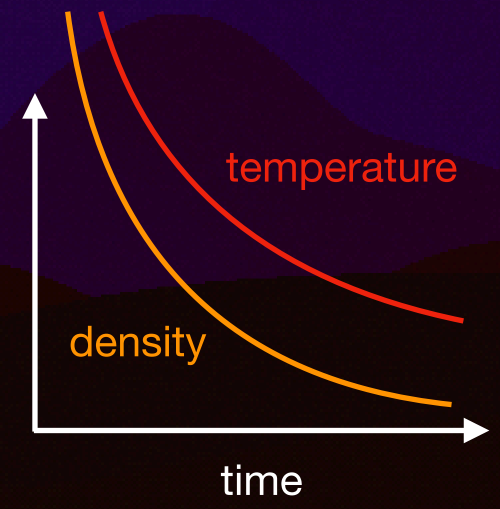
$$ds^2 = - dt^2 + a^2(t)\, dl^2 \quad ,$$

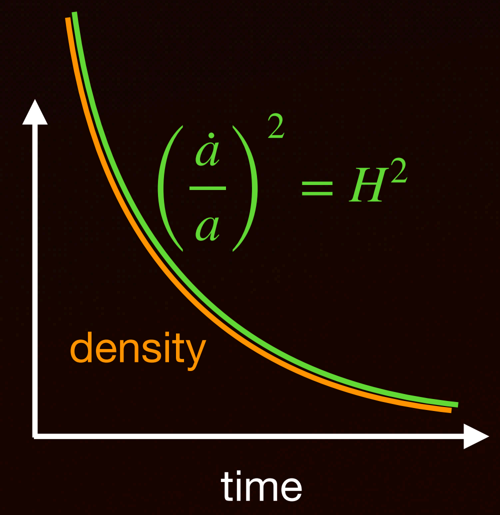
and using the equations of GR we have:

$$G_{00} = 8\pi G T_{00} \quad \Rightarrow \quad \boxed{3\frac{\dot{a}^2}{a^2} + 3\frac{K}{a^2} = 8\pi G\, \rho} \quad ,$$

$$G_{ij} = 8\pi G\, T_{ij} \quad \Rightarrow \quad - 2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{K}{a^2} = 8\pi G\, p \quad ,$$

where $\rho(t)$ is the matter density and $p(t)$ is the matter pressure.

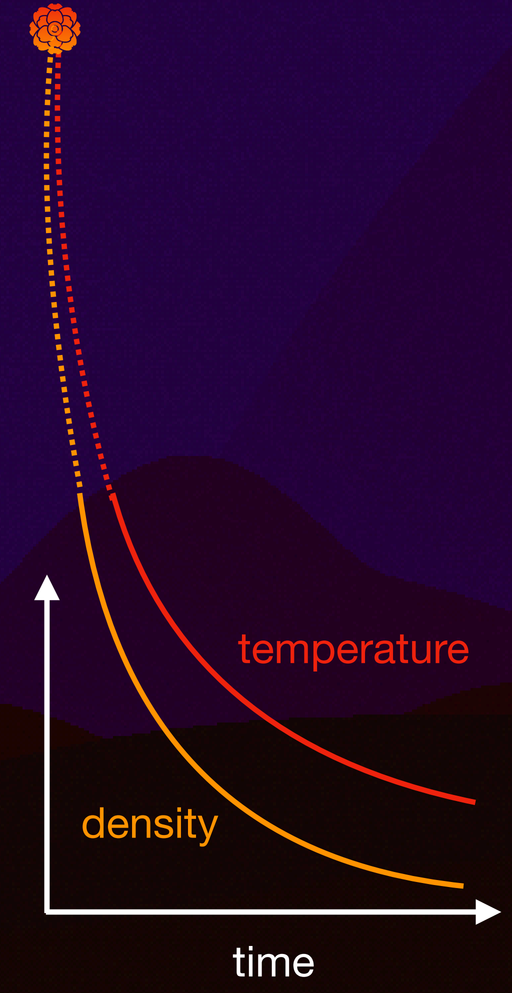




The two Friedmann equations are connected by **the continuity equation,** $\dot{\rho} + 3H(\rho + p) = 0$

The hot Big Bang

Going backwards in time…



- But what this means, of course, is that if we reach **back in time** early enough, there may be an instant when the temperature, the density, the energy, etc., will all become **arbitrarily large — Bang!**

- We can in fact solve the continuity and Friedmann equations in simple cases.

    - For non-relativistic ("cold") matter:

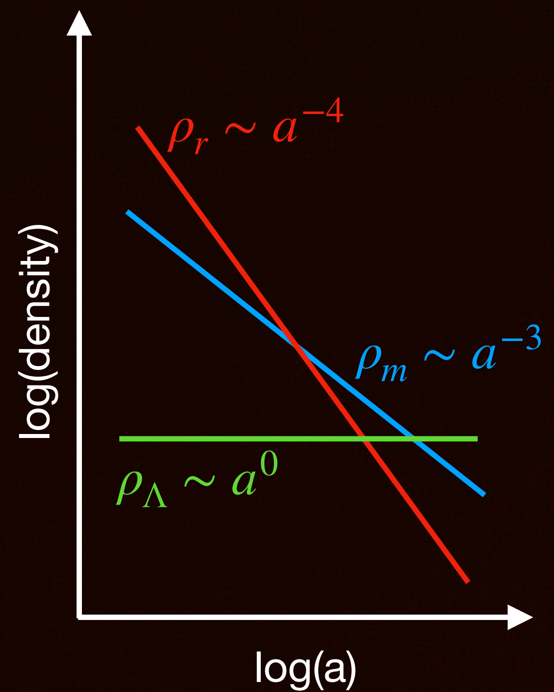
$$p_m = 0 \quad , \quad \rho_m \sim a^{-3} \quad \Longleftrightarrow \quad \dot{\rho}_m + 3H\rho_m = 0$$

    - Relativistic ("hot") matter/radiation:

$$p_r = \frac{1}{3}\rho_r \quad , \quad \rho_r \sim a^{-4} \quad \Longleftrightarrow \quad \dot{\rho}_r + 3H\left(\rho_r + \frac{1}{3}\rho_r\right) = \dot{\rho}_r + 4H\rho_r = 0$$

    - Vacuum energy/cosmological constant (Λ):

$$p_\Lambda = -\rho_\Lambda \quad , \quad \rho_\Lambda \sim a^0 \quad \Longleftrightarrow \quad \dot{\rho}_\Lambda + 3H\left(\rho_\Lambda - \rho_\Lambda\right) = \dot{\rho}_\Lambda = 0$$



- All these components can coexist, and will contribute to the Friedmann equations:

$$\rho = \rho_m + \rho_r + \rho_\Lambda + \ldots$$

The hot Big Bang

Flavors of matter, matters of expansion

- We can easily solve Friedmann's equations for these simple cases of cold matter, radiation (hot matter) and vacuum energy (Λ). Let's assume that spatial curvature (K) is zero, just for simplicity:
  - Cold matter:

$$\rho_m = \rho_{m0}\, a^{-3} \quad \longrightarrow \quad \frac{\dot{a}^2}{a^2} = \frac{8\pi G\, \rho_{m0}}{3} a^{-3} \quad \Longrightarrow \quad a \sim t^{2/3}$$

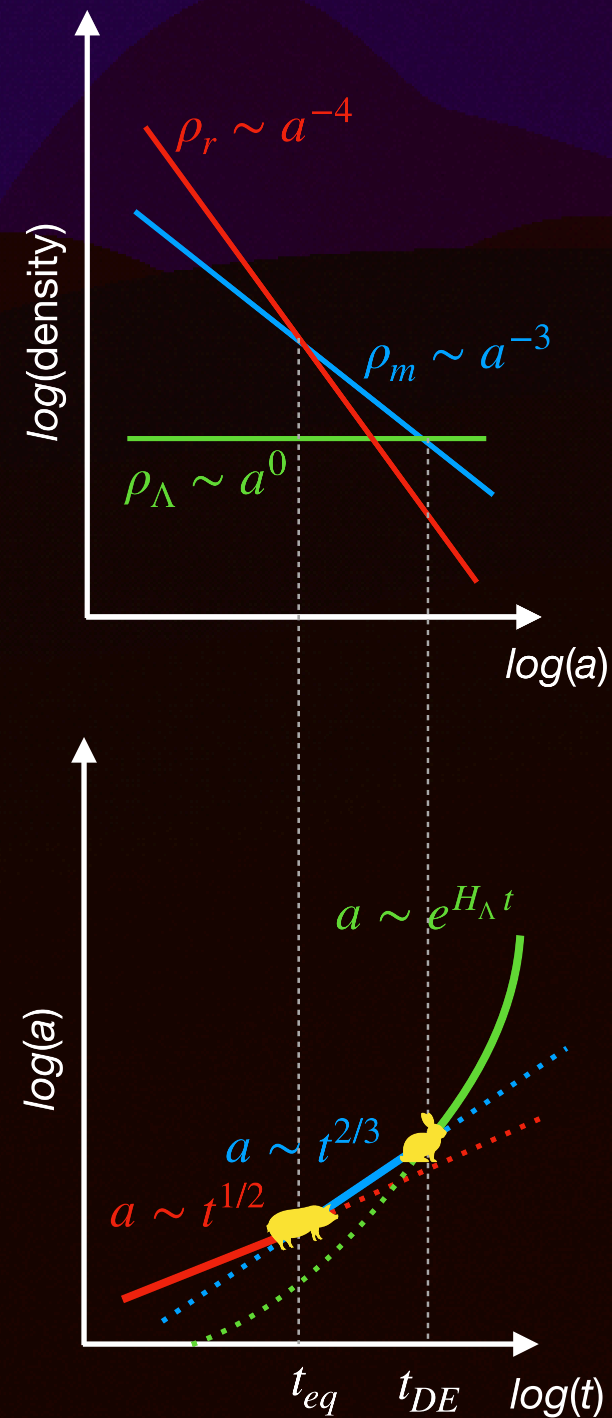
  - Relativistic ("hot") matter/radiation:

$$\rho_r = \rho_{r0}\, a^{-4} \quad \longrightarrow \quad \frac{\dot{a}^2}{a^2} = \frac{8\pi G\, \rho_{r0}}{3} a^{-4} \quad \Longrightarrow \quad a \sim t^{1/2}$$

  - Vacuum energy/cosmological constant (Λ):

$$\rho_\Lambda = \rho_{\Lambda 0} = const\,. \quad \longrightarrow \quad \frac{\dot{a}^2}{a^2} = \frac{8\pi G\, \rho_\Lambda}{3} \quad \Longrightarrow \quad a \sim e^{H_\Lambda t}$$

where $H_\Lambda = \sqrt{\dfrac{8\pi G\, \rho_\Lambda}{3}}$



☞ Question: derive these time dependences of the scale factor

The hot Big Bang

Expansion then and expansion now

- Before we go on and consider the cosmological history that follow from this dynamics, let's look at the way that the expansion rate behaves in this scenario that we painted above. Let's recall that:
  - Cold matter:

$$a \sim t^{2/3} \quad \Rightarrow \quad \frac{\dot a}{a} = \frac{2}{3\,t}$$

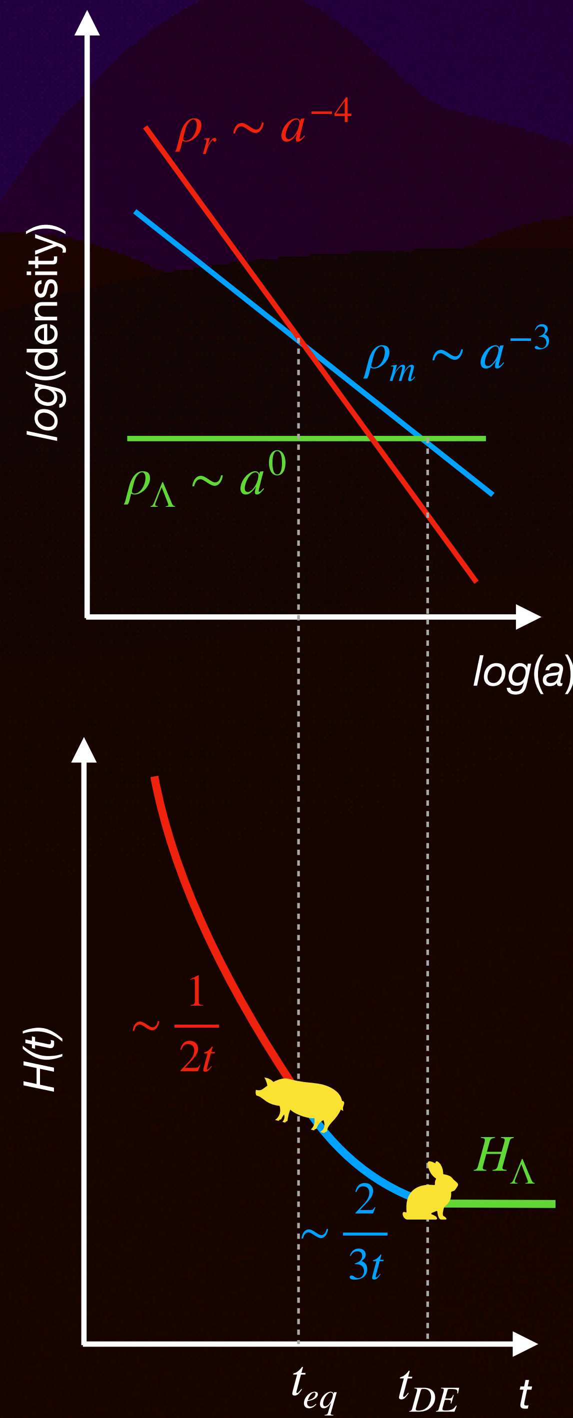
  - Relativistic ("hot") matter/radiation:

$$a \sim t^{1/2} \quad \Rightarrow \quad \frac{\dot a}{a} = \frac{1}{2\,t}$$

  - Vacuum energy/cosmological constant (Λ):

$$a \sim e^{H_\Lambda t} \quad \Rightarrow \quad \frac{\dot a}{a} = H_\Lambda \quad , \quad \text{where } H_\Lambda = \sqrt{\frac{8\pi G\,\rho_\Lambda}{3}}$$

- This means that we can **extrapolate** the Hubble-Lemaître parameter, both from now to the past, as well as from the past until today. We will come back to this when we revisit the Hubble (H_0) tension between the local/late and the distant/early measurements of H_0 .



The hot Big Bang

Expansion then and expansion now

- We should be a bit more specific here. We can write the first Friedmann equation in terms of the **critical density**:

$$\rho_c = \sum_i \rho_i - \frac{3\,K}{8\pi G}\frac{1}{a^2} \quad , \quad \text{where} \quad \rho_c = \frac{3H^2}{8\pi G} \,.$$

- We can then divide by this "critical density" ρ_c and rewrite the Friedmann equation as:

$$1 = \sum_i \frac{\rho_i}{\rho_c} - \frac{3\,K}{8\pi G\,\rho_c\,a^2} \qquad \Rightarrow \qquad \text{Density parameters (ratios):} \quad \Omega_i = \frac{\rho_i}{\rho_c}$$

- We could in fact even assign a density parameter to the **spatial curvature** (not a density at all!):

$$\Omega_K \equiv - \frac{3\,K}{8\pi G\,\rho_c\,a^2} = - \frac{K}{H^2\,a^2}$$

- In that way, the Friedmann equation would be written simply as:

$$\sum_i \Omega_i + \Omega_K = 1 \quad , \text{ or equivalently,} \quad \sum_i \Omega_i = 1 - \Omega_K$$

The hot Big Bang

Expansion then and expansion now

- The expansion rate is then expressed in the very simple form:

$$H^2 = \frac{8\pi G}{3} \sum_i \rho_i - \frac{K}{a^2}$$

- Using the fact that the critical density is basically the expansion rate, and that today $\rho_{c0} = 3H_0^2/(8\pi G)$, we have that:

$$\frac{H^2}{H_0^2} = \sum_i \frac{\rho_i}{\rho_{c0}} - \frac{K}{H_0^2 a^2}$$

- We now use the density parameters to express the expansion rate, and arrive at:

$$\frac{H^2}{H_0^2} = \Omega_{m0}\, a^{-3} + \Omega_{r0}\, a^{-4} + \Omega_{\Lambda 0} + \Omega_{K0}\, a^{-2}$$

Remember that $a(t) = \dfrac{1}{1+z}$

- Or even better:

$$\boxed{H(z) = H_0 \sqrt{\Omega_{m0}\,(1+z)^3 + \Omega_{r0}\,(1+z)^4 + \Omega_{\Lambda 0} + \Omega_{K0}\,(1+z)^2}}$$

The hot Big Bang

Physics where Physics is due

- Let's give some general idea about the physics quantities in these equations.
- The critical density today is simply H_0 in different units. We have:

$$\rho_{c0} = \frac{3H_0^2}{8\pi G} = 1.05 \times 10^4 \left(\frac{H_0}{100\ \mathrm{km\,s^{-1}\,Mpc^{-1}}} \right)^2 \mathrm{eV\,cm^{-3}}$$

i.e., the equivalent (in mass) of about **10 protons per cubic meter**.

