15th IDPASC Summer School

20 - 29 July 2026 Natal, Brazil

Neutrino Physics

Renata Zukanovich Funchal

Instituto de Física, Universidade de São Paulo

Lecture I:

Oscillations & Mass Ordering

"Anyway, in as far as the neutrino masses are negligible compared to the charged lepton masses, the observable effects of leptonic mixing angles are limited to fairly exotic effect such as neutrino oscillations.''

Froggatt & Nielsen

Detecting the First Neutrino

The electron neutrino

70 years ago !

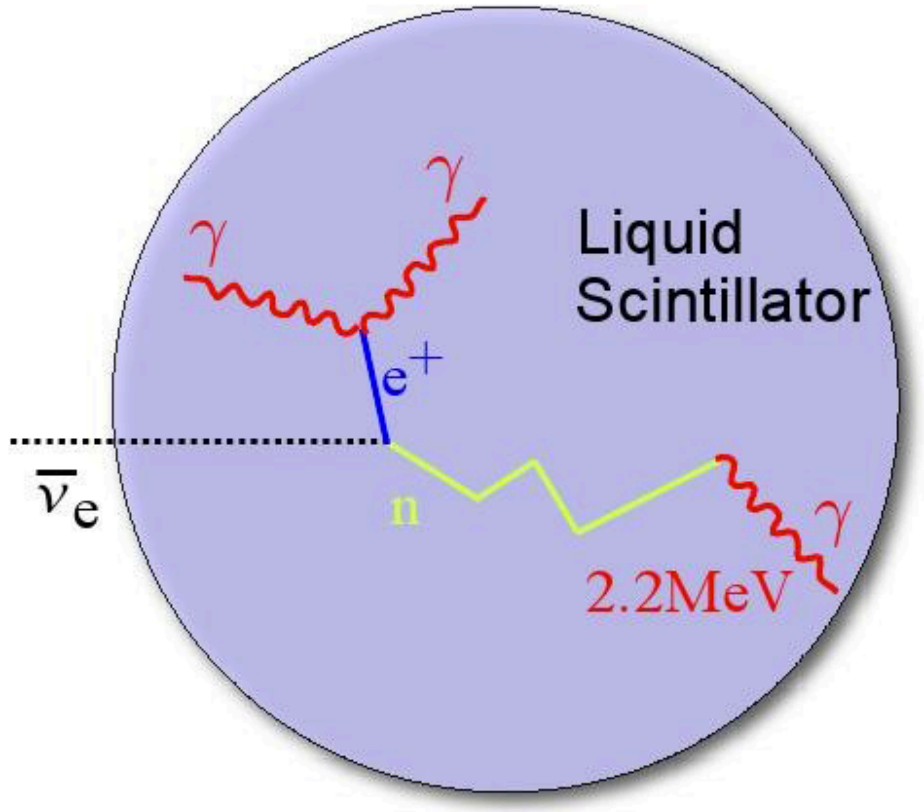
Savannah River, South Carolina, EUA

446 NATURE September 1, 1956 VOL. 178

THE NEUTRINO

By DR. FREDERICK REINES and DR. CLYDE L. COWAN, jun.

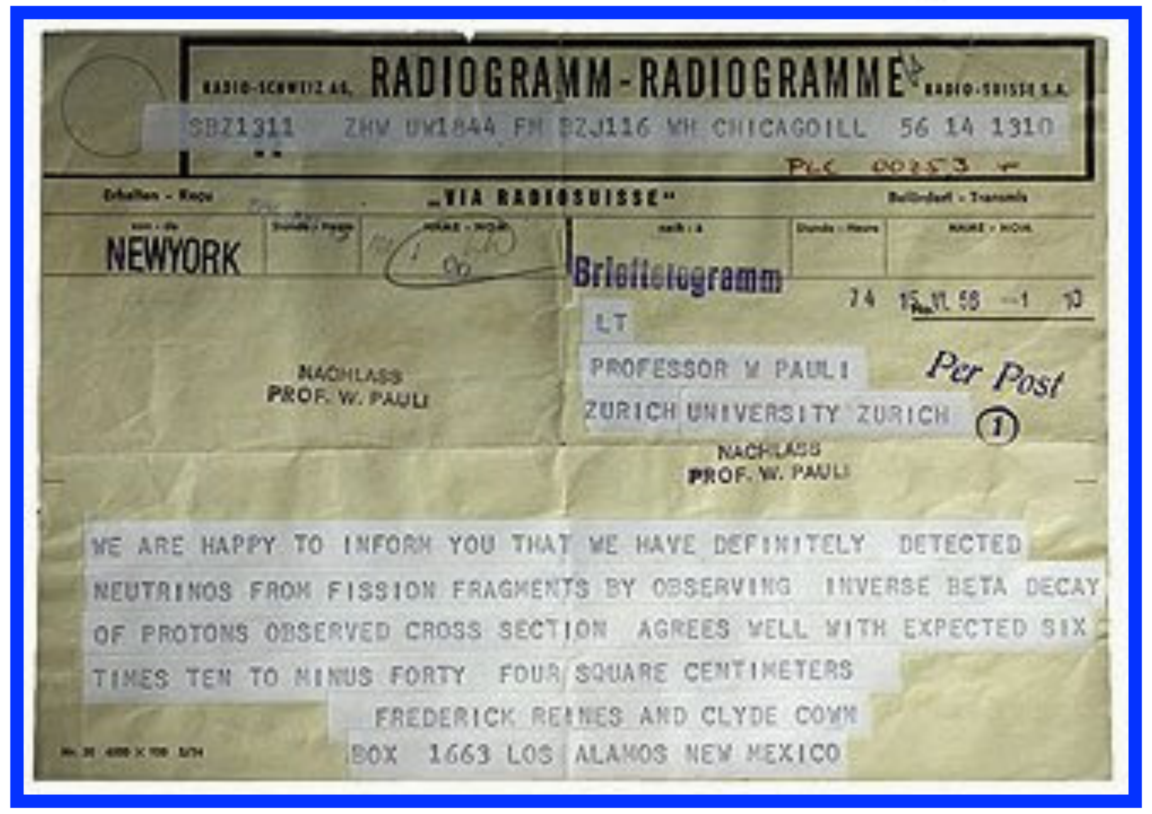
University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico



$$\overline{\nu}_e + p \rightarrow e^+ + n$$

$\downarrow$ 207 μs

$$n + p \rightarrow d + \gamma$$



RADIO-SCHWEIZ AG. RADIOGRAMM - RADIOGRAMME RADIO-SUISSE S.A.

SBZ1311 ZHW UW1844 FM BZJ116 WH CHICAGOILL 56 14 1310

„VIA RADIOSUISSE"

NEWYORK

Briefelegramm

LT

PROFESSOR W PAULI

ZURICH UNIVERSITY ZURICH

Per Post

NACHLASS PROF. W. PAULI

WE ARE HAPPY TO INFORM YOU THAT WE HAVE DEFINITELY DETECTED NEUTRINOS FROM FISSION FRAGMENTS BY OBSERVING INVERSE BETA DECAY OF PROTONS OBSERVED CROSS SECTION AGREES WELL WITH EXPECTED SIX TIMES TEN TO MINUS FORTY FOUR SQUARE CENTIMETERS