- How much of that is in actual "cold" matter?

$$\Omega_{m0} = \frac{\rho_{m0}}{\rho_{c0}} \simeq 0.27 \pm 0.03$$

and of that, the amount in actual "normal" (i.e., baryonic) matter is

$$\Omega_{b0} = \frac{\rho_{b0}}{\rho_{c0}} \simeq 0.036 \pm 0.004$$

- The share of radiation (mostly photons) today is very small:

$$\Omega_{r0} = \frac{\rho_{r0}}{\rho_{c0}} \simeq 2.49 \times 10^{-5}$$

- Even with a rough estimate of these numbers we are able to imagine what happens in such a Universe…

Notation:

$$h = \frac{H_0}{100\ \mathrm{km\,s^{-1}\,Mpc^{-1}}}$$

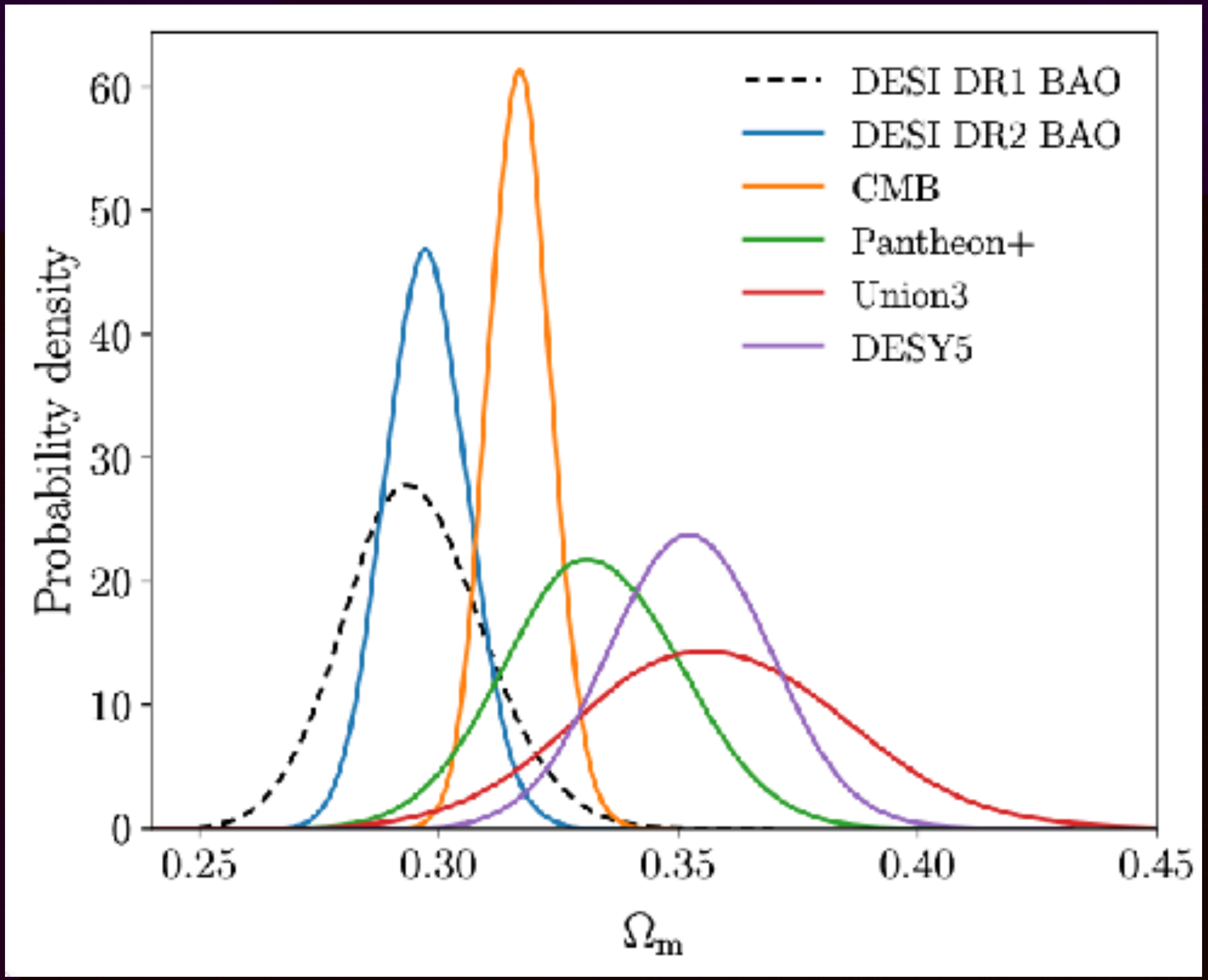
But this share was much larger in the past!

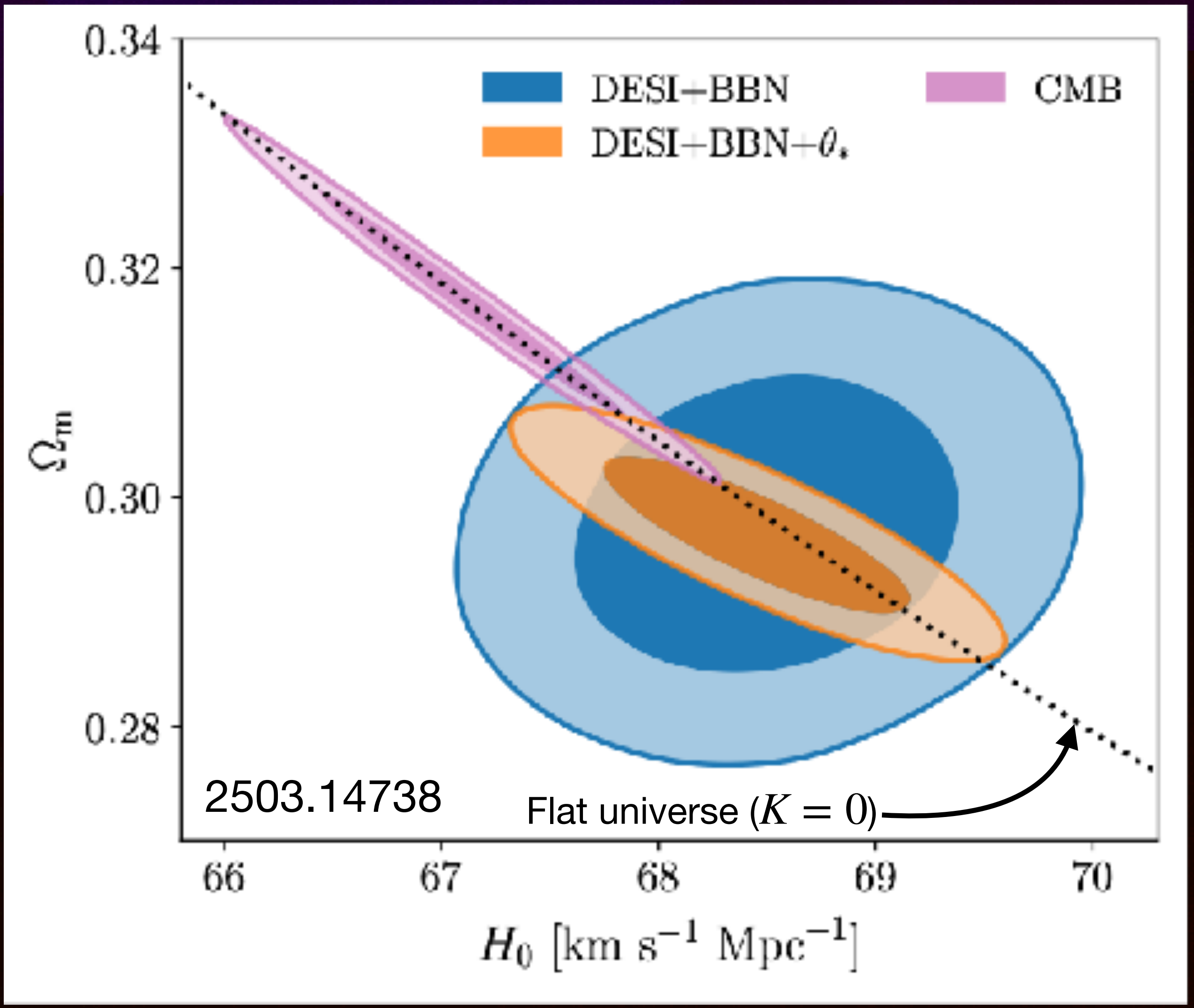
☞ Question: show that radiation dominates the Universe at $z \gtrsim 10^5$

The hot Big Bang

Physics where Physics is due

- Density budget of the Universe: constraints in 2026



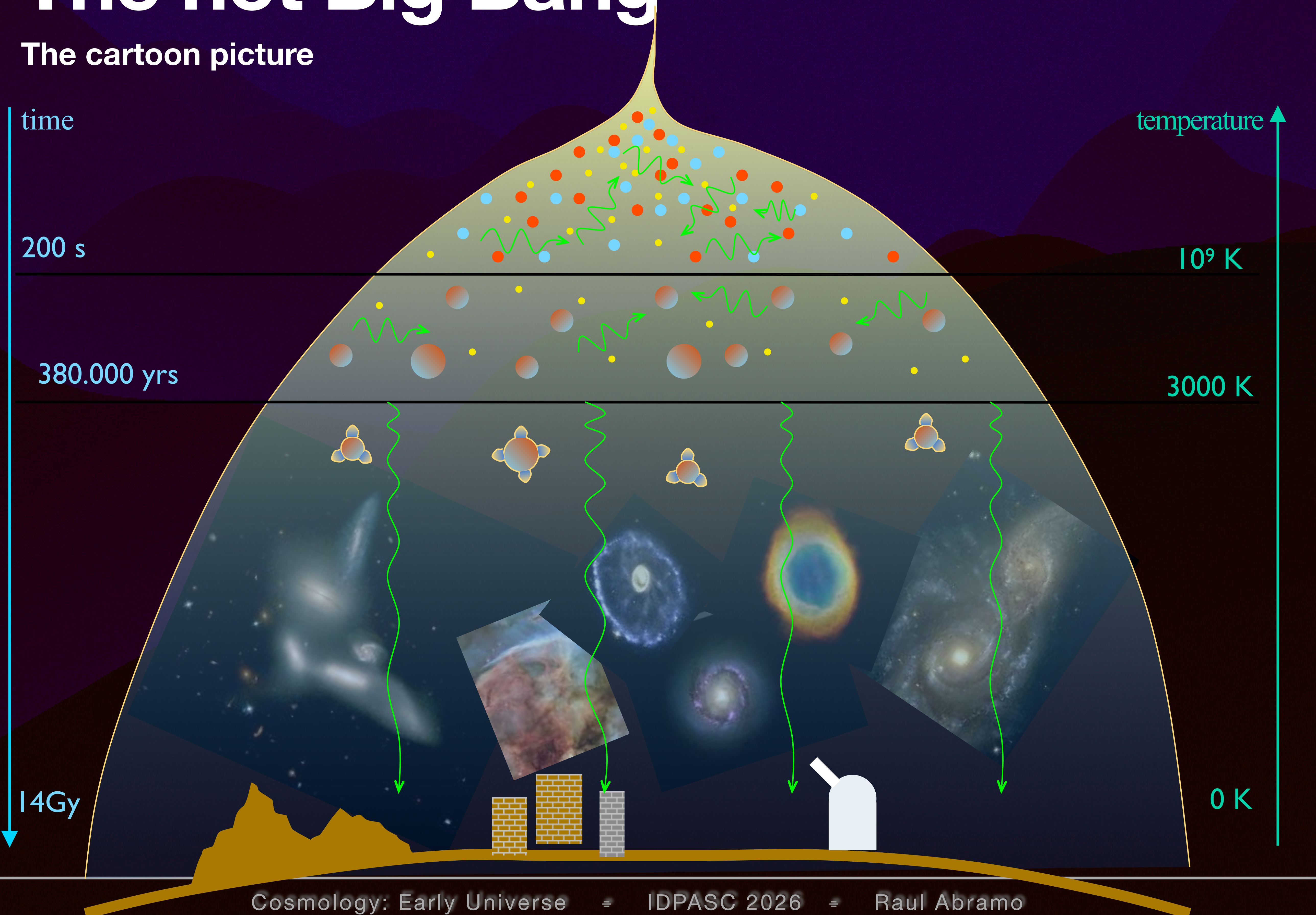


CMB + DESI DR2: $\Omega_k = (2.3 \pm 1) \times 10^{-3}$

- There is "something else": not matter, not curvature $\Rightarrow$ **Dark Energy**!

The hot Big Bang

The cartoon picture



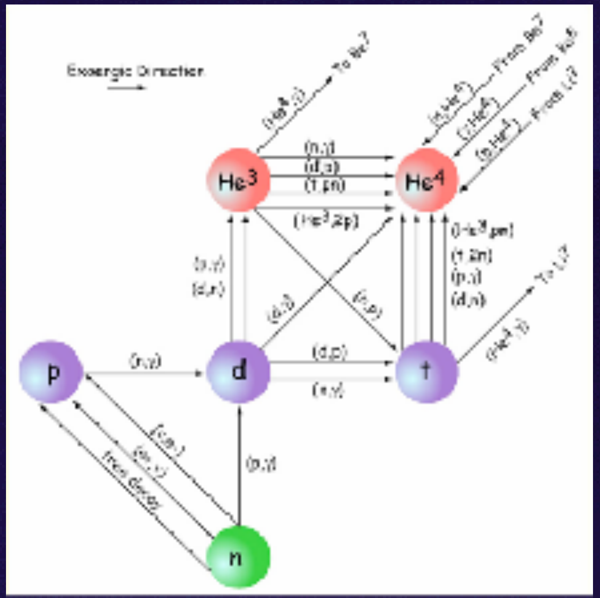
Big Bang Nucleosynthesis

Soup from the cosmic bowl

- In the ~1940s and 50s, George Gamow and his student Ralph Alpher showed that matter in an expanding, cooling universe would go through a series of **phase transitions**. The key to these transitions are the binding energies:
  - Nuclear binding energy (per nucleon): $\sim$ MeV
  - Atomic binding energy (per electron): $\sim$ eV
- So, at **very high energies** ($E \gtrsim$ MeV), nuclear matter would be all made up of protons and neutrons — no atomic nucleus (apart from a single p) would be able to survive!
- As the Universe **cools** down, it then becomes possible for neutrons to be captured into more massive nuclei, thus synthesizing the first atomic nuclei.
- However, some important details needed to be taken into account. First, note that **free neutrons** ($m_n = 0.9396$ GeV) **decai** into protons ($m_n = 0.9383$ GeV) with a half-life of about 880 s, therefore if this process of nucleosynthesis took too long, there would be no **neutrons left**!
- Second, at equilibrium the number of protons and neutrons is regulated by the process:

  $$p + \gamma \leftrightarrow n + e^+ + \nu_e$$

  Since there are many more photons than nucleons (by a factor of ~10^{10} !), it is relatively easy to maintain that equilibrium, even at relatively low energies.
- It turns out that there are neutrons around to keep nucleosynthesis going until the free energy available in the Universe is approximately 0.05 MeV — which takes place when the Universe is approximately 3 mins old. After that, nucleosynthesis is basically over and done.