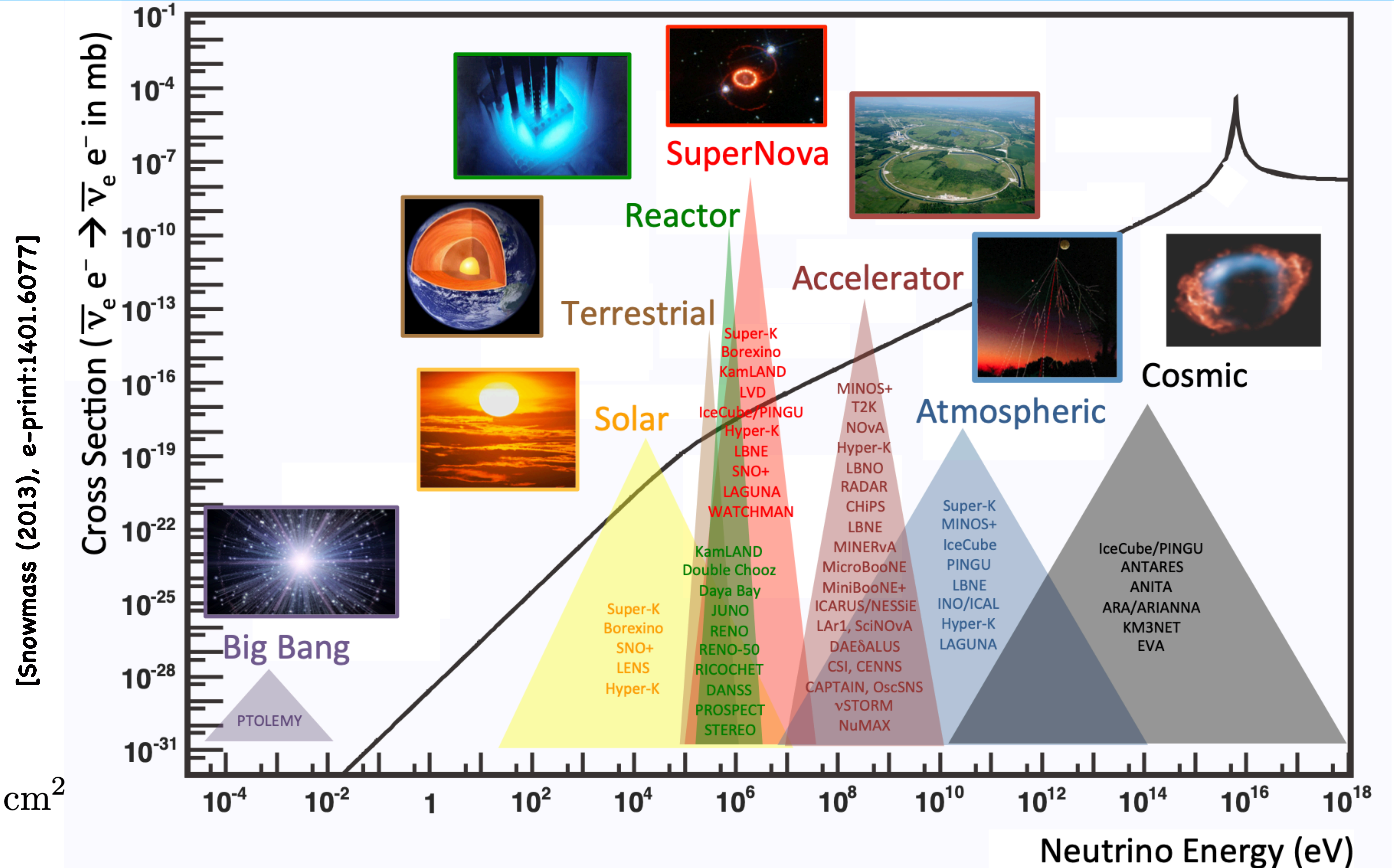
FREDERICK REINES AND CLYDE COWN

BOX 1663 LOS ALAMOS NEW MEXICO

C.L. Cowan Jr, et al. Science 124, 103 (1956)
F. Reines and C.L. Cowan Jr, Nature 178, 446 (1956)

Neutrinos have a Ubiquitous Presence

Span many orders of magnitude in energy and x-section



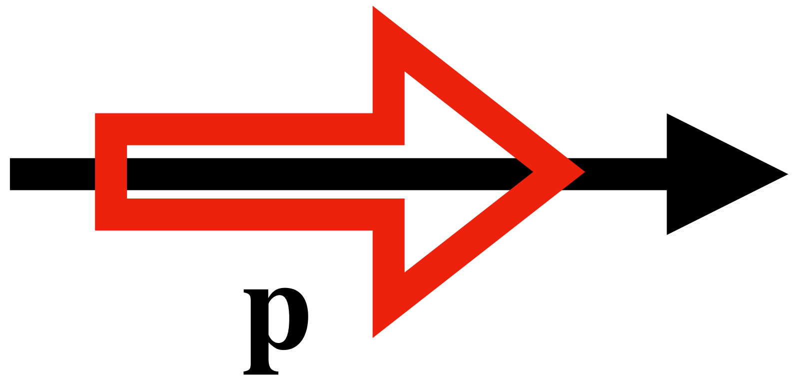
$1\ \mathrm{mb} = 10^{-27}\ \mathrm{cm}^2$

Helicity of Neutrinos

Is the neutrino left- or right-handed?

1958: Experiment by M. Goldhaber, L. Grodzins and A.W. Sunyar

Maurice Goldhaber Lee Grodzins Andrew Sunyar

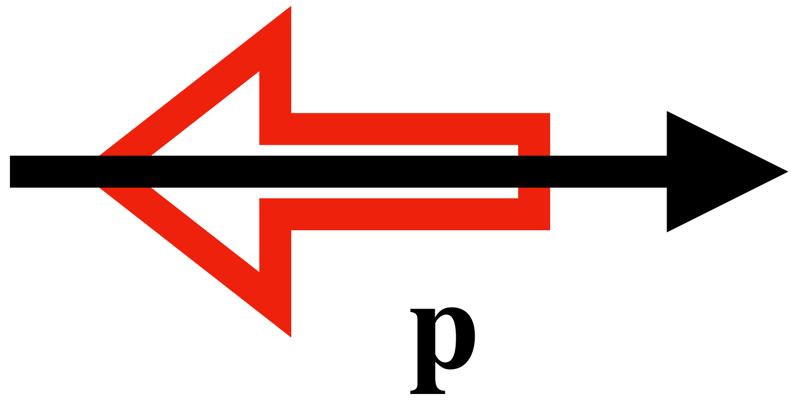


particle momentum

$\sigma \cdot \hat{\mathrm{p}}$ parallel

right-handed = positive helicity (h=+1)

OR



particle momentum

$\sigma \cdot \hat{\mathrm{p}}$ anti-parallel

left-handed = negative helicity (h=-1)

free spin 1/2 particle

Helicity of Neutrinos

The neutrino is left-handed

$$^{152}\mathrm{Eu} + \mathrm{e}^- \rightarrow {}^{152}\,\mathrm{Sm}^* + \nu$$

$$^{152}\mathrm{Sm}^* \rightarrow {}^{152}\,\mathrm{Sm} + \gamma$$

$$J(^{152}\mathrm{Eu}) = \mathrm{J}(^{152}\mathrm{Sm}) = 0 \qquad L(e^-) = 0$$

Helicity of Neutrinos*

M. Goldhaber, L. Grodzins, and A. W. Sunyar
Brookhaven National Laboratory, Upton, New York
(Received December 11, 1957)

A COMBINED analysis of circular polarization and resonant scattering of γ rays following orbital electron capture measures the helicity of the neutrino. We have carried out such a measurement with Eu^{152m}, which decays by orbital electron capture. If we assume the most plausible spin-parity assignment for this isomer compatible with its decay scheme,[1] 0−, we find that the neutrino is "left-handed," i.e., $\boldsymbol{\sigma}_\nu \cdot \hat{p}_\nu = -1$ (negative helicity).

angular momentum conservation $\quad S_z(e^-) = S_z(\nu) + S_z(\gamma) \qquad S_z(\nu) = -\frac{1}{2}S_z(\gamma)$

momentum conservation $\quad \vec{p}(^{152}\mathrm{Eu}) \simeq \vec{\mathrm{p}}(^{152}\mathrm{Sm}^{(*)}) \simeq 0$

$$\vec{p}_\nu = -\vec{p}_\gamma$$

$$\Longrightarrow \quad \overrightarrow{S_\nu} \cdot \vec{p}_\nu = \frac{1}{2}\vec{S}_\gamma \cdot \vec{p}_\gamma$$

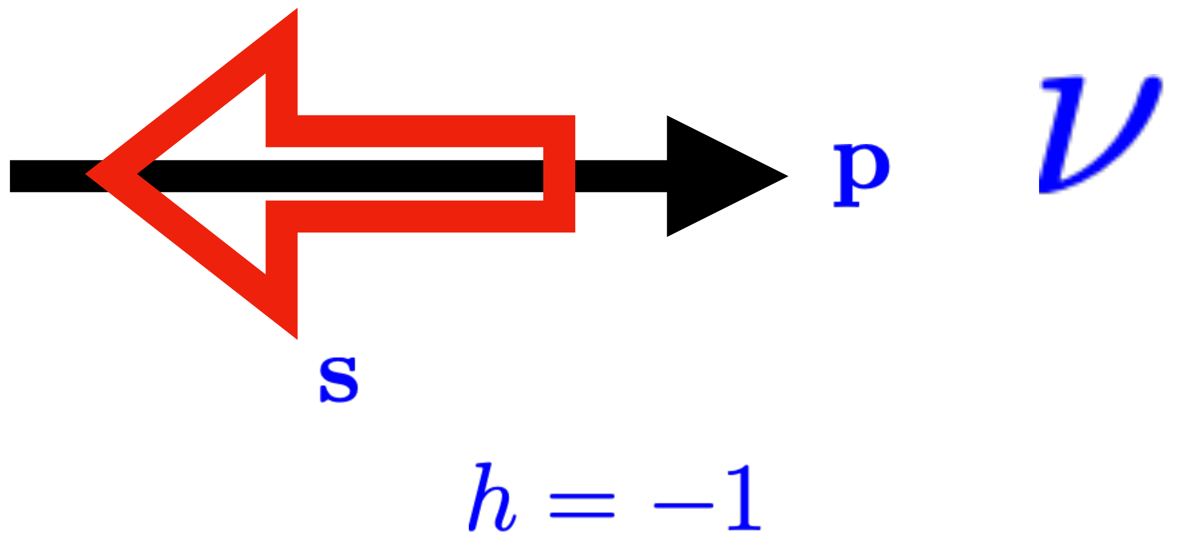
helicity of ν = 1/2 helicity of γ

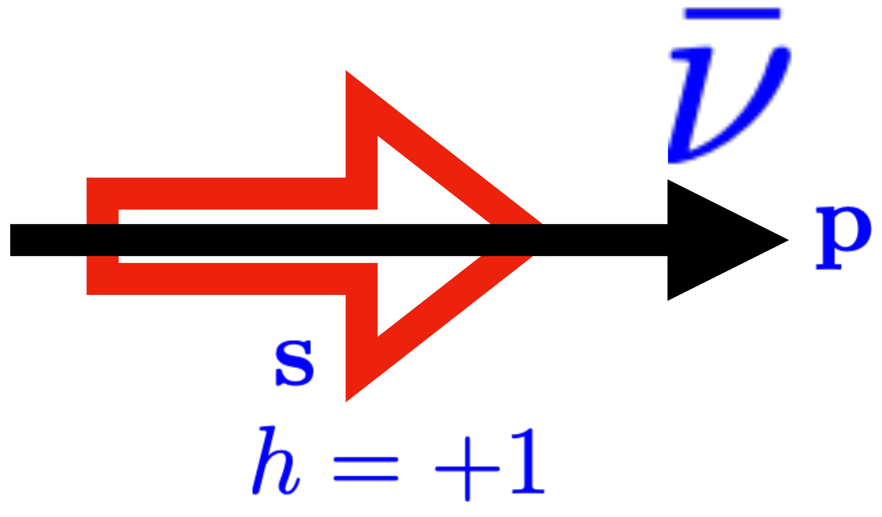
leads to a V-A current

Helicity and Chirality
Neutrino & Antineutrinos

neutrinos have negative helicity

antineutrinos have positive helicity





if massless only need

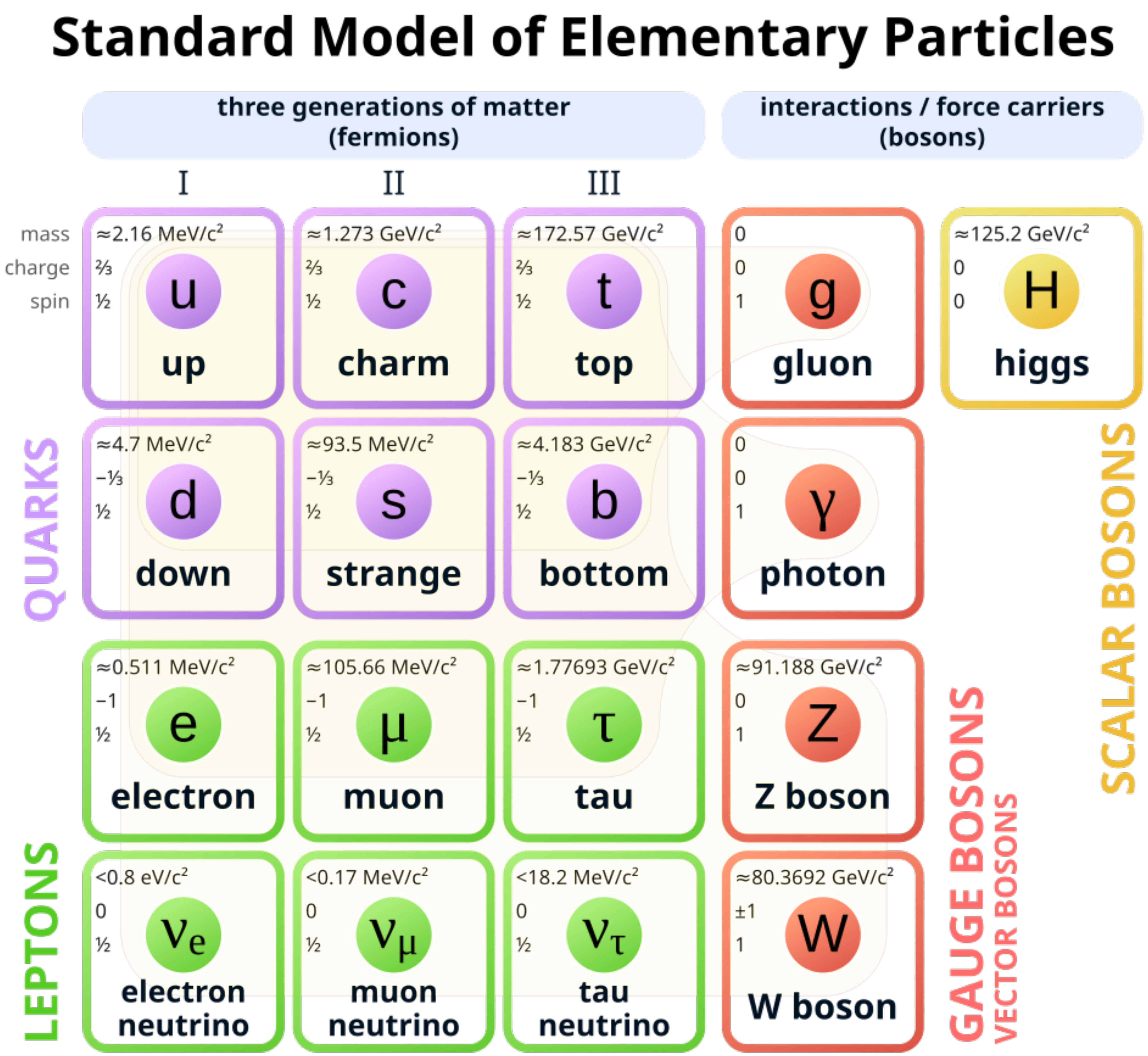
$$\nu_L = \frac{1}{2}(1 - \gamma_5)\nu$$

negative chirality field

Dirac Equation	Chirality	Helicity	Amplitude
particle	L = negative chirality	h = -1	~ 1
		h = +1	$\sim \frac{m}{E}$
		h = -1	$\sim \frac{m}{E}$
antiparticle		h = +1	~ 1

The Neutrino Paved The Way

The best description we have of fundamental particles and interactions



$$SU(3)_c \times \boxed{SU(2)_L \times U(1)_Y}$$

Based on: [see Oscar Eboli]

Lorentz Invariance

Gauge Invariance

With:

spontaneous symmetry breaking

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$$

Neutrinos in the Standard Model

Symmetry Transformations

SM is based on the gauge symmetry group

$$\mathrm{SU(3)_c} \times \boxed{\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}} \rightarrow \mathrm{SU(3)_c} \times \boxed{\mathrm{U(1)_{em}}}$$

I_{1L}, I_{2L}, I_{3L} $\quad [I_{iL}, I_{jL}] = i\epsilon_{ij}I_{kL}$ generators of $\mathrm{SU(2)_L}$ (weak isospin)

Y generator of $\mathrm{U(1)_Y}$ (hypercharge)

$$Q_{\rm em} = Y + I_{3L}$$

Left chiral fermion fields transform as weak isospin doublets

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \begin{array}{l} \rightarrow Q_{\rm em}(\nu_e) = 0 \quad I_{3L} = 1/2 \\ \rightarrow Q_{\rm em}(e) = -1 \quad I_{3L} = -1/2 \end{array} \qquad (\mathbf{1}, \mathbf{2}, -\mathbf{1/2})$$

Neutrinos in the Standard Model

Symmetry Transformations

SM is based on the gauge symmetry group

$$\mathrm{SU(3)_c} \times \boxed{\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}} \rightarrow \mathrm{SU(3)} \times \boxed{\mathrm{U(1)_{em}}}$$

I_{1L}, I_{2L}, I_{3L} $\quad [I_{iL}, I_{jL}] = i\epsilon_{ijk}I_{kL}$ generators of $\mathrm{SU(2)_L}$ (weak isospin)

Y generator of $\mathrm{U(1)_Y}$ (hypercharge)

$$Q_{\mathrm{em}} = Y + I_{3L}$$

Right chiral fermion fields transform as weak isospin singlets

$$e_R \longrightarrow \quad Q_{\mathrm{em}}(e) = -1 \quad I_{3L} = 0 \qquad (\mathbf{1}, \mathbf{1}, -\mathbf{1})$$

neutrino fields enter only in $SU(2)_L$ doublets $\cancel{\nu_{eR}}$

Neutrinos in the Standard Model

3 generations of fermions

LEPTONS

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} = L_\alpha \quad (\mathbf{1}, \mathbf{2}, -\mathbf{1/2})$$

$$e_R \quad \mu_R \quad \tau_R = E_\alpha \quad (\mathbf{1}, \mathbf{1}, -\mathbf{1})$$

Neutrinos in the Standard Model

3 generations of fermions

LEPTONS

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} = L_\alpha \qquad (1, 2, -1/2)$$

$$e_R \quad \mu_R \quad \tau_R = E_\alpha \qquad (1, 1, -1)$$

QUARKS

$$\begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix} \quad \begin{pmatrix} c^i_L \\ s^i_L \end{pmatrix} \quad \begin{pmatrix} t^i_L \\ b^i_L \end{pmatrix} = Q^i_\alpha \qquad (3, 2, 1/6)$$

$$u^i_R \quad c^i_R \quad t^i_R = U^i_\alpha \qquad (3, 1, 2/3)$$

$$d^i_R \quad s^i_R \quad b^i_R = D^i_\alpha \qquad (3, 1, -1/3)$$

Masses & Mixing in the SM

Fermion Masses

$$m\overline{\psi}\psi = m\overline{\psi}(P_L + P_R)\psi = m\overline{\psi}_R\psi_L + \mathrm{h.c.}$$

chirality flipping term (needs both L & R)

This is not gauge invariant!