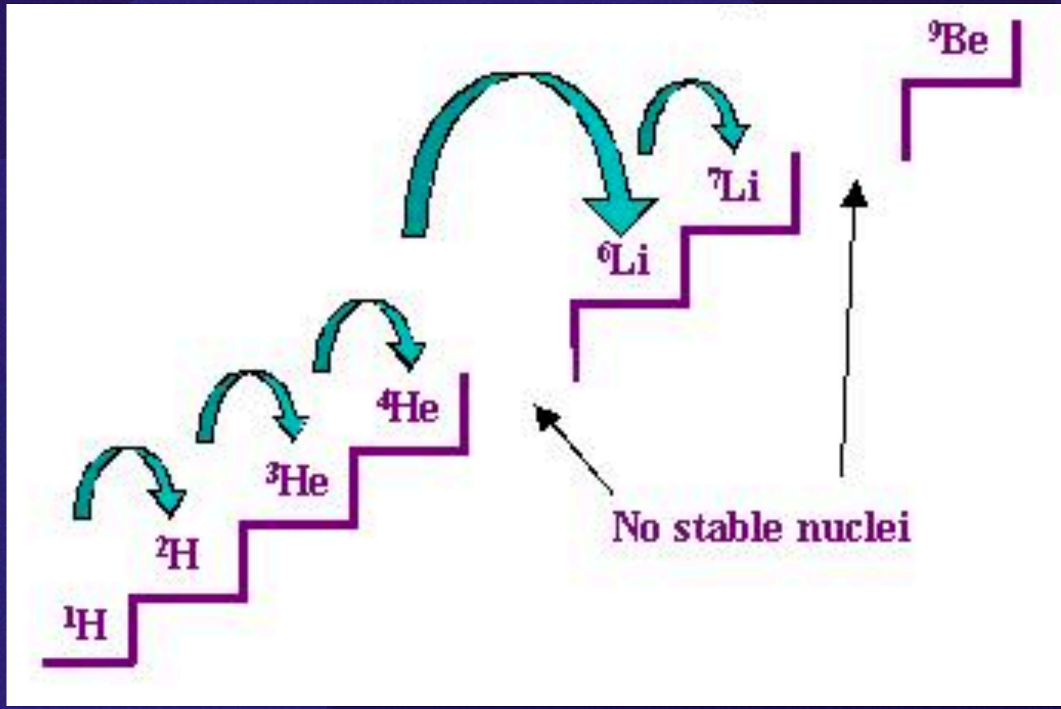


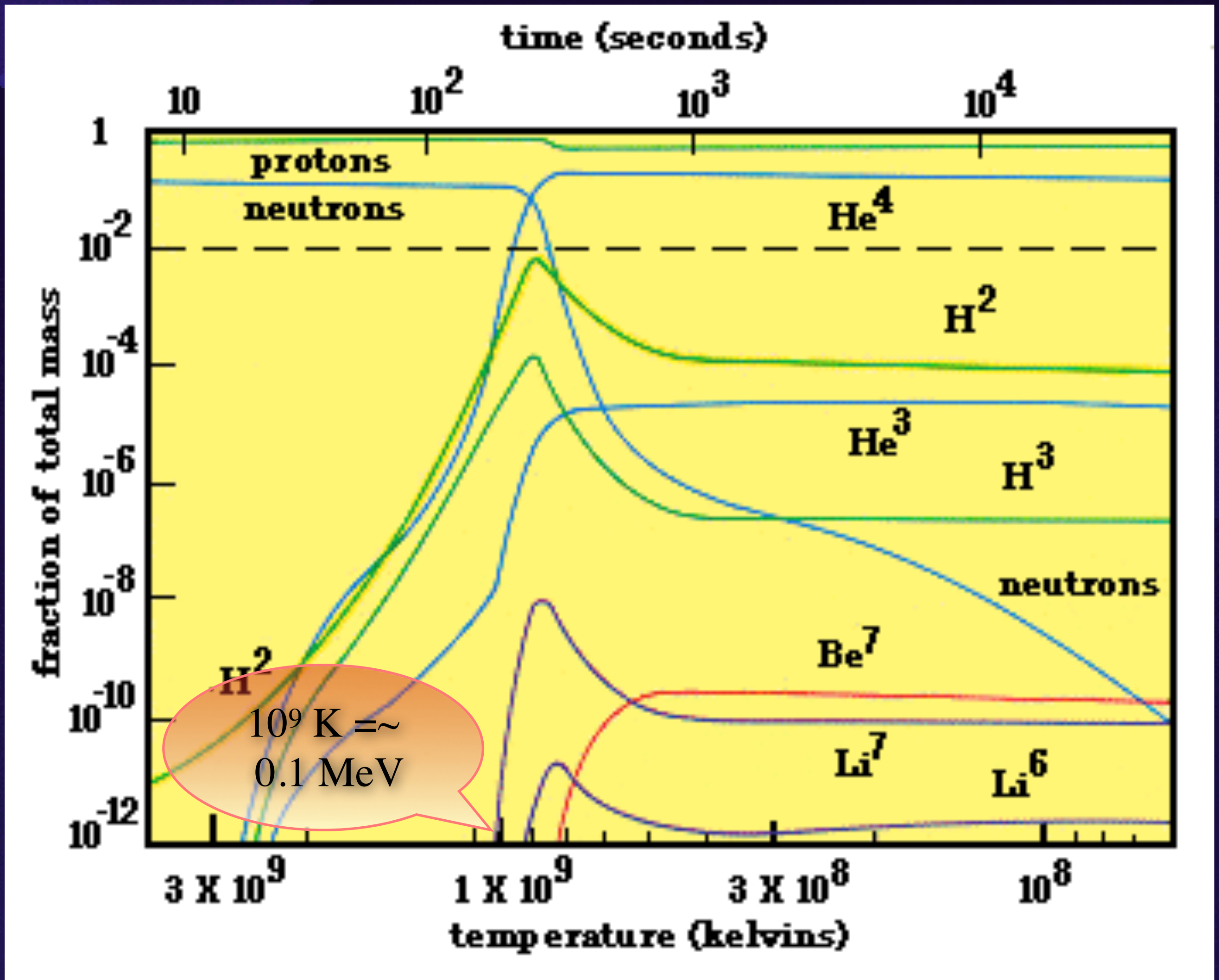
☞ Question: show that when the energy in the radiation was $E = k_B T = 0.05$ MeV, the age of the Universe was approximately 100s .

Big Bang Nucleosynthesis

Soup from the cosmic bowl

- The sequence of events can be summarized in the plots below:



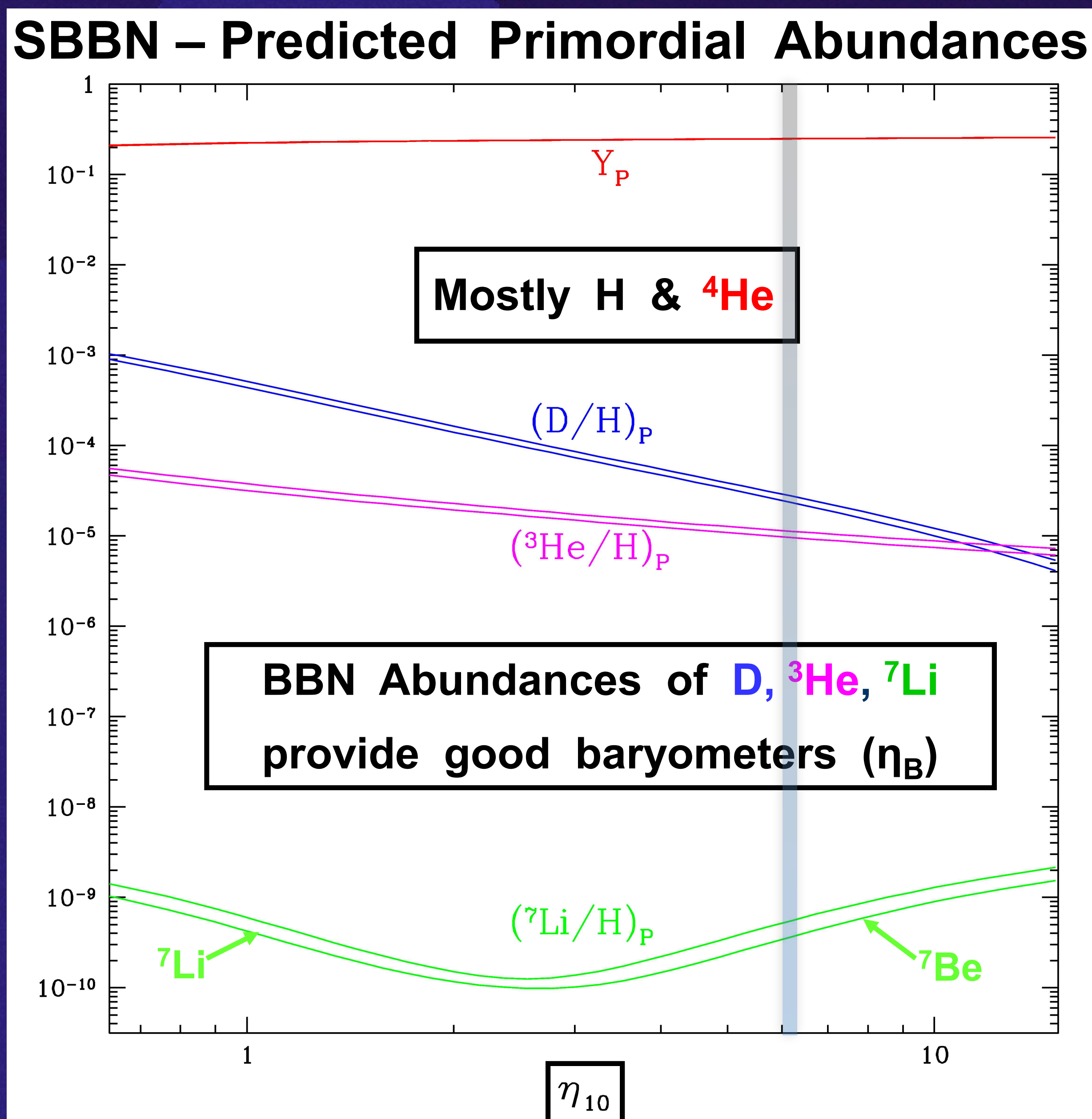


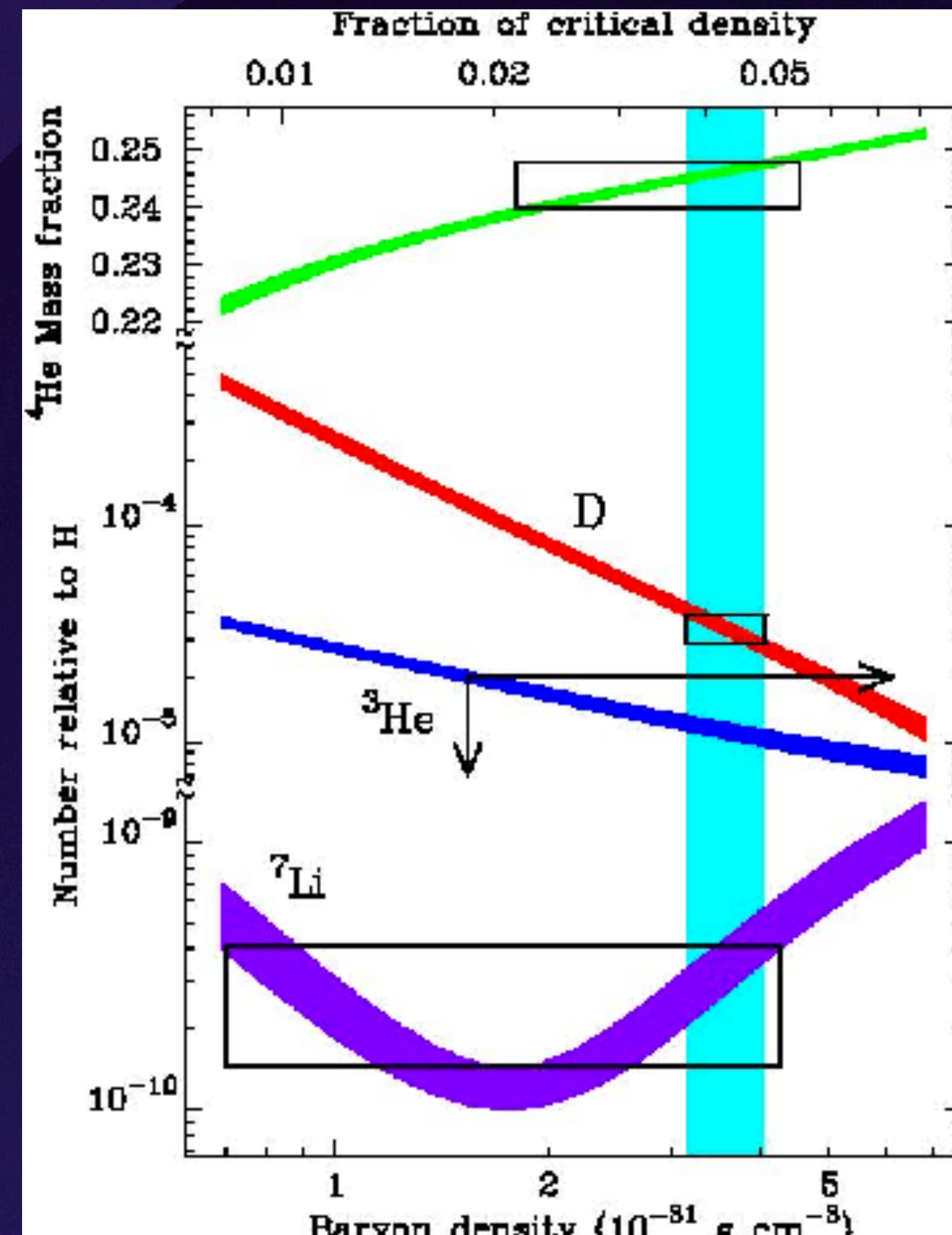
- The absence of stable nuclei with atomic number A=5 and A=8 acts as a bottleneck, shutting off the production of heavier nuclei.

For more details see, e.g., the textbook by V. Mukhanov, "Physical Cosmology", or the review paper by G. Steigman, astro-ph/0712.1100
Another nice review is the one by Tytler et al. (2000), found in the link: http://ned.ipac.caltech.edu/level5/Tytler2/Tytler_contents.html

Big Bang Nucleosynthesis

Relics from the past, today





$$\eta_B = \frac{n_B}{n_\gamma} \quad , \quad \eta_{10} = 10^{10}\,\eta_B \simeq 274\,\Omega_B\,h^2$$

$$\Omega_B h^2 = 0.0221 \pm 0.0003 \quad \Rightarrow \quad \eta_{10} = 274\,\Omega_B h^2 = 6.05 \pm 0.08$$

The cosmic microwave background

As far as the eye can see

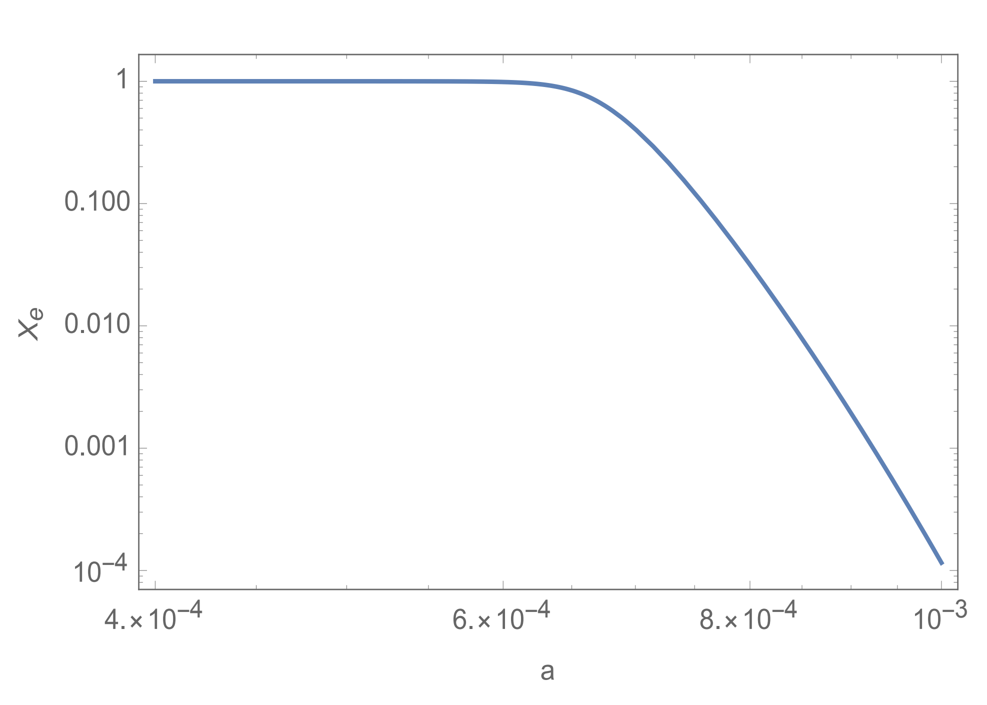
- Let's now travel back to the time when the mean temperature in the radiation field was still several thousands degrees, so the energy was $E = k_B T \gtrsim$ eV .
- At such high energies, no atom can remain neutral: all nuclei are ionized, all electrons are moving freely. Baryonic matter is in a **plasma state**, and is therefore **tightly coupled** to the photons.
- This plasma + radiation fluid is maintained until $t \sim 10^5$ yrs after the BB. Therefore, the typical time scale associated with the expansion of the Universe at that time is ~10^5 yrs — much, much greater than the typical time scales for the **collisions** between electrons, photons and nuclei.

- Hence we should have **thermodynamic equilibrium** at that time, even as the Universe "adiabatically" expands and cools down.
- In fact, we can use these equilibrium arguments to make a simple calculation for the fraction of ionized (free) electrons ($X_e = n_{fe}/n_e$), assuming that the only element is Hydrogen (a good approximation!). We then get that:

$$\frac{X_e^2}{1+X_e} = \frac{1}{n_b}\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{3/2} e^{-B/k_B T}$$

$k_B = 8.62 \times 10^{-5}\,\mathrm{eV\,K^{-1}}$

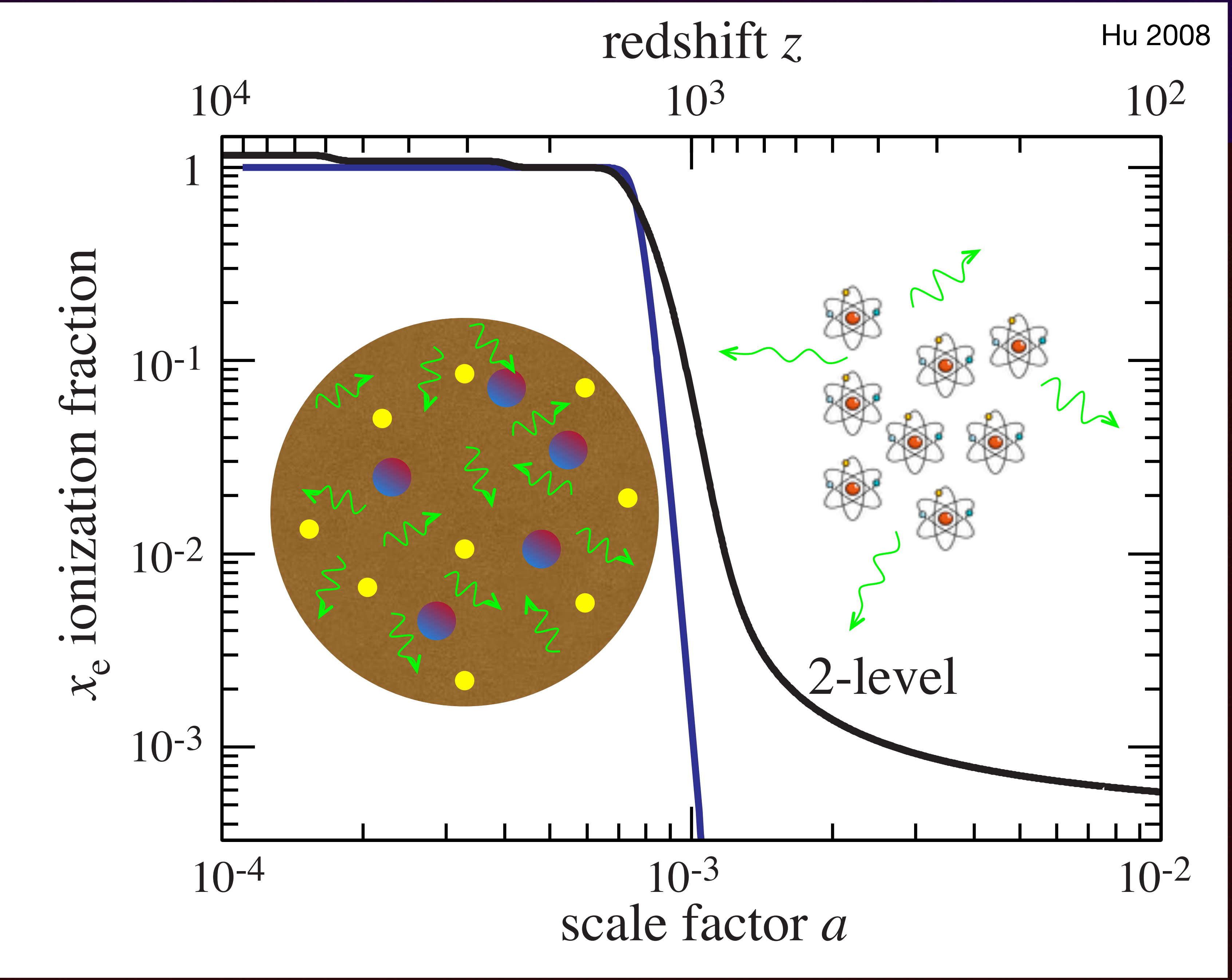
This is known as the **Saha equation**. Here, n_b is the number density of protons, m_e is the electron mass, and B is the binding energy for the H atom, 13.6 eV.