Masses & Mixing in the SM

Fermion Masses

$$m\overline{\psi}\psi = m\overline{\psi}(P_L + P_R)\psi = m\overline{\psi}_R\psi_L + \mathrm{h.c.}$$

chirality flipping term (needs both L & R)

This is not gauge invariant!

Fortunately the ϕ can also fix this via Yukawa couplings to fermions

$$-\mathscr{L}_Y = Y^{\ell}_{\alpha\beta}\overline{L}_\alpha E_\beta \phi + Y^{d}_{\alpha\beta}\overline{Q}_\alpha D_\beta \phi + Y^{u}_{\alpha\beta}\overline{Q}_\alpha U_\beta \tilde{\phi} + \mathrm{h.c.}$$

This is gauge invariant !

Masses & Mixing in the SM

Fermion Masses

$$m\overline{\psi}\psi = m\overline{\psi}(P_L + P_R)\psi = m\overline{\psi}_R\psi_L + \mathrm{h.c.}$$

chirality flipping term (needs both L & R)

This is not gauge invariant!

Fortunately the ϕ can also fix this via Yukawa couplings to fermions

$$-\mathscr{L}_Y = Y^{\ell}_{\alpha\beta}\overline{L}_\alpha E_\beta \phi + Y^{d}_{\alpha\beta}\overline{Q}_\alpha D_\beta \phi + Y^{u}_{\alpha\beta}\overline{Q}_\alpha U_\beta \tilde{\phi} + \mathrm{h.c.}$$

This is gauge invariant !

$$\tilde{\phi} = i\sigma_2\phi^*$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} \xrightarrow{\text{EWSB}} \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

Masses & Mixing in the SM

Biunitary Transformation

down quarks | Yukawa Matrix (complex) | down quarks

$$\overline{\begin{pmatrix}\circ & \circ & \circ\end{pmatrix}_L}\, V_L^{d\dagger}\; V_L^{d}\begin{pmatrix}\circ & \circ & \circ\\ \circ & \circ & \circ\\ \circ & \circ & \circ\end{pmatrix}V_R^{d\dagger}\; V_R^{d}\begin{pmatrix}\circ\\ \circ\\ \circ\end{pmatrix}_R$$

rotated

L fields

diagonal mass matrix

rotated

R fields

and similar to up quarks

Masses & Mixing in the SM

Fermion Masses

$$m\,\overline{\psi}\psi = m\,\overline{\psi}(P_L + P_R)\psi = m\,\overline{\psi}_R\psi_L + \mathrm{h.c.}$$

chirality flipping term (needs both L & R)

This is not gauge invariant!

Fortunately the ϕ can also fix this via Yukawa couplings to fermions

$$-\mathscr{L}_Y = Y^{\ell}_{\alpha\beta}\,\overline{L}_\alpha E_\beta\,\phi + Y^{d}_{\alpha\beta}\,\overline{Q}_\alpha D_\beta\,\phi + Y^{u}_{\alpha\beta}\,\overline{Q}_\alpha U_\beta\,\tilde{\phi} + \mathrm{h.c.}$$

This is gauge invariant !

$$\tilde{\phi} = i\sigma_2\phi^*$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} \xrightarrow{\text{EWSB}} \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

$$m_u\,\overline{u}_L u_R + m_d\,\overline{d}_L d_R + m_c\,\overline{c}_L c_R + \ldots + h.c.$$

$$m_{U\alpha} = \frac{v}{\sqrt{2}}y^U_{\alpha\alpha} \quad m_{D_\alpha} = \frac{v}{\sqrt{2}}y^D_{\alpha\alpha}$$

after diagonalization of $Y^{u,d}$

QUARK MASSES

$$V_{CKM} = V^{u\dagger}_L V^d_L$$

will impact CC interactions

Masses & Mixing in the SM

Biunitary Transformation

$$m\overline{\psi}\psi = m\overline{\psi}(P_L + P_R)\psi = m\overline{\psi}_R\psi_L + \mathrm{h.c.}$$

chirality flipping term (needs both L & R)

This is not gauge invariant!

Fortunately the ϕ can also fix this via Yukawa couplings to fermions

$$-\mathcal{L}_Y = Y^{\ell}_{\alpha\beta}\overline{L}_\alpha E_\beta \phi + Y^{d}_{\alpha\beta}\overline{Q}_\alpha D_\beta \phi + Y^{u}_{\alpha\beta}\overline{Q}_\alpha U_\beta \tilde{\phi} + \mathrm{h.c.}$$

$$m_e \overline{e}_L e_R + m_\mu \overline{\mu}_L \mu_R + m_\tau \overline{\tau}_L \tau_R + h.c.$$

$$m_\alpha = \frac{v}{\sqrt{2}} y^{\ell}_{\alpha\alpha}$$

after diagonalization of Y^{ℓ}

$$\tilde{\phi} = i\sigma_2\phi^*$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} \xrightarrow{\text{EWSB}} \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

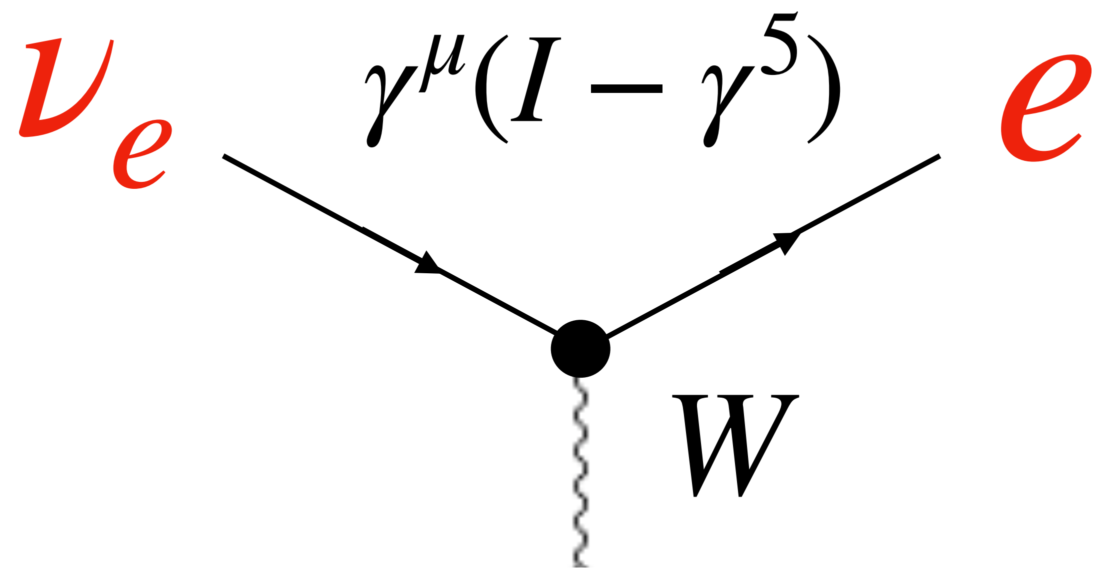
CHARGED LEPTON MASSES

Neutrinos do not have mass (no R component)

Neutrinos in the Standard Model

Neutrino Interactions with Gauge Bosons

Charged Current Interaction (CC)



Flavor diagonal in the

interaction basis

allows the concept of neutrino flavor = flavor of the charged lepton connected to the same CC vertex

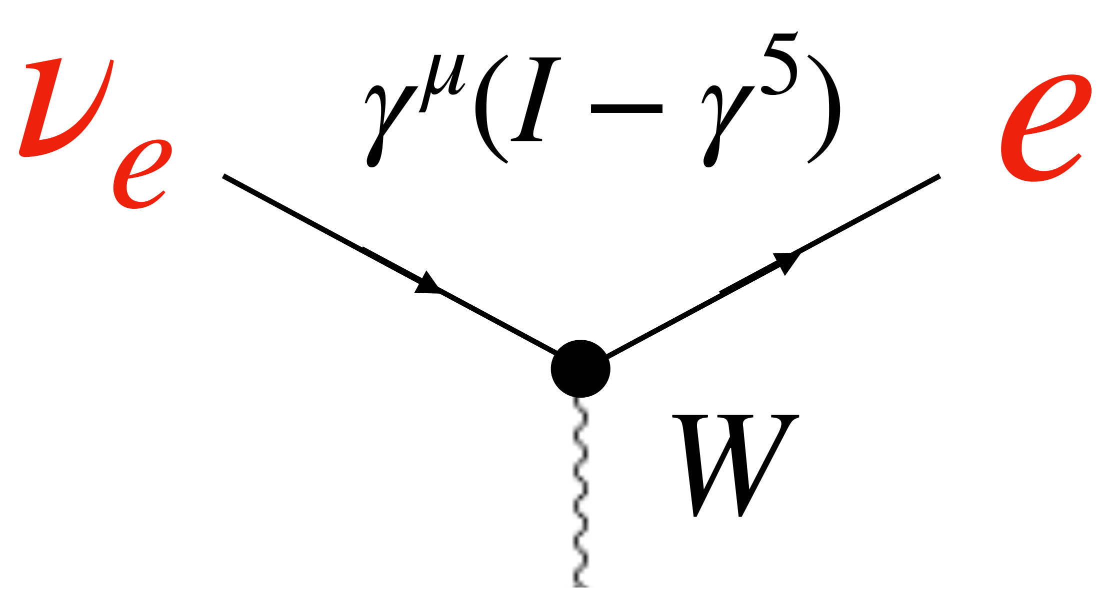
Neutrinos in the Standard Model

Neutrino Interactions with Gauge Bosons

$$L_\alpha \to e^{i\theta_\alpha} L_\alpha \qquad E_\alpha \to e^{i\theta_\alpha} E_\alpha$$

3 accidental global $U(1)_{L\alpha}$ symmetries of the SM

Charged Current Interaction (CC)



Flavor diagonal in the interaction basis

particles (antiparticles)	L_e	L_μ	L_τ
$e^-, \nu_e \; (e^+, \bar{\nu}_e)$	-1 (+1)	0	0
$\mu^-, \nu_\mu \; (\mu^+, \bar{\nu}_\mu)$	0	-1 (+1)	0
$\tau^-, \nu_\tau \; (\tau^+, \bar{\nu}_\tau)$	0	0	-1 (+1)

$$L \equiv L_e + L_\mu + L_\tau$$ Total Lepton Number

define neutrino (antineutrino) according to the convention above (conserve Family Lepton Number)

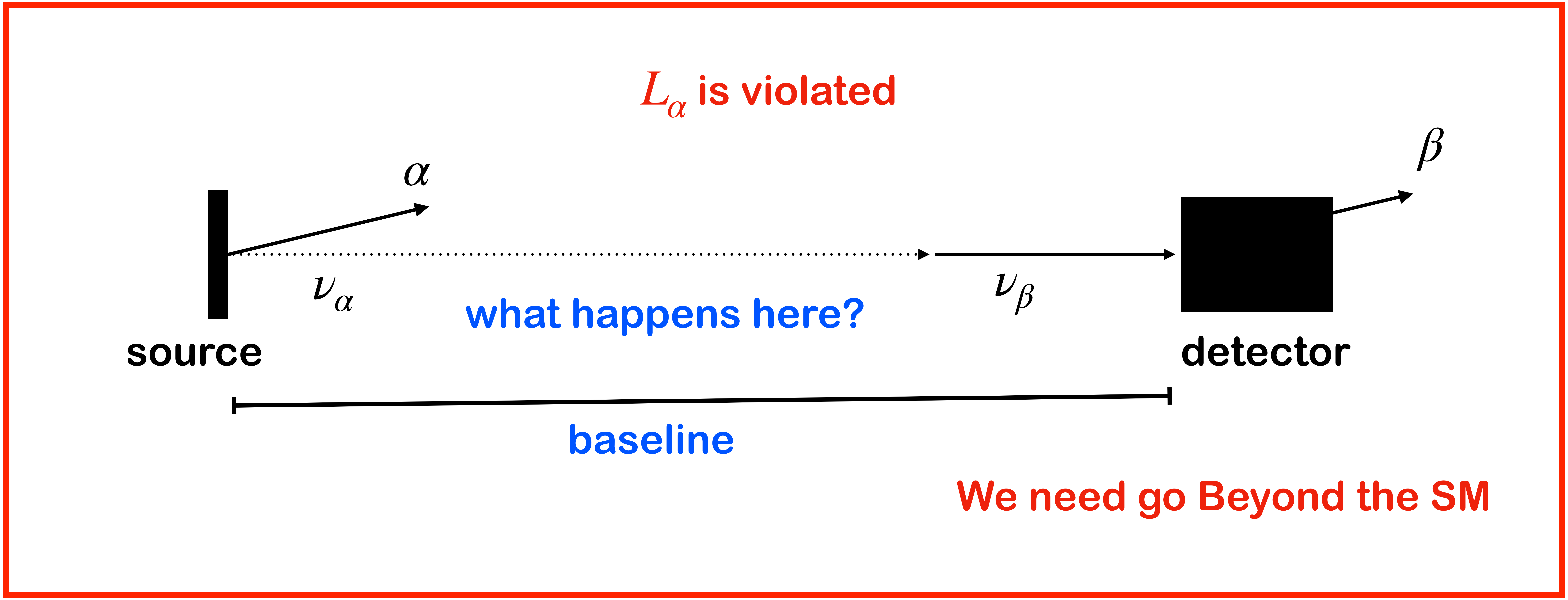
Many Experimental Evidences

Neutrinos Change Flavor in Nature

- Solar ν_e transitioning to ν_μ / ν_τ (Cl, Ga, SK, SNO, Borexino)
- Atmospheric ν_μ & $\bar{\nu}_\mu$ disappearing mostly to ν_τ (SK, DeepCore, ORCA)
- Accelerator ν_μ & $\bar{\nu}_\mu$ disappearing (K2K, T2K, MINOS, NOνA)
- Accelerator $\nu_\mu / \bar{\nu}_\mu$ reappearing as $\nu_e / \bar{\nu}_e$ (T2K, MINOS, NOνA)
- Reactor $\bar{\nu}_e$ disappearing (KamLAND, DC, Daya Bay, RENO, SNO+, JUNO)

This is Evidence of Neutrino Mass

Neutrino Change Flavor in Nature



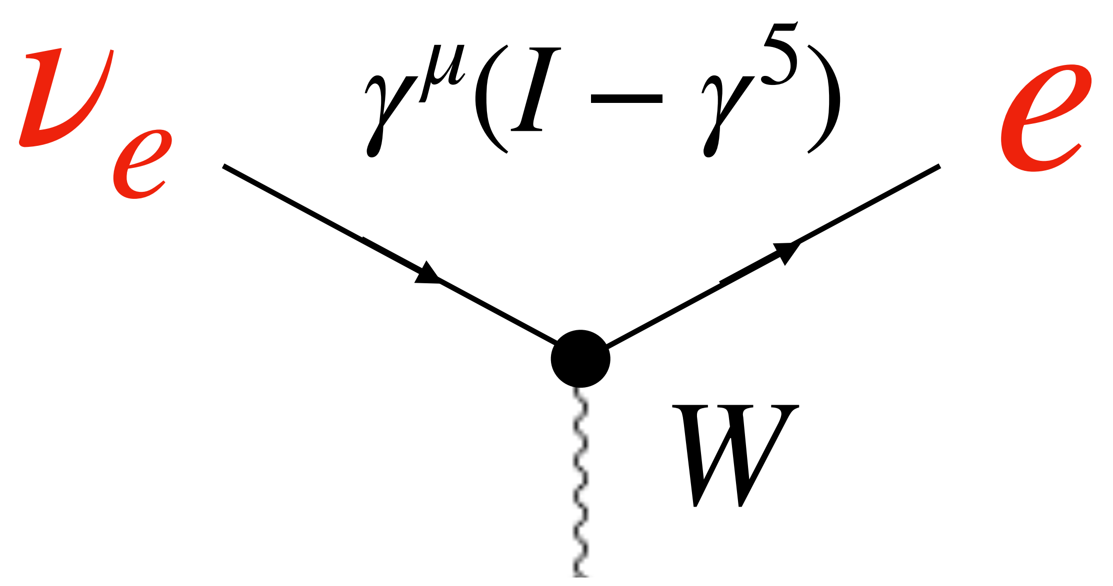
Neutrinos in the Standard Model

Neutrino Interactions with Gauge Bosons

$$L_\alpha \to e^{i\theta_\alpha} L_\alpha \qquad E_\alpha \to e^{i\theta_\alpha} E_\alpha$$

3 accidental global $U(1)_{L\alpha}$ symmetries of the SM

Charged Current Interaction (CC)



Flavor diagonal in the

interaction basis

particles (antiparticles)	L_e	L_μ	L_τ
$e^-, \nu_e \ (e^+, \bar{\nu}_e)$	-1 (+1)	0	0
$\mu^-, \nu_\mu \ (\mu^+, \bar{\nu}_\mu)$	0	-1 (+1)	0
$\tau^-, \nu_\tau \ (\tau^+, \bar{\nu}_\tau)$	0	0	-1 (+1)

$$L \equiv L_e + L_\mu + L_\tau$$

Total Lepton Number

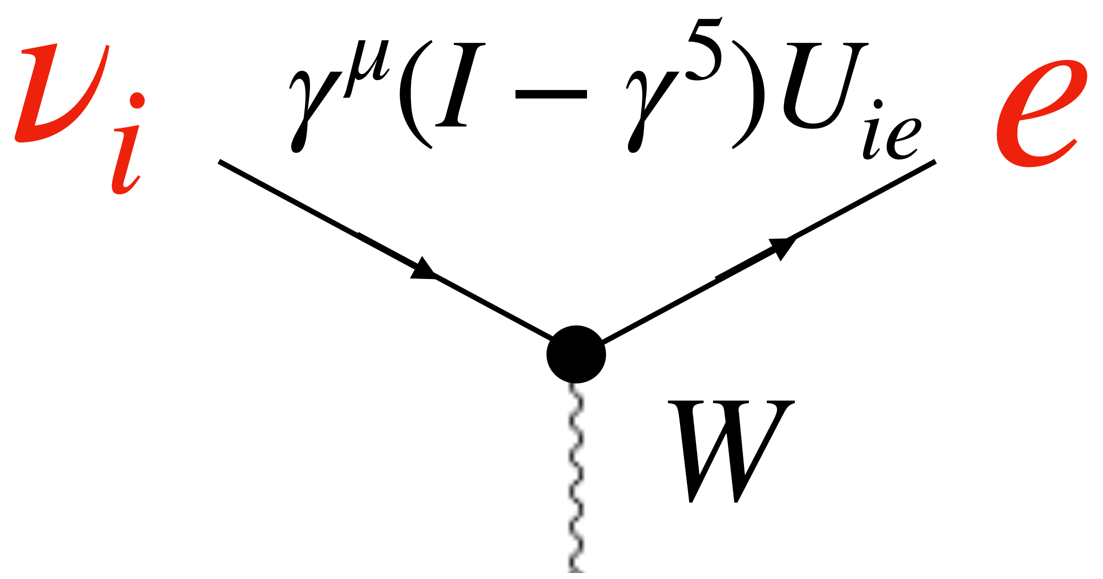
Family Lepton Number is violated by Oscillations - but total Lepton Number is conserved