The cosmic microwave background

As far as the eye can see

- Here is a much more accurate calculation of the ionized fraction of electrons:

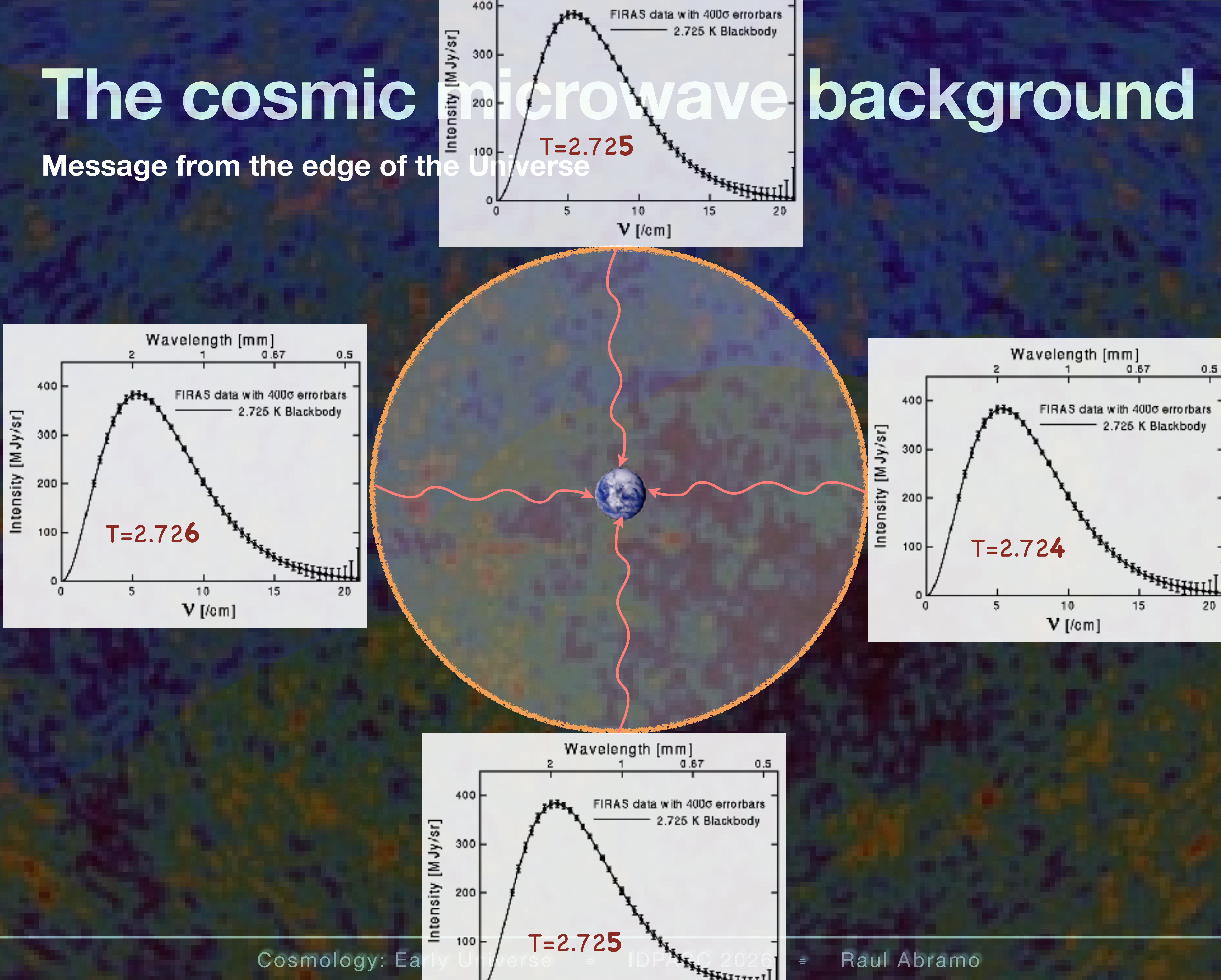


Interestingly, the energy below which the electrons "recombine" with protons is $k_B T \simeq 0.25$ eV, or approx. 3000 K!

☞ Question: provide a good argument for **why** this energy is so much **below** 13.6 eV

The cosmic microwave background

Message from the edge of the Universe



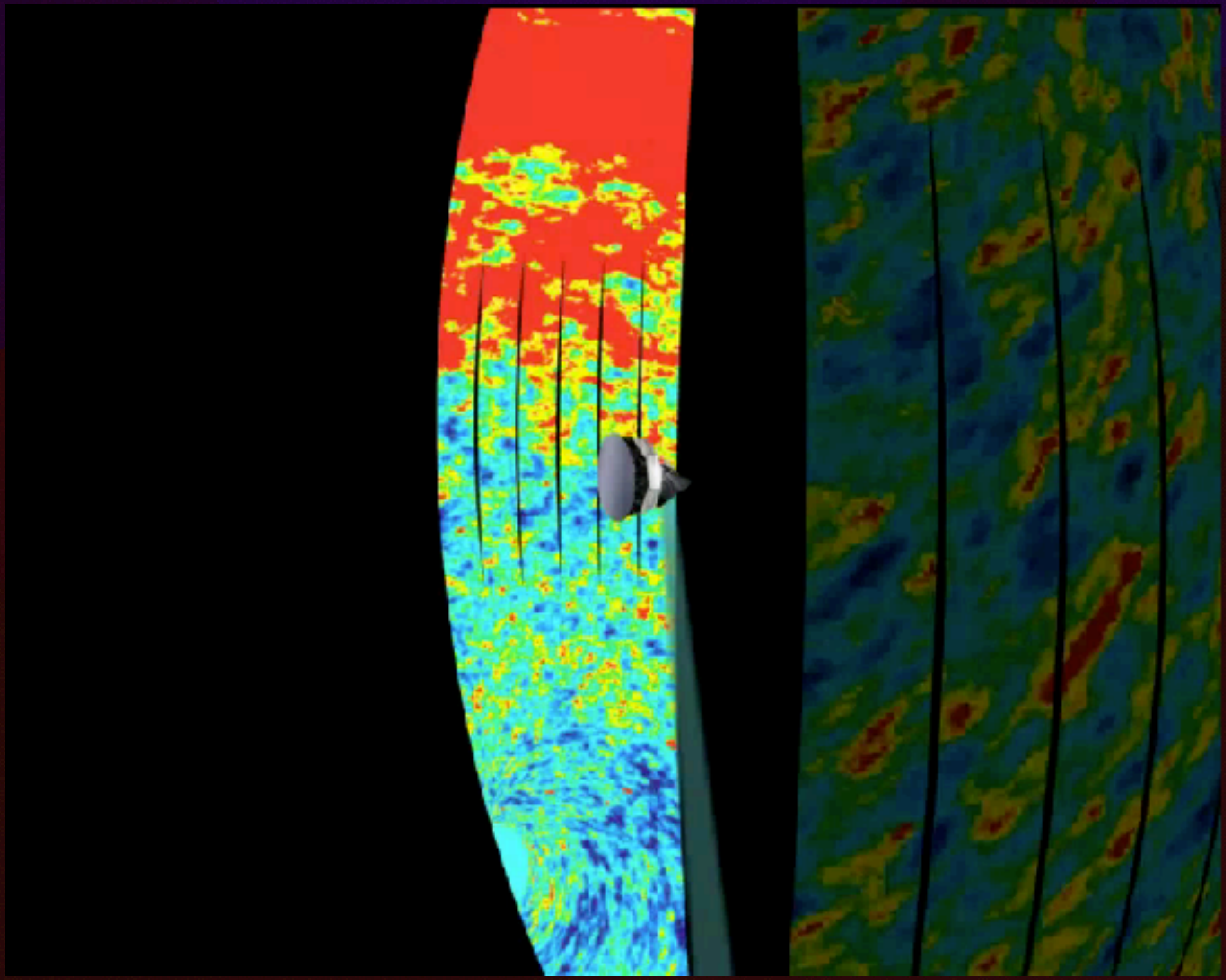
The cosmic microwave background

Message from the edge of the Universe

- The CMB tells us that the Universe was much simpler at $z \sim 10^3$, when $t \sim 4. \times 10^5$ yrs.
- Radiation and matter were in thermodynamic equilibrium
- Temperature and density fluctuations were very small:

$$\frac{\delta T}{T} \sim \frac{\delta \rho}{\rho} \sim 10^{-4} \; - 10^{-5}$$

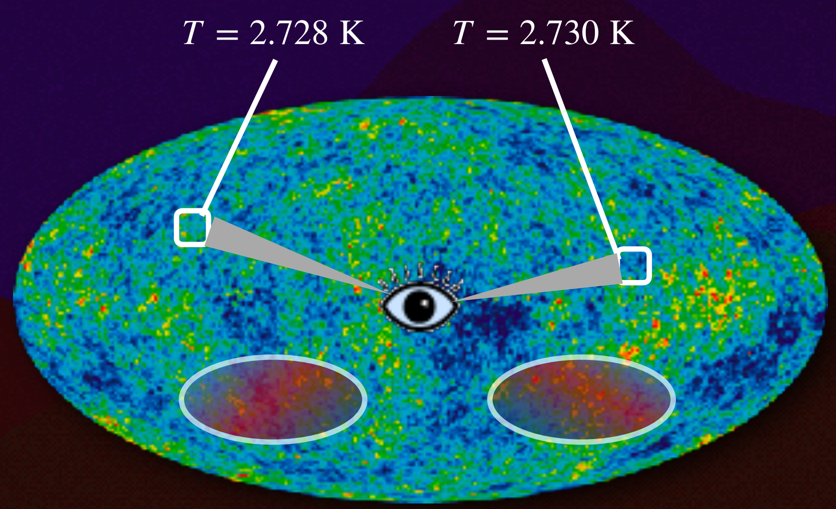
- The CMB is a picture of what the Universe looked like at that time: it gives us a sort of template for the initial conditions.

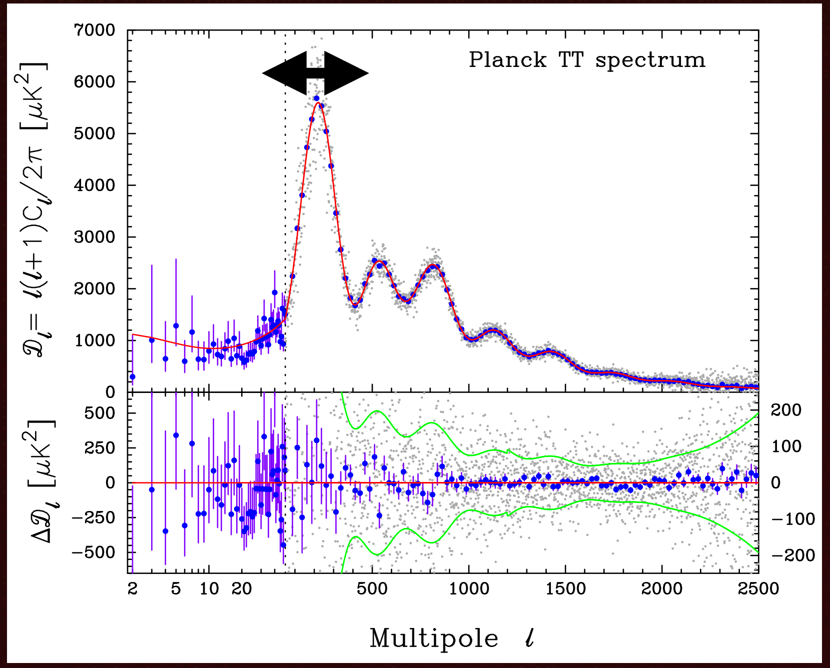


The cosmic microwave background

and the problems of homogeneity, horizon & flatness

- In particular, recent experiments such as Planck, ACT and others show that the initial density field (at $t = 400{,}000$ yrs , $z \simeq 1100$) shows some remarkable features:
    - The density and temperature fluctuations are **tiny**: $\delta\rho/\rho \sim \delta T/T \sim 10^{-5}$ ("homogeneity")
    - Not only that, but the **pattern** of the anisotropies of the CMB is exactly ***the same***, no matter ***where*** in the sky we observe them ("horizon")
    - That pattern is rather curious: a **Gaussian** random field with a **nearly flat spectrum** ("initial conditions")
    - Spatial **curvature is negligible**: even today it's **less than 1%** — and it was vanishingly small at t_{dec} ("flatness")
- None of these features can be **predicted** or **explained** by the "old" hot Big Bang model: in that scenario they must be **encoded in the initial state** of the Universe.





The hot Big Bang model

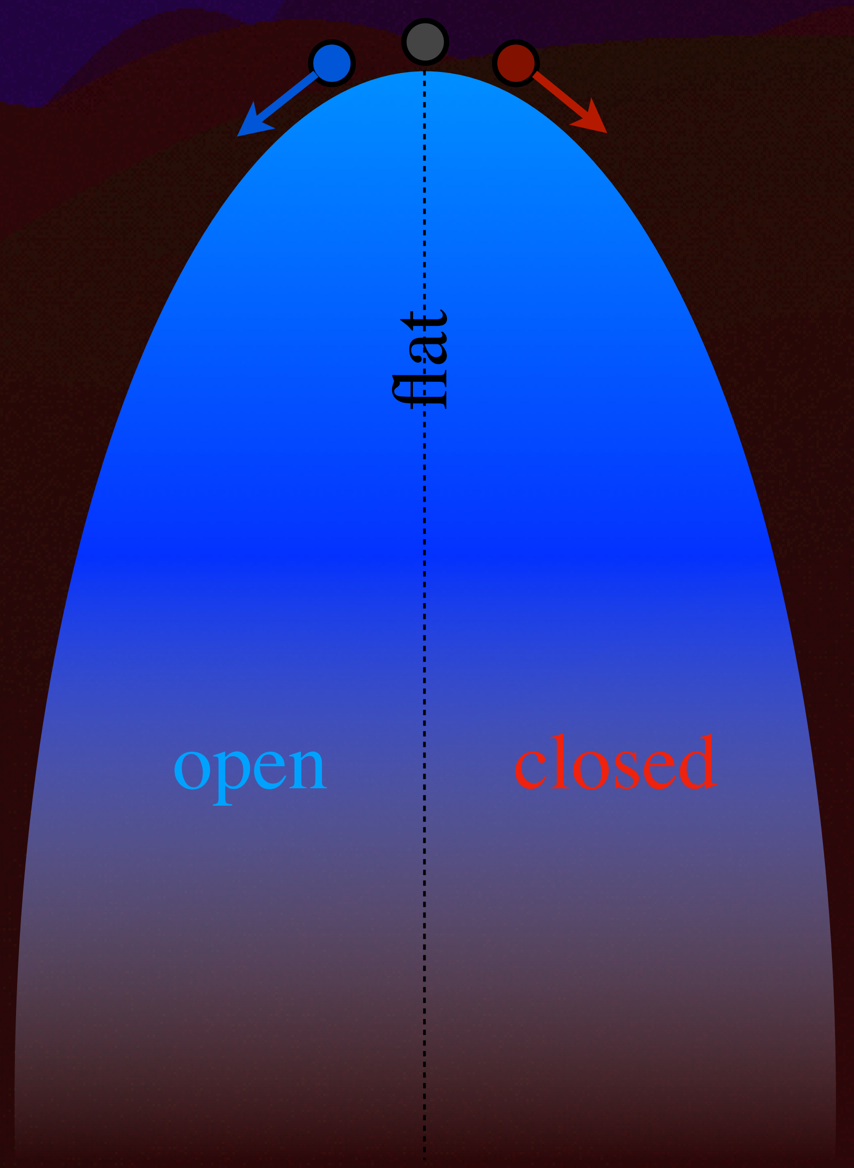
Spatial curvature and acceleration

- Let's start with the issue of spatial curvature: why is it so small?

$$\frac{d\ln\Omega_K}{d\ln a} = 8\pi G\frac{\rho+3p}{H^2} \sim -\frac{\ddot{a}}{a}\frac{1}{H^2}$$

- Therefore, if the Universe is **deccelerating** ($\ddot{a}<0$), the importance of **spatial curvature grows** — fast, usually like a positive power law of the scale factor.