If neutrino = antineutrino (Majorana) : L also should be violated

Neutrinos in the Mass Basis

Neutrino Interactions with Gauge Bosons

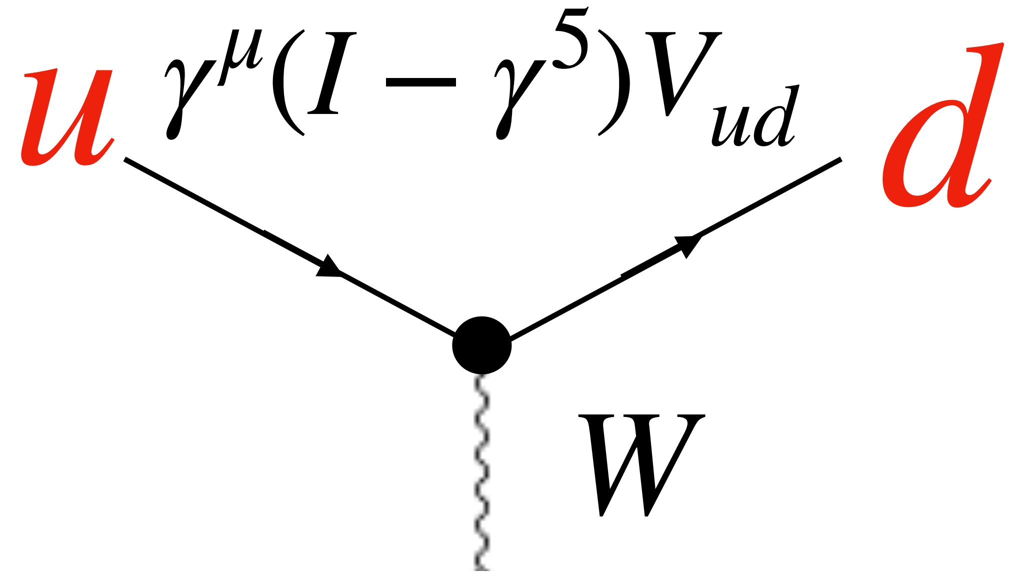
Charged Current Interaction (CC) in the mass basis



Flips up $\longleftrightarrow$ down components of the L doublets

Via PMNS mixing matrix (massive neutrinos!)

mass basis $\neq$ interaction basis



Flips up $\longleftrightarrow$ down components of the L doublets

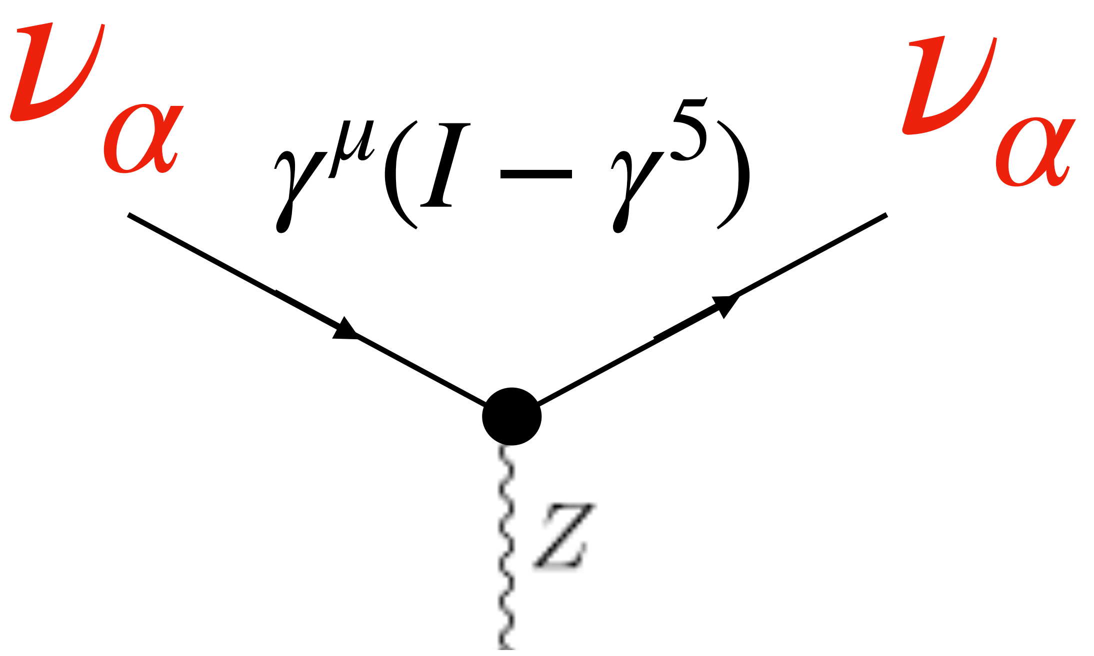
Via CKM mixing matrix

mass basis $\neq$ interaction basis

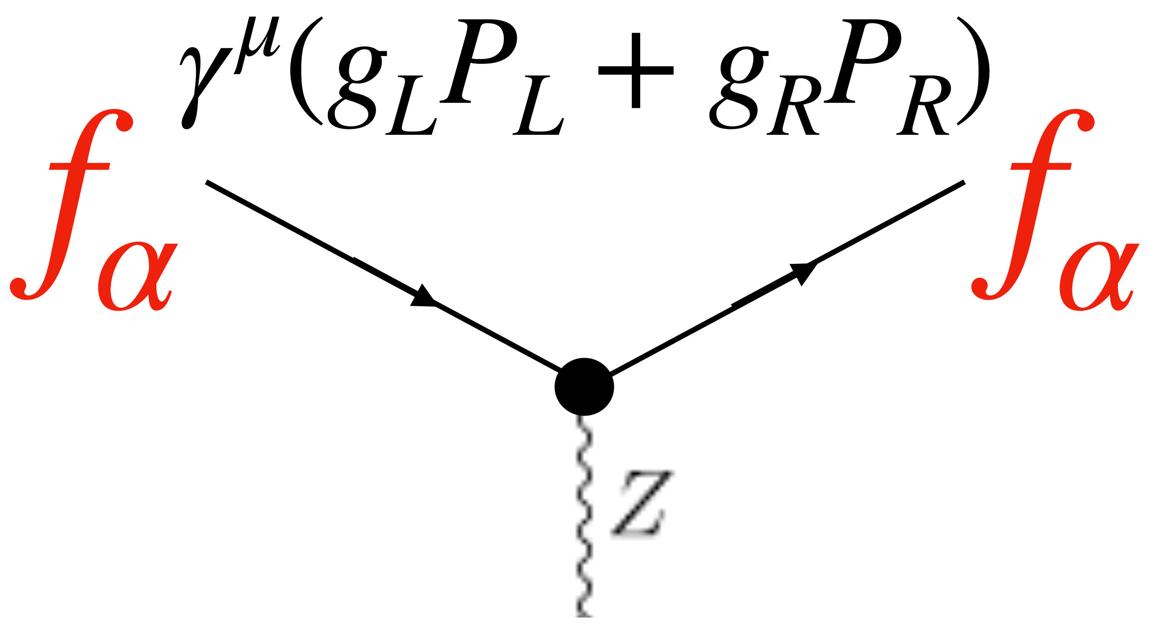
Neutrinos in the Standard Model

Neutrino Interactions with Gauge Bosons

Neutral Current (NC)



Couples only to L component



$$g_L = I_{3L} - Q\sin^2\theta_W$$
$$g_R = -\,Q\sin^2\theta_W$$

Couples only to both L & R components

No flavor changing neutral currents

only neutrinos/antineutrinos of left chirality participate of CC or NC interactions
This will not change with mass!