- However, if the Universe is **accelerating** ($\ddot{a}>0$), then **spatial curvature is suppressed**. So, if the Universe undergoes a prolonged or particularly strong **period of accelerated expansion**, then we can make $\Omega_K \to 0$.



The hot Big Bang model

Light cones and causal structure of spacetime

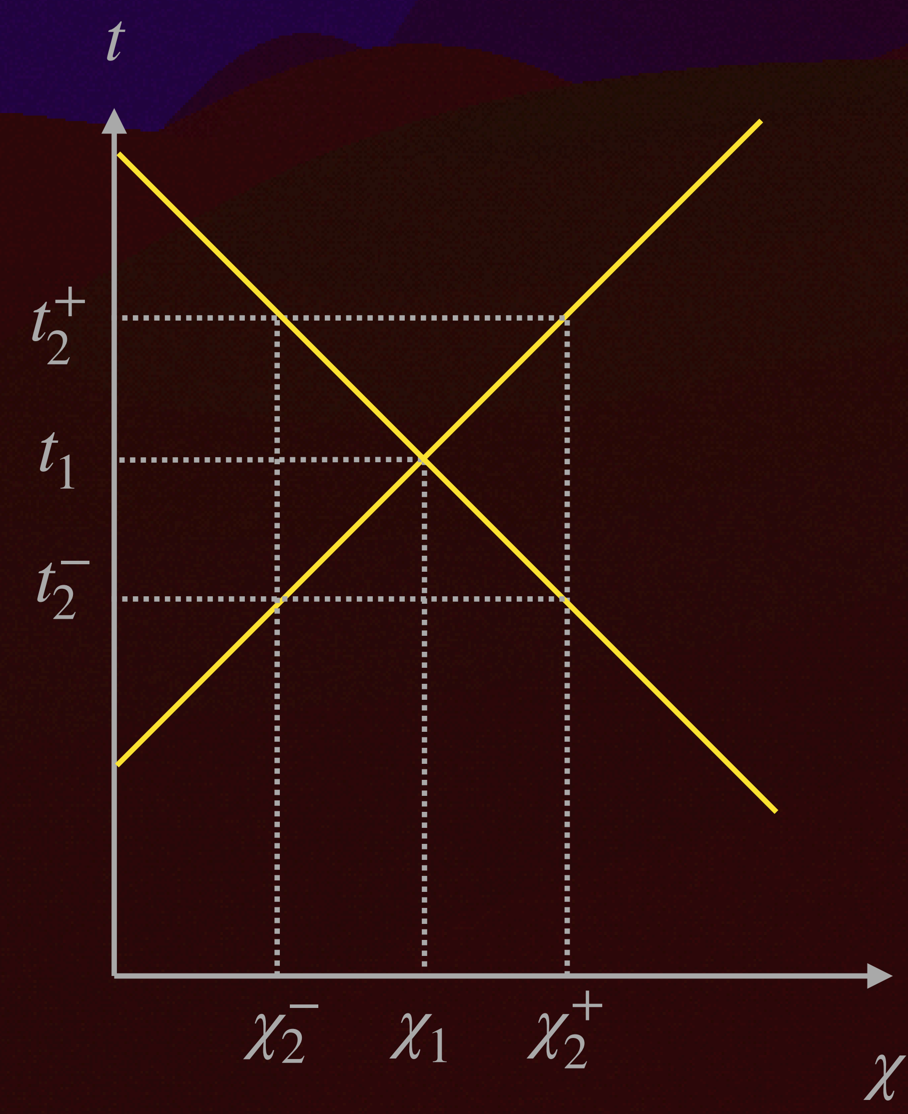
- For the **homogeneity** and **horizon** problems are closely connected. In order to tackle them, we should first address the issue of the **light cone** in a FLRW Universe.

- Let's start with the simplest possible case: no expansion, $a \to 1$ $(\alpha = 0)$ — i.e., a static Universe with the **Minkowski** metric.

- The radial null rays are then given by:

$$\chi_2 - \chi_1 = \pm \int_{t_1}^{t_2} \frac{dt}{a(t)} = \pm \int_{t_1}^{t_2} dt = \pm (t_2 - t_1)$$

- This gives then the **light cone** (LC) in Minkowski spacetime.



The hot Big Bang model

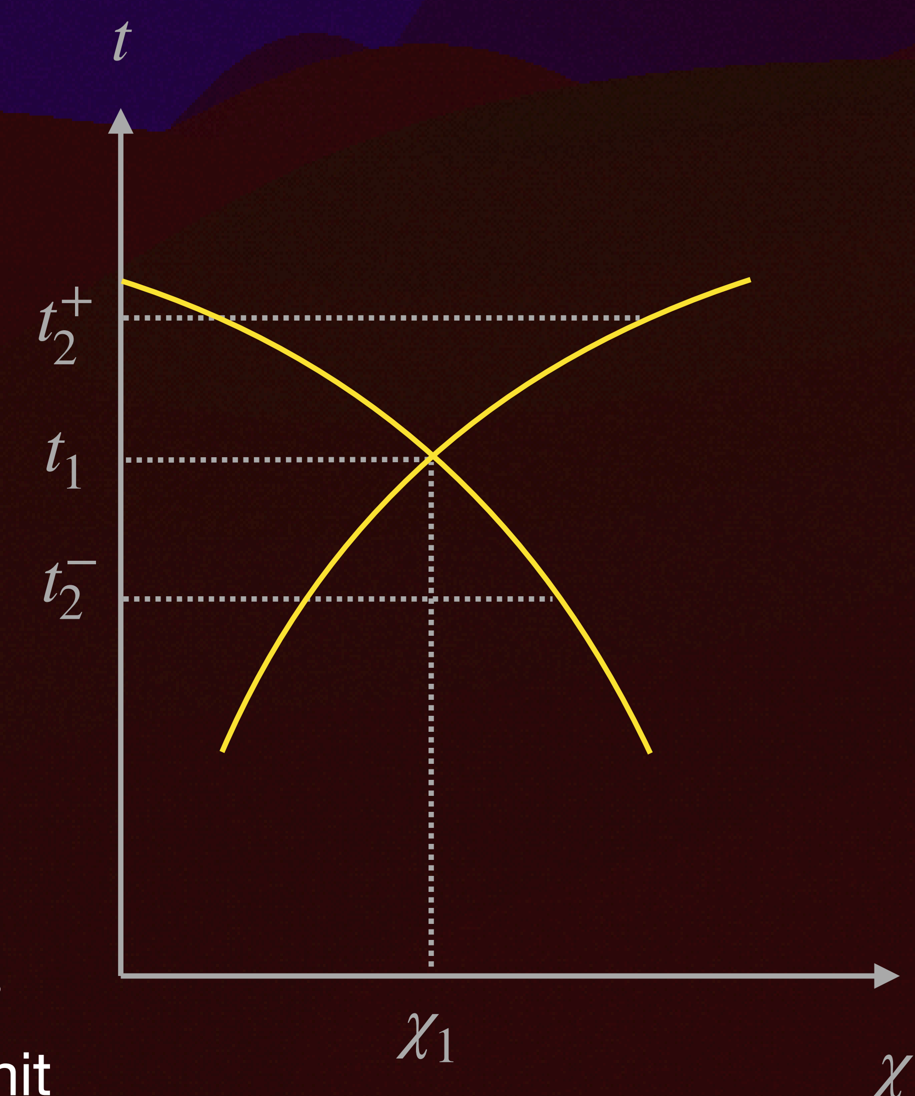
Light cones and causal structure of spacetime

- Now let's assume an Universe dominated by normal matter ("dust"), i.e., $\alpha = 2/3$ — a **deccelerating** Universe

- Using $a = (t/t_0)^{2/3}$ we get the LC:

$$\chi_2 - \chi_1 = \pm \int_{t_1}^{t_2} \frac{dt}{a(t)} = \pm\, t_0 \int_{t_1}^{t_2} \frac{dt}{t_0} \left(\frac{t}{t_0}\right)^{-2/3}$$

$$= \pm\, t_0\, 3 \left[\left(\frac{t_2}{t_0}\right)^{1/3} - \left(\frac{t_1}{t_0}\right)^{1/3} \right]$$



- So, the LC becomes "warped" **downwards** — i.e., towards the **past**.

- Notice, however, that at the "center" the LC has a $\pm 45^\circ$ inclination, as it should! (Check this out by taking the limit $t_2 \to t_1$.)

The hot Big Bang model

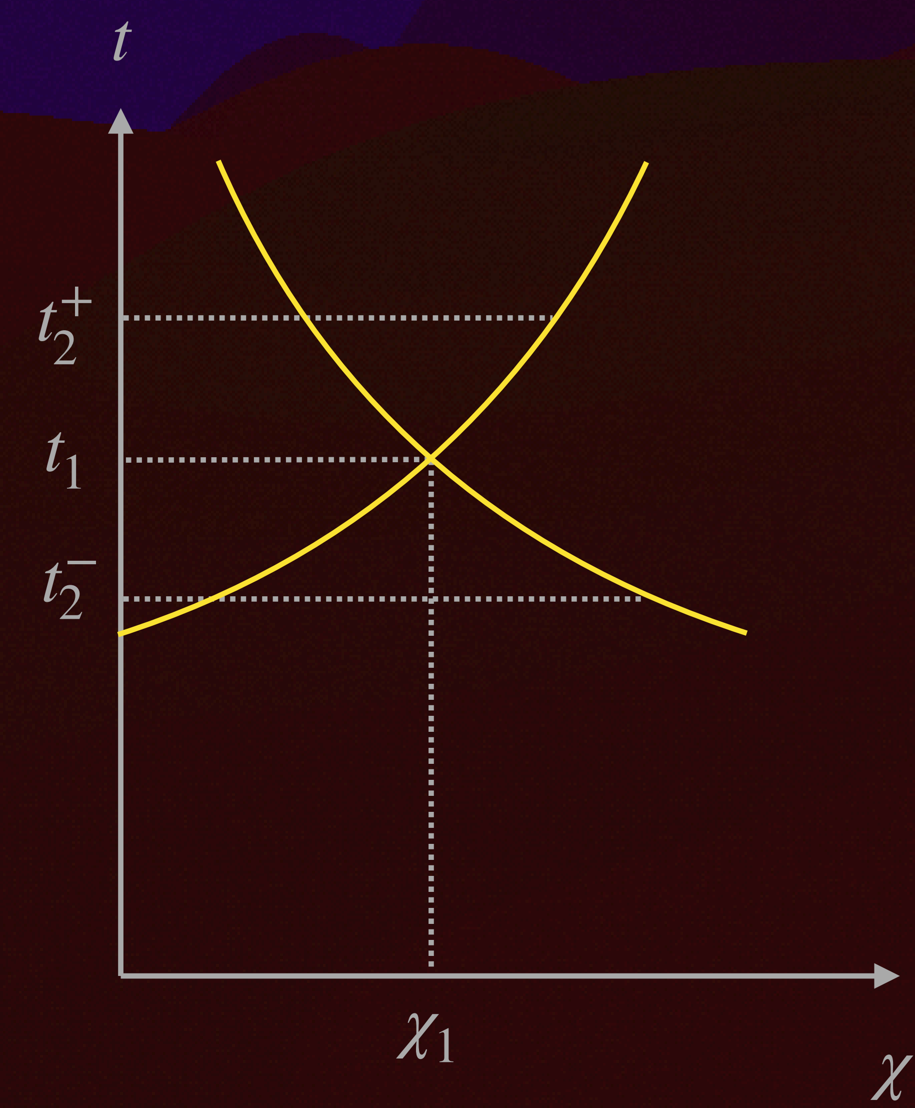
Light cones and causal structure of spacetime

- Now let's take a rather different expanding Universe, with a power-law that leads to **accelerated** expansion, $\alpha = 2$.
- Using $a = (t/t_0)^2$ we get the LC:

$$\chi_2 - \chi_1 = \pm \int_{t_1}^{t_2} \frac{dt}{a(t)} = \pm\, t_0 \int_{t_1}^{t_2} \frac{dt}{t_0} \left(\frac{t}{t_0}\right)^{-2}$$

$$= \pm\, t_0 \,(-1) \left[\left(\frac{t_2}{t_0}\right)^{-1} - \left(\frac{t_1}{t_0}\right)^{-1} \right]$$

$$= \pm\, t_0 \left[\frac{t_0}{t_1} - \frac{t_0}{t_2} \right]$$



- The LC is again warped, but now **upwards** — towards the **future**.
- In the case $\alpha \to \infty$ the power-law becomes an exponential, but qualitatively the LC is very similar to this figure.

The hot Big Bang model

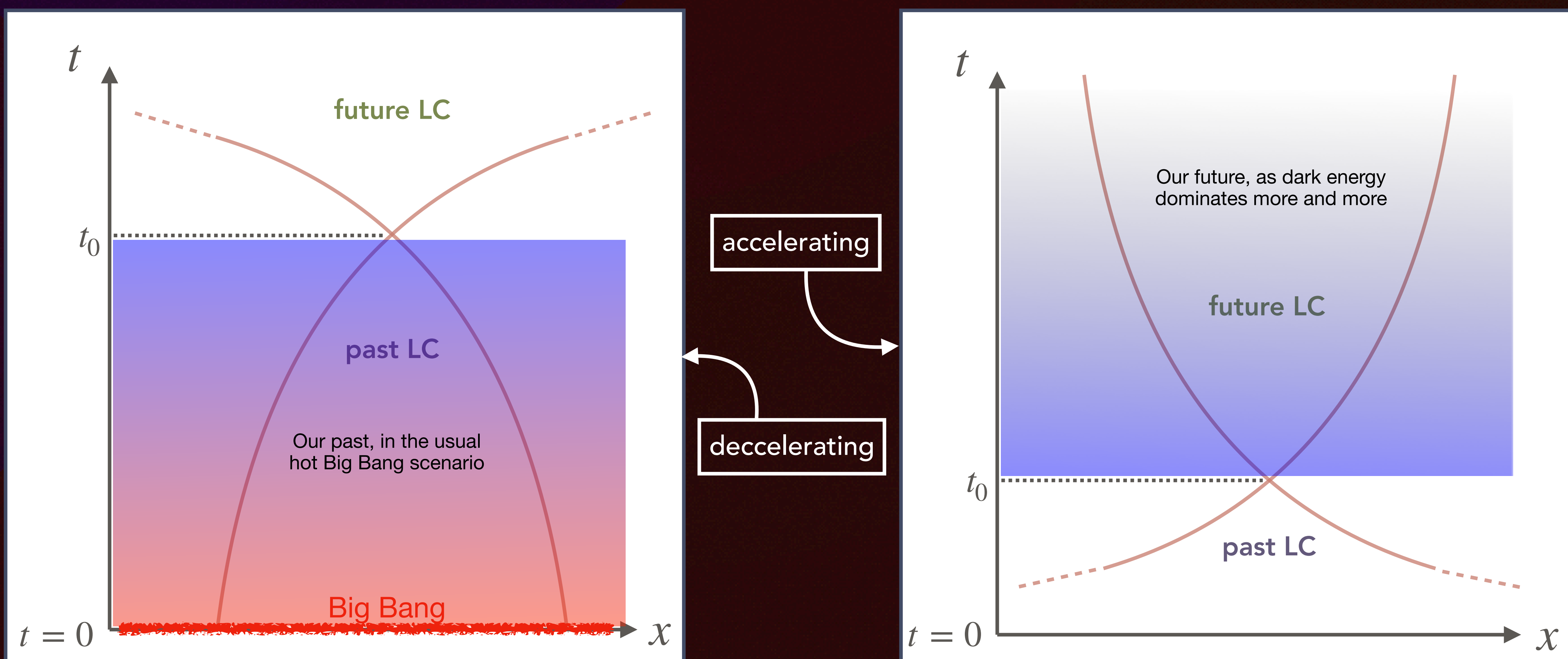
Light cones and causal structure of spacetime

- Hence, for the general case $a = (t/t_0)^\alpha$ we have:

$$\chi_2 - \chi_1 = \pm\, t_0 \frac{1}{1-\alpha}\left[\left(\frac{t_2}{t_0}\right)^{1-\alpha} - \left(\frac{t_1}{t_0}\right)^{1-\alpha}\right] \quad ,$$

where deceleration corresponds to $\alpha < 1$, while acceleration is $\alpha > 1$.