Neutrino Flavor Oscillations

How they are induced by mass and mixing

(1) if neutrinos have mass

flavor eigenstates
interaction states

mass eigenstate
propagating states

$$|\nu_e\rangle, |\nu_\mu\rangle, |\nu_\tau\rangle \neq |\nu_1\rangle, |\nu_2\rangle, |\nu_3\rangle$$

CC Interactions are not diagonal in the mass basis (as for quarks!)

$$\frac{g}{\sqrt{2}} W_\mu^- \sum_{\alpha,j} (U_{\alpha j}\, \bar{e}_\alpha \gamma^\mu P_L \nu_j) + \mathrm{h.c.}$$

mixing matrix element

Neutrino Flavor Oscillations

How they are induced by mass and mixing

If neutrinos have mass

flavor eigenstates
interaction states

mass eigenstate
propagating states

$$|\nu_e\rangle, |\nu_\mu\rangle, |\nu_\tau\rangle \neq |\nu_1\rangle, |\nu_2\rangle, |\nu_3\rangle$$

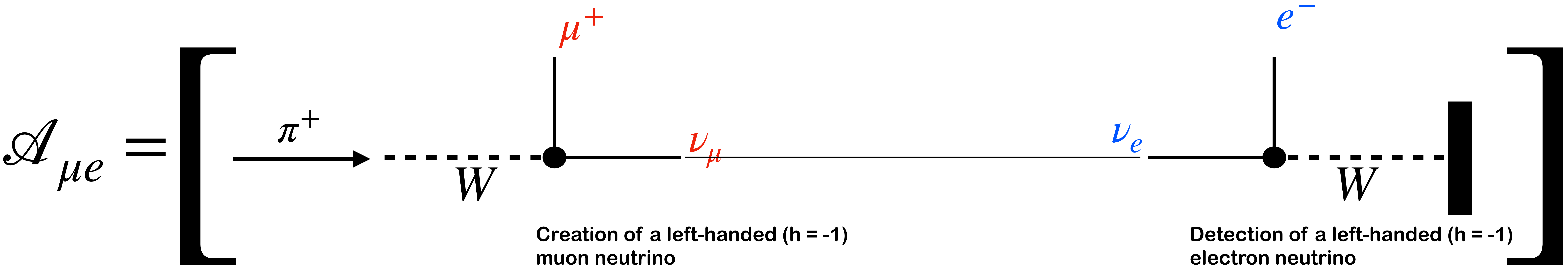
a flavor eigenstate is a superposition of mass eigenstates

$$\boxed{|\nu_\alpha\rangle = \sum_{j=1}^{3} U^*_{\alpha j} |\nu_j\rangle \quad \alpha, \beta = e, \mu, \tau}$$

unitary mixing matrix elements

Neutrino Flavor Oscillations

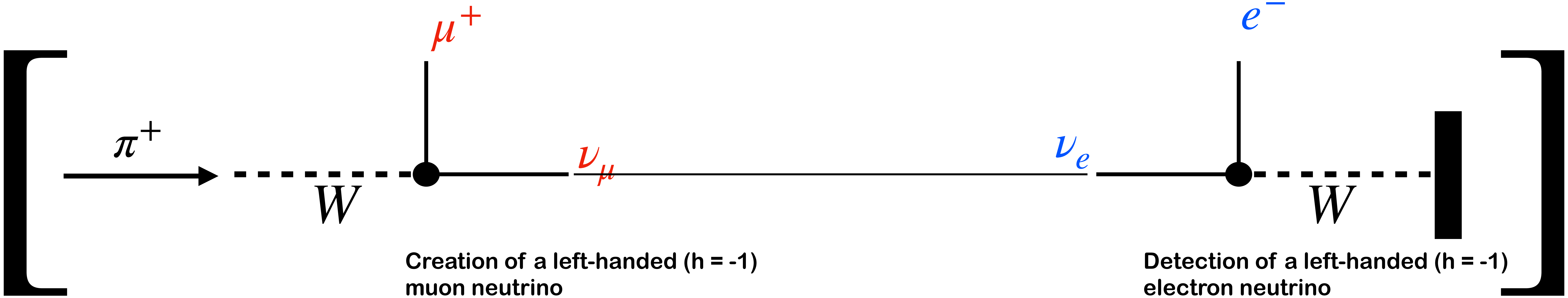
How they are induced by mass and mixing



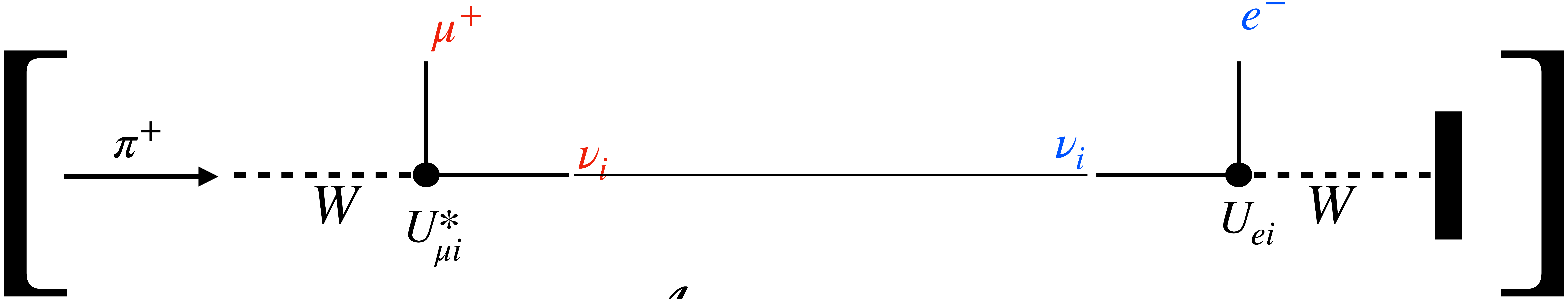
Neutrino Flavor Oscillations

How they are induced by mass and mixing

$\mathscr{A}_{\mu e} =$



$= \sum_{i=1}^{3}$

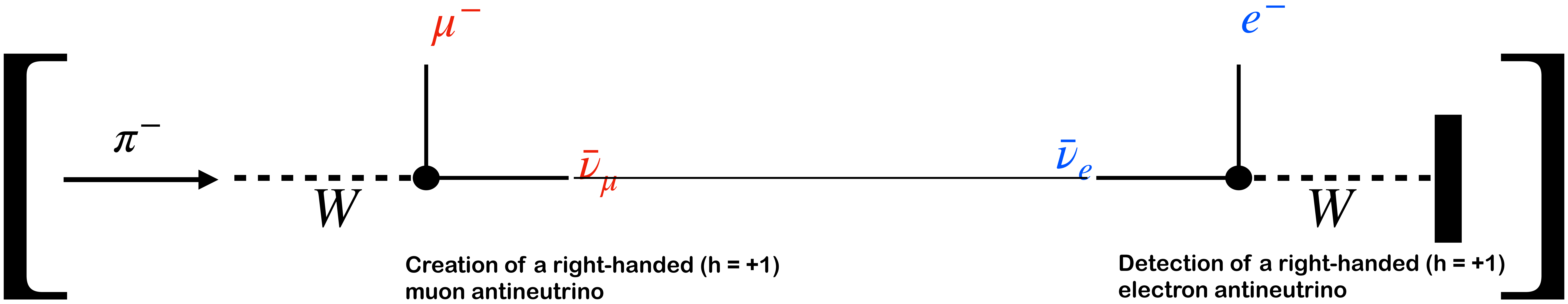


$\mathscr{A}_{ii}$

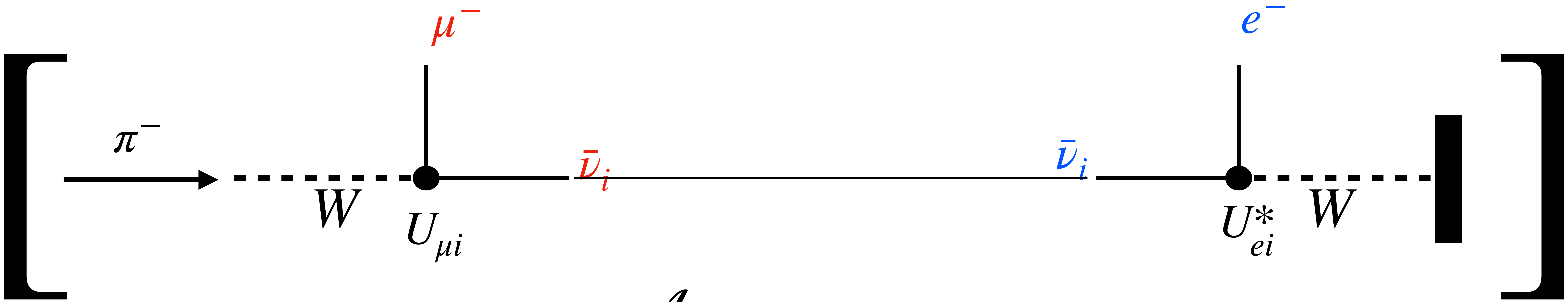
Neutrino Flavor Oscillations

How they are induced by mass and mixing

$\mathcal{A}_{\bar{\mu}\bar{e}} =$



$= \sum_{i=1}^{3}$



$\mathcal{A}_{\bar{i}\bar{i}}$

Neutrino Flavor Oscillations

How they are induced by mass and mixing

(2) Neutrinos are ultra relativistic

for neutrinos

$$p \gg m \qquad \rightarrow E = \sqrt{p^2 + m^2} \simeq p + \frac{m^2}{2p}$$

So ν_i and ν_j with masses m_i and m_j in the same beam ($p_i = p_j$) experience an energy difference

$$\Delta E = E_i - E_j = \frac{m_i^2 - m_j^2}{2p} \simeq \frac{m_i^2 - m_j^2}{2E}$$

because neutrinos have very small masses

this is a very small number

Neutrino Flavor Oscillations

How they are induced by mass and mixing

as we will see the oscillations depend on the combination

$$\frac{m_i^2 - m_j^2}{2E} L = \frac{\Delta m_{ij}^2}{2E} L$$

L = baseline (distance of propagation between production and detection)

this combination can be sizable @ large baselines

Neutrino Flavor Oscillations

How they are induced by mass and mixing

as we will see the oscillations depend on the combination

$$\frac{m_i^2 - m_j^2}{2E} L = \frac{\Delta m_{ij}^2}{2E} L$$

L = baseline (distance of propagation between production and detection)

this combination can be sizable @ large baselines

The observation of oscillations will depend on the Energy of the neutrinos (source) and on the baseline (detector position)