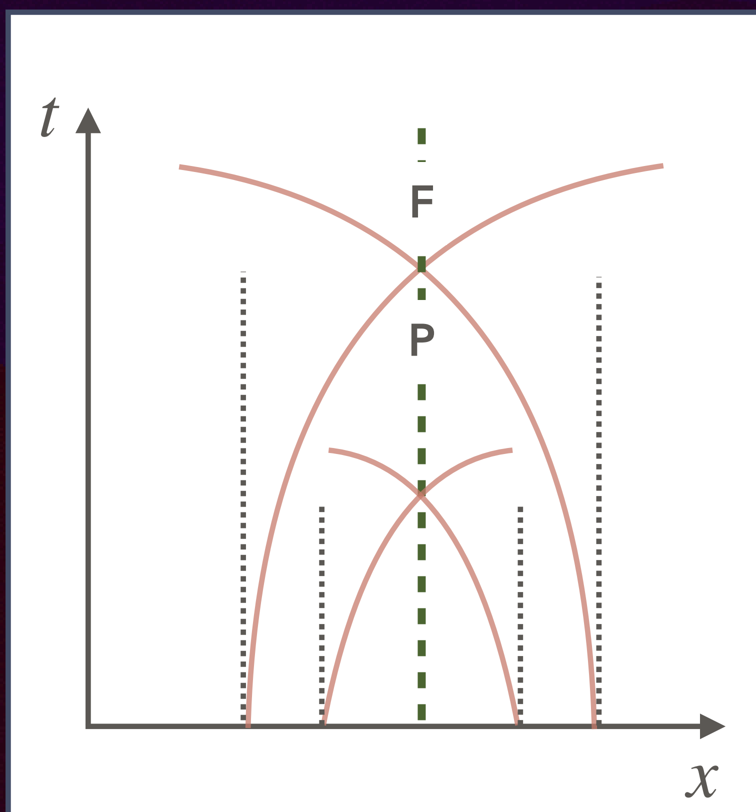
- If we continue the LC to the past and to the future, we have the two qualitative pictures below:



The hot Big Bang model

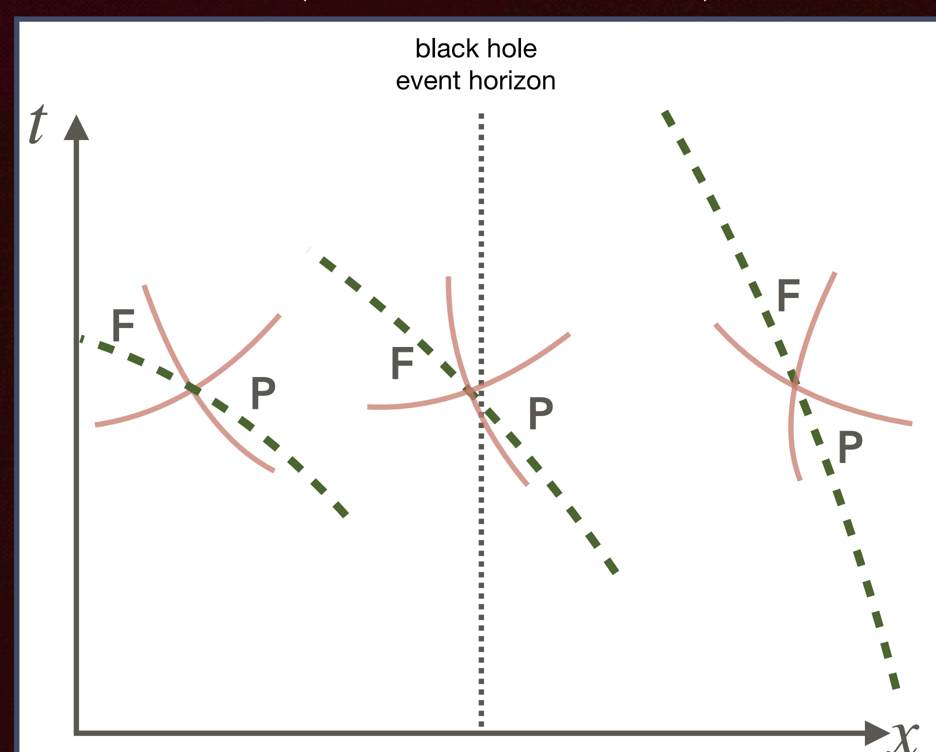
The light cone and cosmological horizons

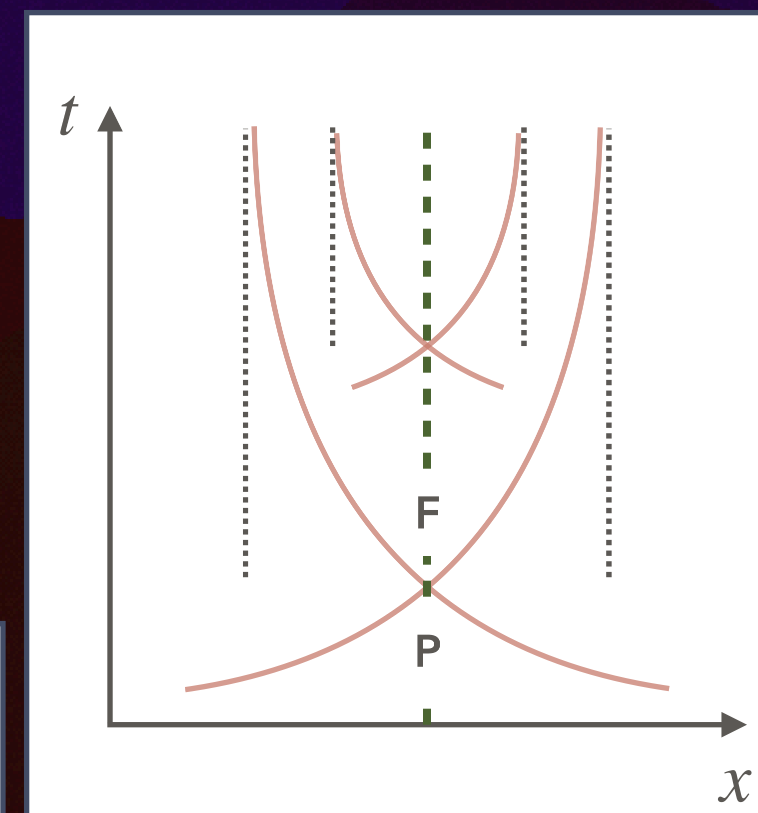
- There is a remarkable pattern to these horizons:



Deccelerating Universe:
particle horizon
time-like horizon
(**past** LC is limited)

Schwarzschild:
event horizon
space-like horizon
(future LC is limited)





Accelerating Universe:
event horizon
time-like horizon
(**future** LC is limited)

The hot Big Bang model

The light cone and cosmological horizons

- With the decceleration scenario in mind (i.e., the traditional hot Big Bang), let's revisit the **homogeneity problem**.
- When we observe light from the CMB that arrives from two opposite directions, we get a picture from two regions that, before that, were never in any kind of **causal contact**.
- In fact, the past LCs of those two regions were separated by a huge spatial distance. Let's take a look at all the involved distances.
- If we assume that the Universe was matter-dominated for most of its history, we obtain that the radius from us (here, now, $t_0 = 13.8$ Gyr) to the surface of last scattering of the CMB (t_{dec}) is:

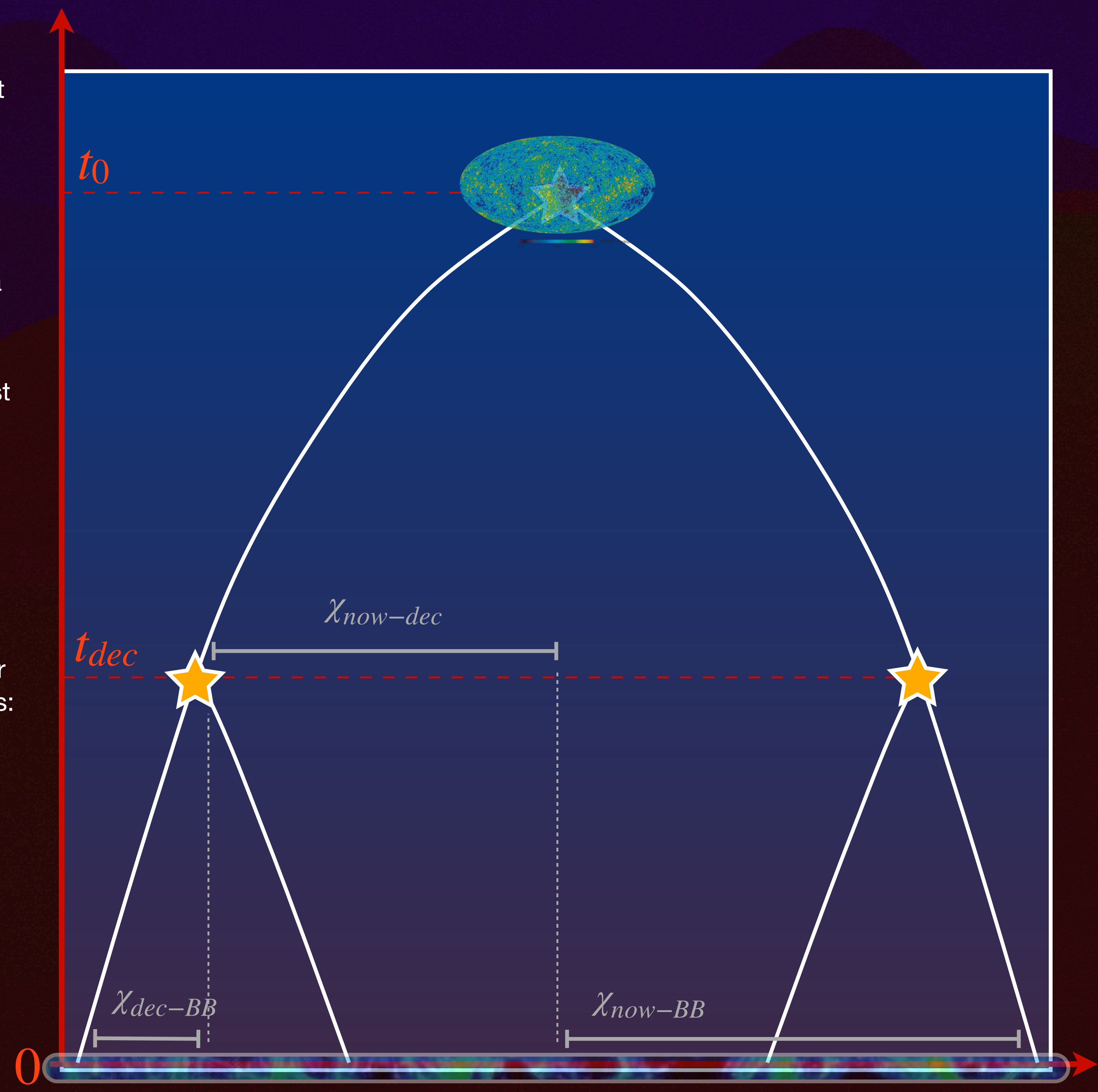
$$\chi_{now-dec} = t_0\, 3 \left[\left(\frac{t_0}{t_0} \right)^{1/3} - \left(\frac{t_{dec}}{t_0} \right)^{1/3} \right] \simeq 3 t_0$$

- The **maximal distance** that light traveled to/from any observer at t_{dec} since the Big Bang ($t = 0$), the particle horizon at t_{dec}, is:

$$\chi_{dec-BB} = t_{dec}\, 3 \left[\left(\frac{t_{dec}}{t_{dec}} \right)^{1/3} - \left(\frac{0}{t_{dec}} \right)^{1/3} \right] = 3 t_{dec}$$

- Finally, the **maximal distance** that light traveled to/from us, here & now, since the BB, is very nearly the same as the one we computed above for the last scattering surface, i.e., the particle horizon today, is:

$$\chi_{now-BB} = \simeq 3 t_0$$



The hot Big Bang model

The light cone and cosmological horizons

- The previous calculation shows that the **entire past** of the region we observe on the “left" in the CMB was **never even remotely in causal contact** with the region we observe on the "right".

- In fact, the **number of causally disconnected regions** with radius

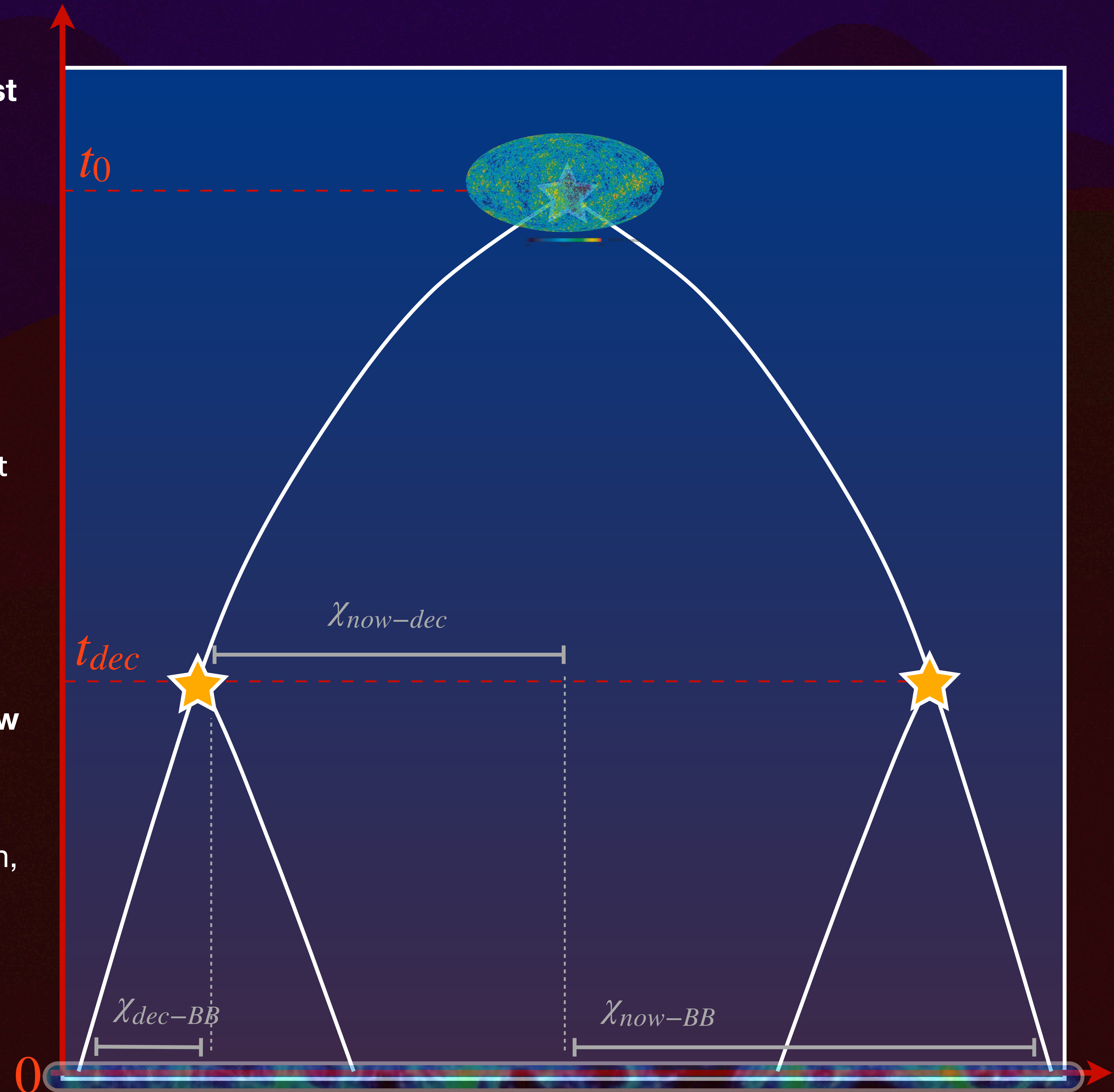
$$\chi_{dec-BB} = 3t_{dec}$$

between the points on the left and on the right is approximately:

$$\frac{\chi_{now-BB}}{\chi_{dec-BB}} \simeq \frac{t_0}{t_{dec}} \simeq \frac{13.8 \times 10^9\,\mathrm{yr}}{4. \times 10^5\,\mathrm{yr}} = 3.45 \times 10^4$$

- This sums up nicely the homogeneity problem: **how could** all those regions, which in the hot Big Bang scenario were **completely independent**, all have almost exactly **the same properties** — density, temperature, relative amounts of baryons, radiation, DM — as well as **the same pattern of small fluctuations**?

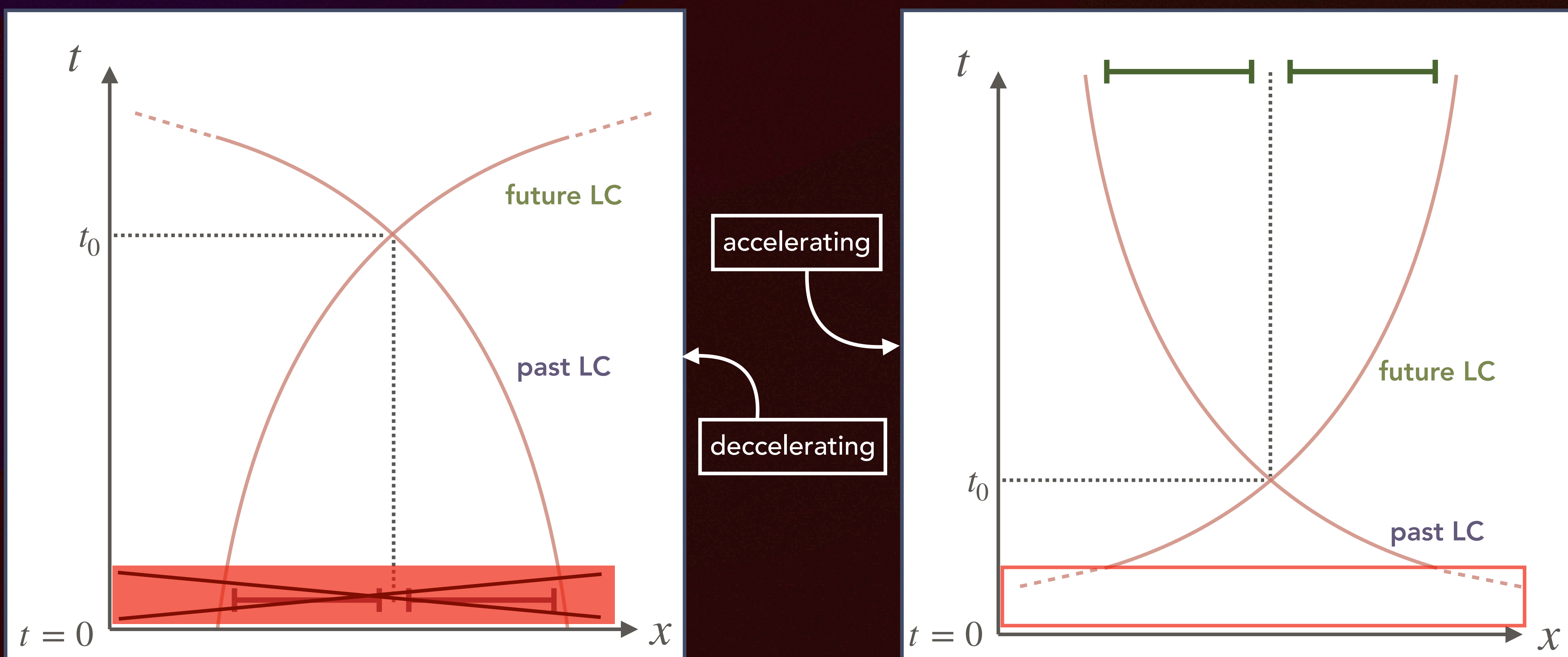
- Any solution to this problem **must address the issue of causality** and the **past LC**.



The hot Big Bang model

The light cone and cosmological horizons

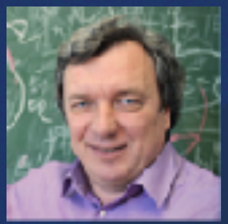
- Now remember what happens when in an **accelerating** Universe: the past LC is **blown up**, and a sufficiently long period of acceleration can dramatically increase the reach of the causal connection between regions that may seem far apart today.
- In fact, between 1978 and 1982 Alexei Starobinski, Alan Guth, Viatcheslav Mukhanov, Andrei Linde and others found that an **early period** of accelerated expansion (***inflation***) could explain many of these problems of the hot Big Bang scenario:
  - The curvature problem
  - The homogeneity problem
  - The problem of the origin of the density/temperature fluctuations (initial conditions)



Inflation

Once upon a time, before the Big Bang…

A. Starobinsky

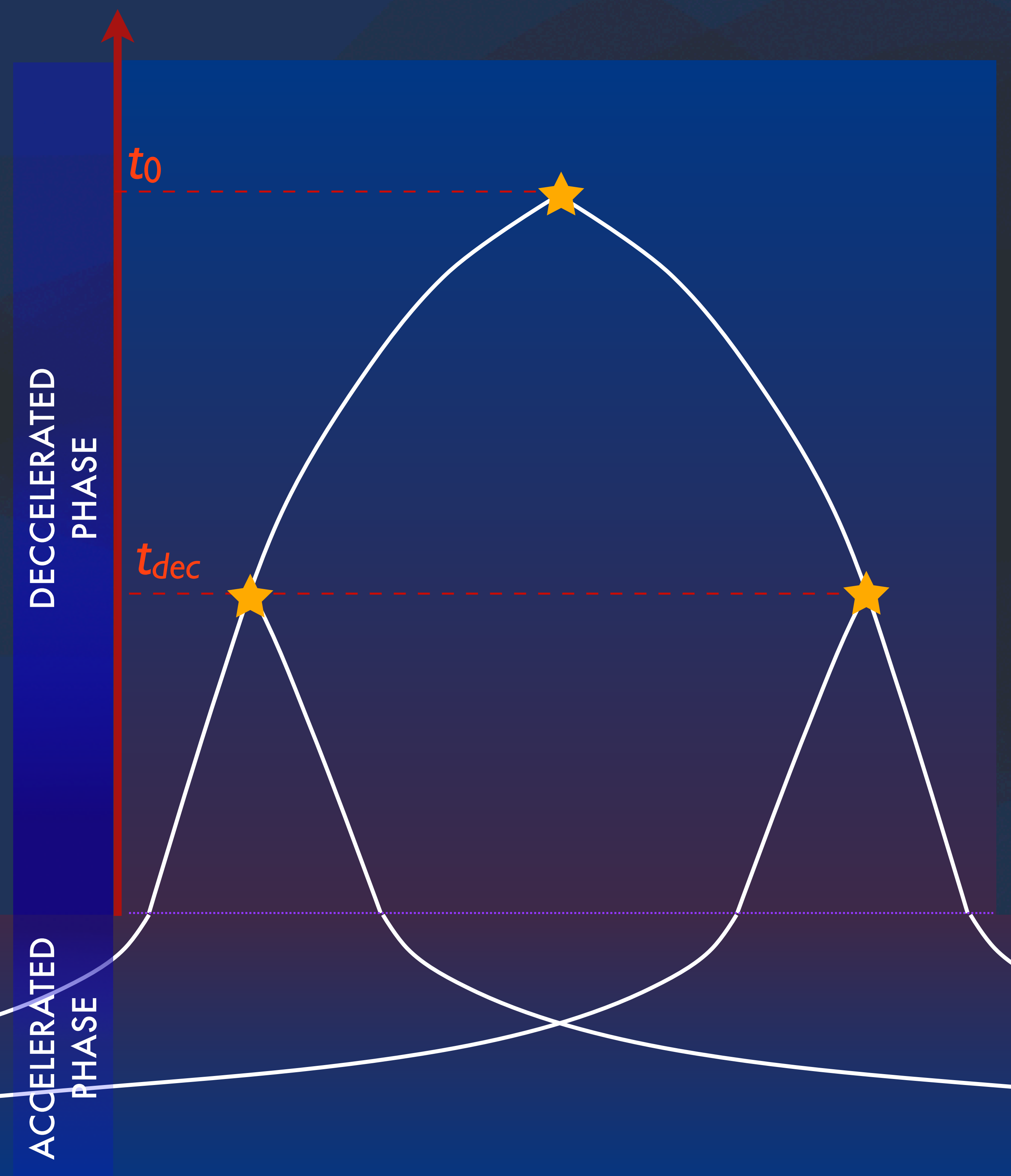


V. Mukhanov

A. Linde

A. Guth

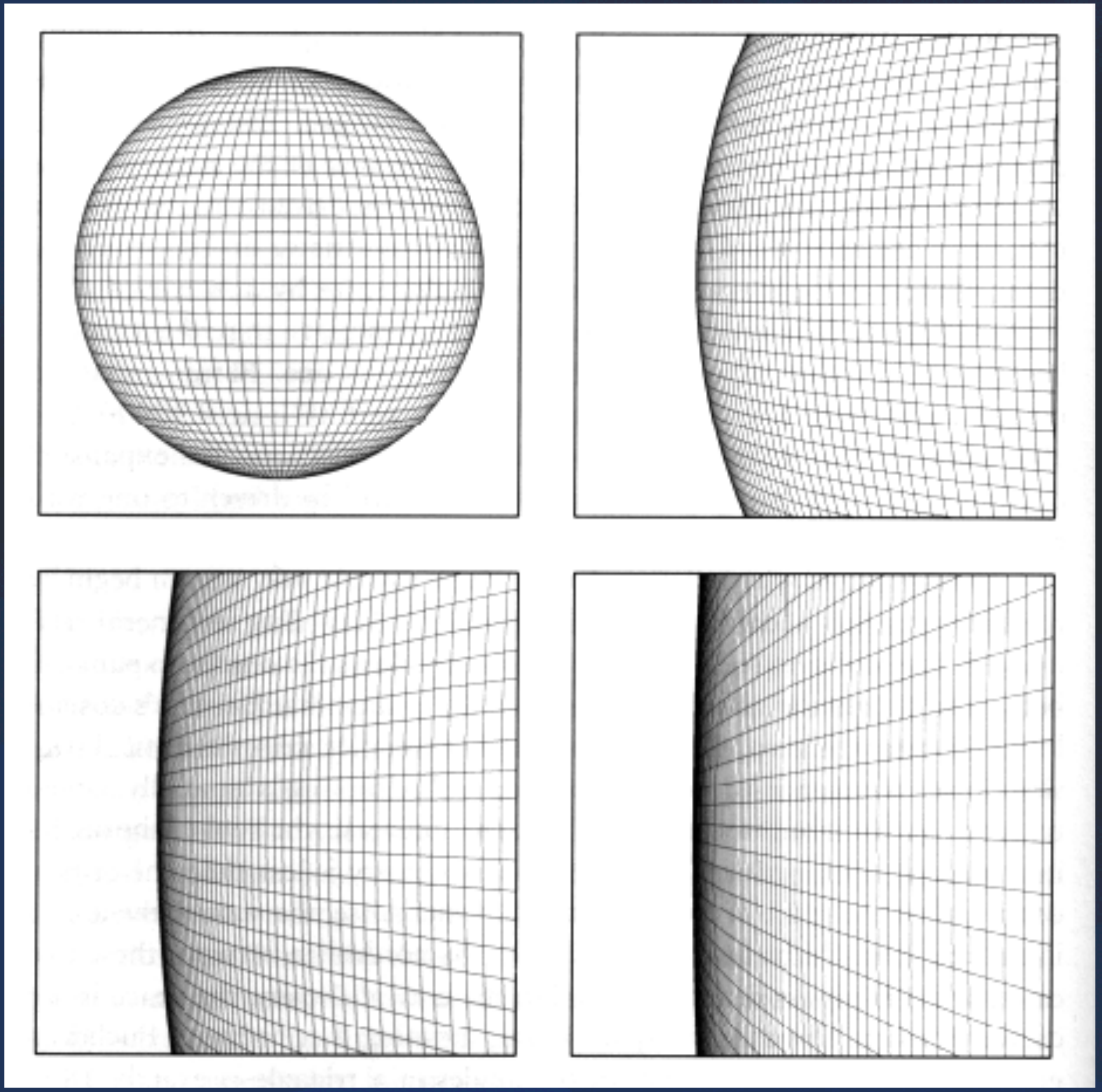
- The basic idea of inflation is very simple — and most models are in fact extremely simple and robust.
- Suppose that in the very, **very early Universe**, the some form of matter/energy led to an **accelerated expansion**. In that case, the past LC would have behaved in the form shown in the Figure.
- With this early (but rather short) burst of accelerated expansion (***inflation***), the past LC **expands** dramatically. We can think of this as being the result of the fact that, with accelerated expansion, the speed of the expansion in the beginning is much slower than if we had deccelerated expansion.
- Another way to think about this is to consider the size of a small, causally connected region in those early instants: inflation quickly blows up that region, which then becomes extremely large.
- With inflation lasting sufficiently long, we can easily solve the **homogeneity problem**.



Inflation

Once upon a time, before the Big Bang…

- Another effect of inflation is that it **suppresses spatial curvature**.
- Since the energy density during inflation decays very slowly, the accelerated expansion means that the density-like term K/a^2 of the Friedmann equation decays (almost exponentially).
- At the end of the inflationary era the contribution of curvature has decayed so much (sometimes by a fator $> e^{70}$) that spatial curvature is completely negligible — and it remains irrelevant until now, and almost certainly well into the future.



Inflation

The ultimate free lunch

- Ok, but we have not explained how inflation is able to produce those **initial density fluctuations.**

- In fact, with very **powerful** yet **simple** arguments, in the ~1970s Harrison and Zel'dovich argued that the initial spectrum of density fluctuations should be very **nearly scale invariant**.

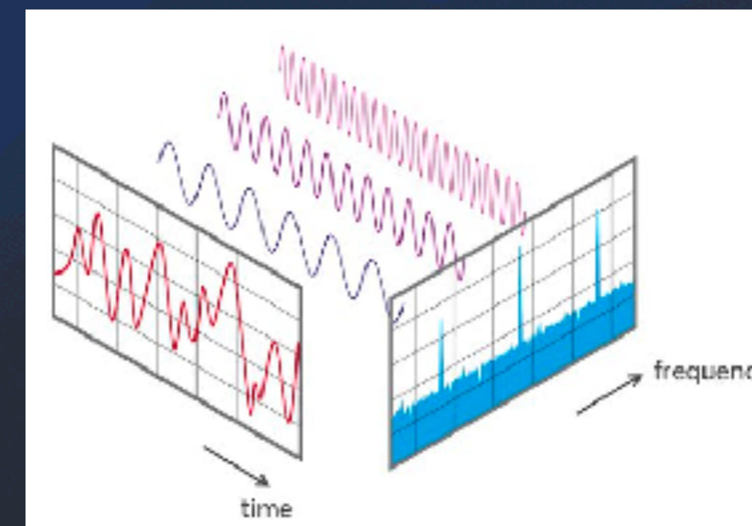
- First, let's remember that the spectrum gives us the ***amplitudes*** of the Fourier modes:

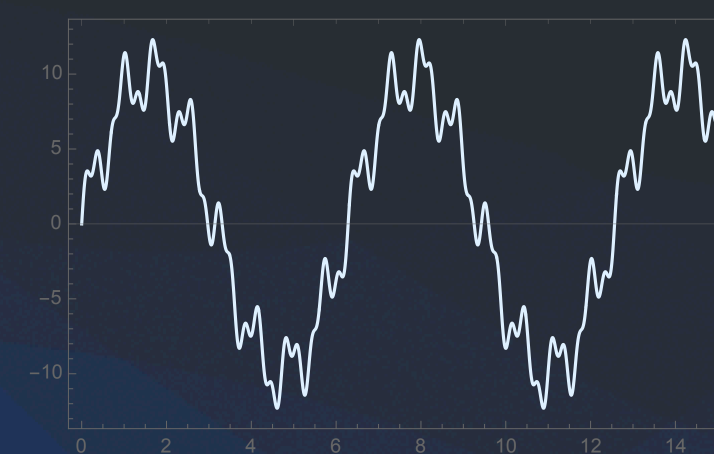
$$f(t) \quad \to \quad \tilde{f}(\omega) = \int dt\, e^{i\omega t} f(t) \quad \to \quad f(t) = \int \frac{d\omega}{2\pi}\, e^{-i\omega t}\, \tilde{f}(t)$$

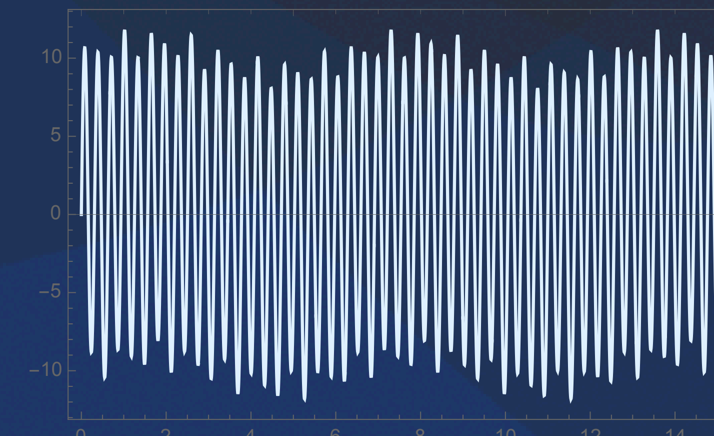
$$P_f(\omega) = |\tilde{f}(\omega)|^2$$

- When we put a lot of power in the low-frequency modes, the modulations of the function happen mostly at long time scales.

- When we put most of the power on high-frequency modes, the modulations of the function are mostly on small time scales.



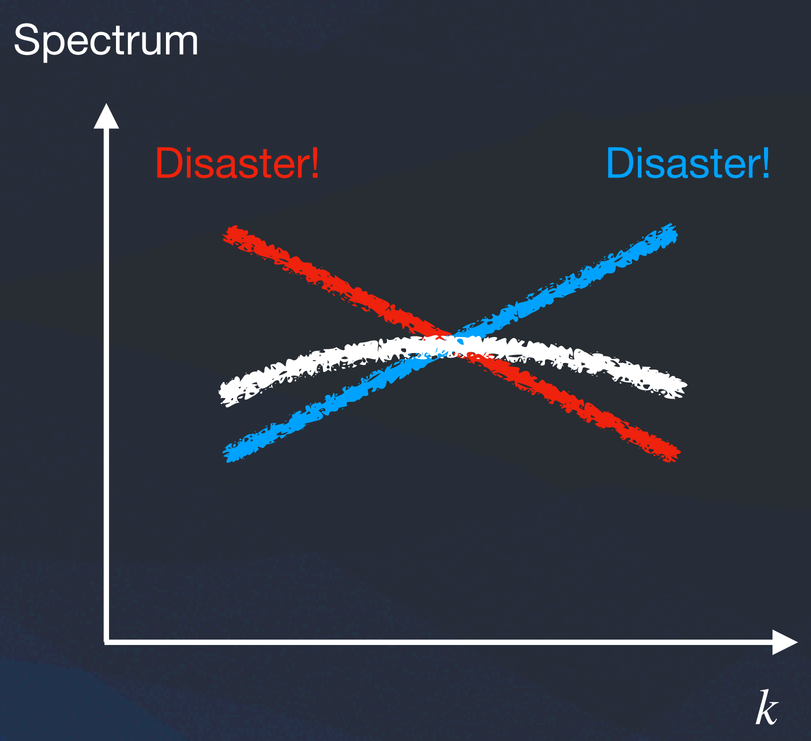




Inflation

The ultimate free lunch

- Harrison and Zel'dovich argued that the **continuum of modes** of the density fluctuations must reach **all possible scales**, from the smallest, $\lambda \to 0$ $(k \to \infty)$, to the largest, $\lambda \to \infty$ $(k \to 0)$
- They argued that the spectrum should not be too "red" (too much power on large scales/low frequencies), otherwise the Universe would be **inhomogeneous** on largest scales.
- But the reverse could not be true either: too much power on small scales would make the Universe "brittle", and could lead to **runaway production of primordial black holes** in the early Universe.
- The only way out is for the initial spectrum of density fluctuations to have a **nearly "scale-invariant" spectrum**, with neither too much power on large scales, nor on small scales.
- And, as it turns out, **inflation** is able to produce exactly that: a **nearly scale-invariant spectrum of Gaussian random fluctuations** — just what we need to explain the pattern seen in the CMB.



Inflation

The ultimate free lunch

- The main ingredient that we need to add to the rapid accelerated expansion during inflation is… **quantum mechanics** (i.e., Quantum Field Theory)!

Quantum Mechanics:

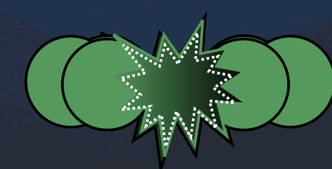
$$\Delta E \, \Delta t > \frac{1}{2}\hbar$$

Vacuum is filled with **virtual pairs** of particles, which appear for brief moments until they are annihilated back to vacuum

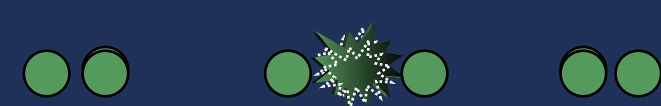
Inflation

The ultimate free lunch

- Consider this process in a familiar setting, without considering any expansion. These particles are appearing and disappearing all the time — and even if they live precarious, brief existences, their effect is critical to explain all the experiments at, e.g., the LHC, and the Standard Model.



$$\Delta E \; {\scriptstyle \Delta t} > \frac{1}{2}\hbar$$

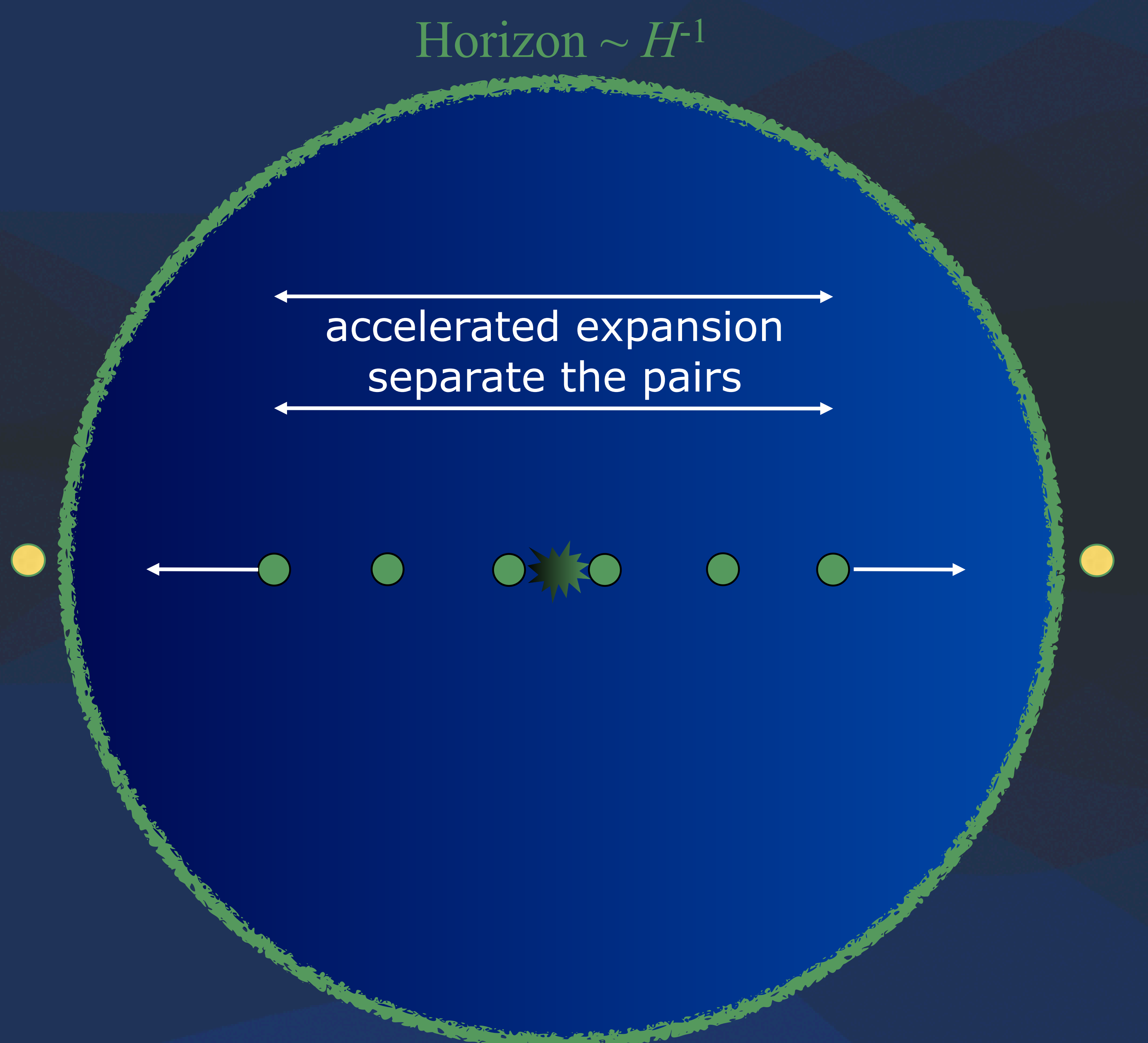


$${\scriptstyle \Delta E} \; \Delta t > \frac{1}{2}\hbar$$

Inflation

The ultimate free lunch

- But consider now what happens if the Universe is **expanding in an accelerated way** — with a future-like horizon that limits the size of the future light cone.
- The accelerated expansion converts **virtual pairs** into **real pairs**, in a process similar to Hawking radiation.
- This process generates **density perturbations** as well as **gravitational waves,** with precisely the features that we need to explain the patterns in the CMB!

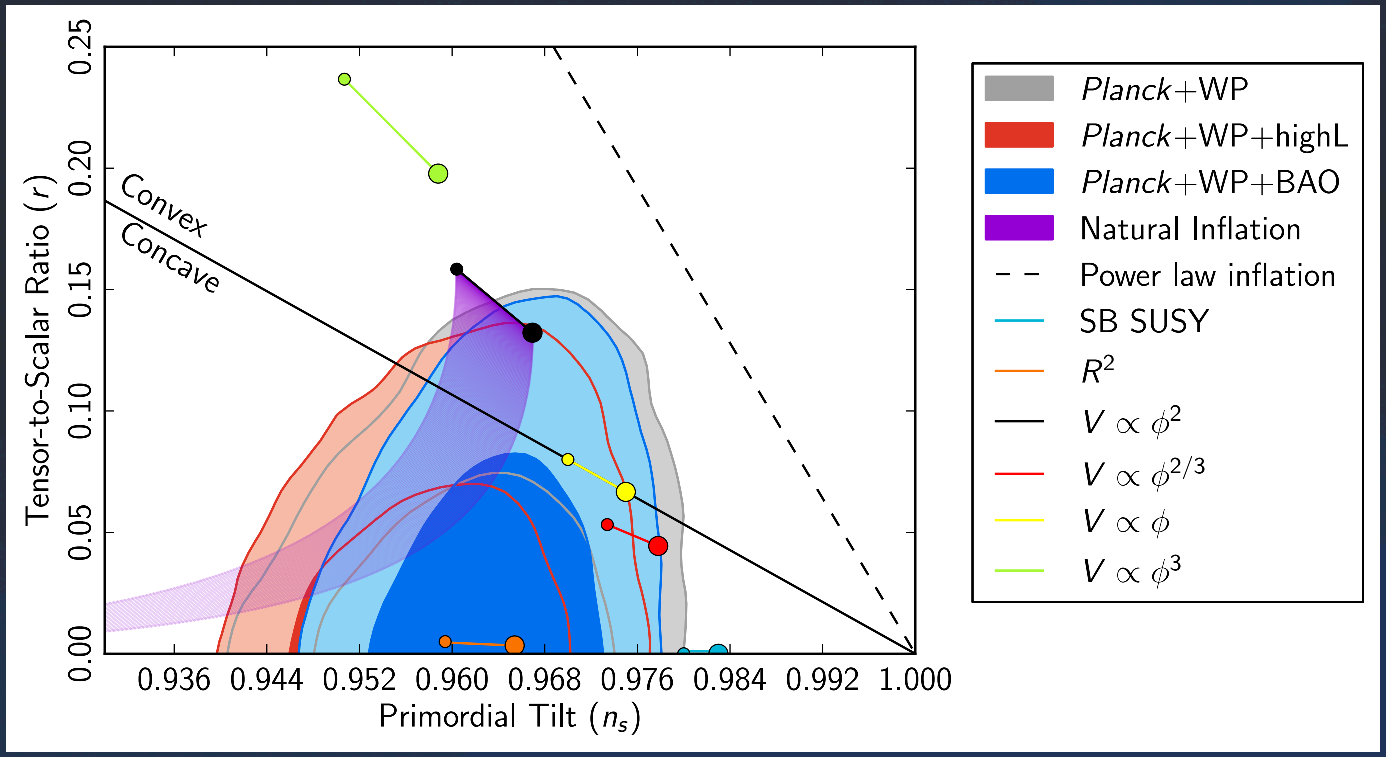


Inflation

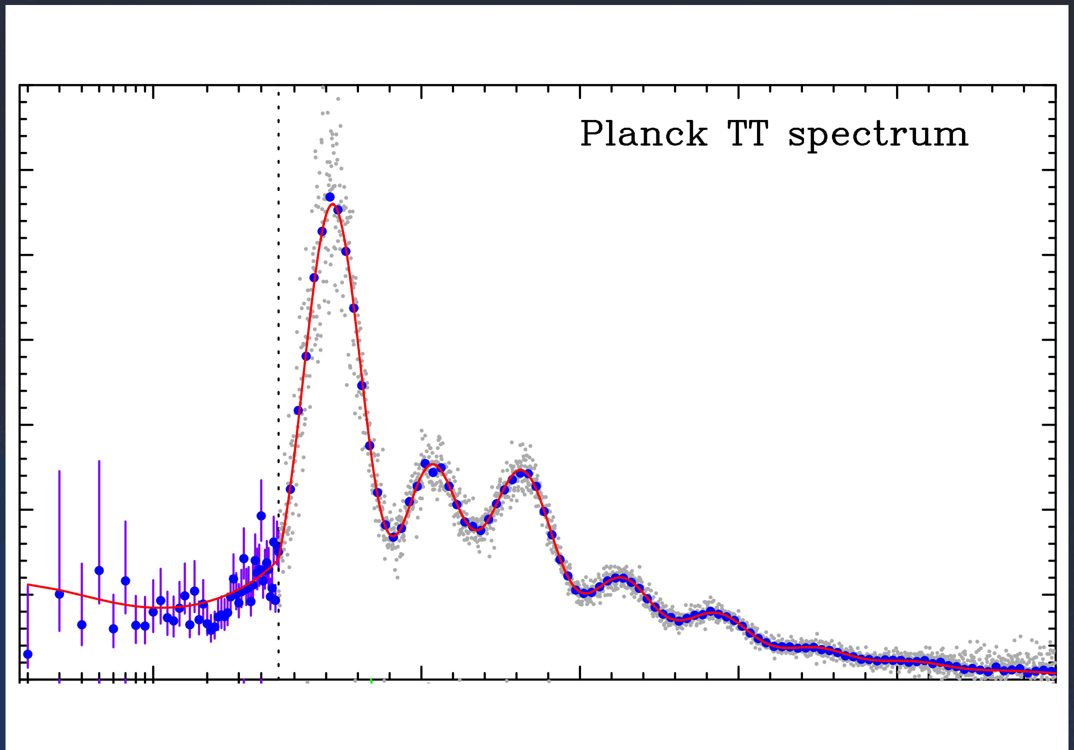
Παν μέτρον άριστον

- Observations are fully consistent with the idea of inflation, even if until now the data is not powerful enough to prove that inflation really did happen

Relative amount of gravitational waves



Deviations from exact scale invariance $(n_s = 1)$

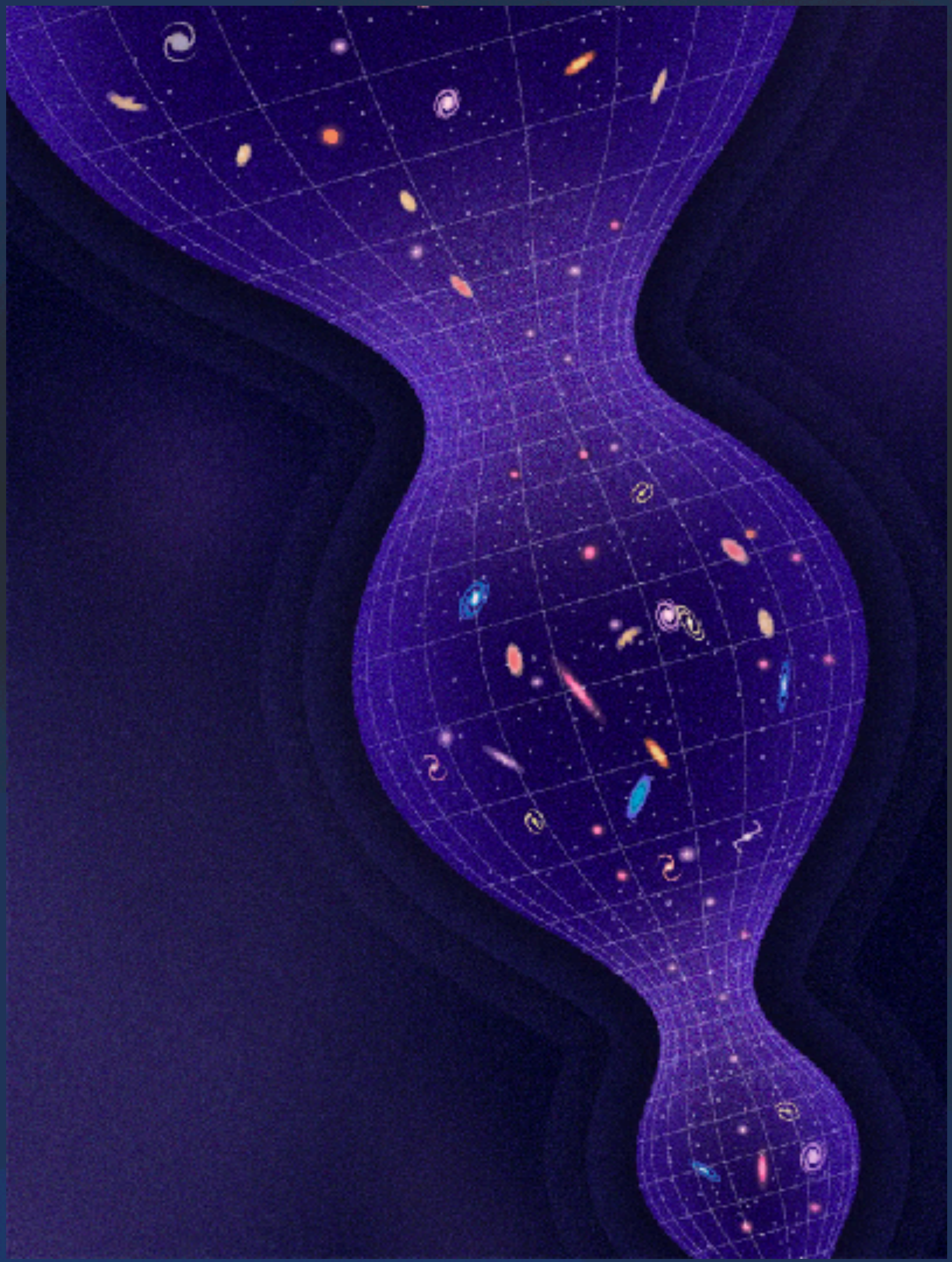


Planck 2018

Inflation

or not?

- But is inflation the only way to get the Universe started on the right track?
- Perhaps not: some cosmologists (Paul Steinhardt, Neil Turok, Patrick Peter and others) have proposed a scenario where the Universe first **contracts**, then expands — a **bounce**.
- In some versions, the Universe goes through **cyclic phases** of **contraction** and **expansion**, with a “bottleneck" in between (“ekpyrosis”).
- However, these models are often contrived and unnatural: the transition from contraction to expansion is extremely delicate, and tiny deviations can lead to **singularities**.
- As of today, the most widely and most robust explanation for the initial conditions of our Universe is still the idea that **inflation** somehow replaced the Big Bang as the “real" beginning of our story.



Inflation

Where our story really starts

- Inflation gives us a **template** for the **initial conditions**.
- The next step is to figure out: given these initial conditions and the physical laws as we know them, do we arrive at the Universe we know today, with stars, galaxies, galaxy clusters, black holes, supernovas…?