If we know Δm_{ij}^2 we can calculate the appropriate baseline for $\neq$ neutrino sources to observe oscillation

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

@ t=0 neutrino flavor eigenstates are produced as a superposition of mass eigenstates

$$|\nu_\alpha(t=0)\rangle = \sum_{j=1}^{3} U^*_{\alpha j}\,|\nu_j(t=0)\rangle \qquad \alpha = e, \mu \text{ or } \tau$$

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

@ t=0 neutrino flavor eigenstates are produced as a superposition of mass eigenstates

$$|\nu_\alpha(t=0)\rangle = \sum_{j=1}^{3} U^*_{\alpha j}\,|\nu_j(t=0)\rangle \qquad \alpha = e, \mu \text{ or } \tau$$

@ t (after propagating a distance L) they have evolved to

$$|\nu_\alpha(t)\rangle = \sum_{j=1}^{3} U^*_{\alpha j}\,|\nu_j(t)\rangle$$

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

@ t=0 neutrino flavor eigenstates are produced as a superposition of mass eigenstates

$$|\nu_\alpha(t=0)\rangle = \sum_{j=1}^{3} U^*_{\alpha j}\,|\nu_j(t=0)\rangle \qquad \alpha = e, \mu \text{ or } \tau$$

@ t (after propagating a distance L) they have evolved to

$$|\nu_\alpha(t)\rangle = \sum_{j=1}^{3} U^*_{\alpha j}\,|\nu_j(t)\rangle$$

The transition amplitude to a different flavor ν_β is

$$\mathscr{A}_{\alpha\beta} = \langle \nu_\beta | \nu_\alpha(t)\rangle$$

Probability of the transition

$$P_{\alpha\beta}(t) = |\mathscr{A}_{\alpha\beta}|^2 = |\langle \nu_\beta | \nu_\alpha(t)\rangle|^2$$

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

@ t=0 neutrino flavor eigenstates are produced as a superposition of mass eigenstates

$$|\nu_\alpha(0)\rangle = \sum_{j=1}^{3} U^*_{\alpha j}\,|\nu_j(0)\rangle \qquad \alpha = e, \mu \text{ or } \tau$$

@ t (after propagating a distance L) they have evolved to

$$|\nu_\alpha(t)\rangle = \sum_{j=1}^{3} U^*_{\alpha j}\,|\nu_j(t)\rangle$$

Probability of the transition

$$\boxed{P_{\alpha\beta}(t) = \left| \sum_i \sum_j U_{\beta i} U^*_{\alpha j} \langle \nu_i(0)\,|\,\nu_j(t)\rangle \right|^2}$$

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

Under the plane wave approximation

$$|\nu_j(t)\rangle = e^{-iHt}|\nu_j(0)\rangle = e^{-iE_j t}|\nu_j(0)\rangle$$

E_j is the energy of the free streaming neutrino

$$\mathscr{A}_{\alpha\beta}(t) = \sum_i \sum_j U_{\beta i} U^*_{\alpha j} e^{-iE_j t}\langle \nu_i(0)|\nu_j(0)\rangle = \sum_i U_{\beta j} U^*_{\alpha j} e^{-iE_j t}$$

as $\langle \nu_i(0)|\nu_j(0)\rangle = \delta_{ij}$

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

Under the plane wave approximation

$$|\nu_j(t)\rangle = e^{-iHt}|\nu_j(0)\rangle = e^{-iE_jt}|\nu_j(0)\rangle$$

E_j **is the energy of the free streaming neutrino**

$$\mathcal{A}_{\alpha\beta}(t) = \sum_i \sum_j U_{\beta i} U^*_{\alpha j} e^{-iE_jt} \langle \nu_i(0) | \nu_j(0) \rangle = \sum_i U_{\beta j} U^*_{\alpha j} e^{-iE_jt}$$

as $\langle \nu_i(0) | \nu_j(0) \rangle = \delta_{ij}$

survival/oscillation probability

$$P_{\alpha\beta}(t) = \delta_{\alpha\beta} - 4\sum_{j<k} \mathrm{Re}[U^*_{\beta k} U_{\alpha k} U_{\beta j} U^*_{\alpha j}] \sin^2\left(\frac{\Delta_{kj}}{2}\right) + 2\sum_{j<k} \mathrm{Im}[U^*_{\beta k} U_{\alpha k} U_{\beta j} U^*_{\alpha j}] \sin(\Delta_{kj})$$

$$\Delta_{kj} = (E_k - E_j)t$$

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

$$P_{\alpha\beta}(t) = \boxed{\delta_{\alpha\beta} - 4\sum_{j<k} \mathrm{Re}[U^*_{\beta k}U_{\alpha k}U_{\beta j}U^*_{\alpha j}]\sin^2\left(\frac{\Delta_{kj}}{2}\right)} + 2\sum_{j<k} \mathrm{Im}[U^*_{\beta k}U_{\alpha k}U_{\beta j}U^*_{\alpha j}]\sin(\Delta_{kj})$$

this term conserves CP (same for $\bar{\nu}$)

$$U \rightarrow U^*$$

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

$$P_{\alpha\beta}(t) = \boxed{\delta_{\alpha\beta} - 4\sum_{j<k} \mathrm{Re}[U^*_{\beta k} U_{\alpha k} U_{\beta j} U^*_{\alpha j}] \sin^2\left(\frac{\Delta_{kj}}{2}\right)} + \boxed{2\sum_{j<k} \mathrm{Im}[U^*_{\beta k} U_{\alpha k} U_{\beta j} U^*_{\alpha j}] \sin(\Delta_{kj})}$$

this term conserves CP (same for $\bar{\nu}$)

this term violates CP (changes sign for $\bar{\nu}$)

$$U \to U^*$$

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

$$P_{\alpha\beta}(t) = \boxed{\delta_{\alpha\beta} - 4\sum_{j<k} \mathrm{Re}[U^*_{\beta k} U_{\alpha k} U_{\beta j} U^*_{\alpha j}] \sin^2\left(\frac{\Delta_{kj}}{2}\right)} + \boxed{2\sum_{j<k} \mathrm{Im}[U^*_{\beta k} U_{\alpha k} U_{\beta j} U^*_{\alpha j}] \sin(\Delta_{kj})}$$

this term conserves CP (same for $\bar{\nu}$)

$U \rightarrow U^*$

this term violates CP (changes sign for $\bar{\nu}$)

If $\beta = \alpha$ there is no CP violation as $\mathrm{Im}[\,|U_{\alpha k}|^2 |U_{\alpha j}|^2] = 0$

(no CP violation in disappearance experiments)

CP violation can only manifest if $\beta \neq \alpha$ (appearance experiments)

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

$$P_{\alpha\beta}(t) = \delta_{\alpha\beta} - 4\sum_{j<k} \mathrm{Re}[U^*_{\beta k}U_{\alpha k}U_{\beta j}U^*_{\alpha j}]\sin^2\left(\frac{\Delta_{kj}}{2}\right) + 2\sum_{j<k} \mathrm{Im}[U^*_{\beta k}U_{\alpha k}U_{\beta j}U^*_{\alpha j}]\sin(\Delta_{kj})$$

phase differences

ultra-relativistic

mass squared difference

$$\frac{\Delta_{kj}}{2} \equiv \frac{(E_k - E_j)}{2\hbar}\frac{L}{c} = \frac{\Delta m^2_{kj}c^4}{4E}\frac{L}{\hbar c} = 1.27\left(\frac{\Delta m^2_{kj}}{\mathrm{eV}^2}\right)\left(\frac{\mathrm{GeV}}{E}\right)\left(\frac{L}{\mathrm{km}}\right) \qquad \Delta m^2_{kj} = m^2_k - m^2_j$$

[only here we do not use natural units]

this is useful for calculations

$$\boxed{\frac{\Delta_{kj}}{2} \equiv \frac{\Delta m^2_{kj}}{4E}L}$$

(back to natural units)

- only depend on mass squared differences (no sensitivity to the absolute mass scale)

neutrino energy (source) & baseline (source-detector)

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

$$P_{\alpha\beta}(t) = \delta_{\alpha\beta} - 4\sum_{j<k} \mathrm{Re}[U^*_{\beta k} U_{\alpha k} U_{\beta j} U^*_{\alpha j}] \sin^2\left(\frac{\Delta_{kj}}{2}\right) + 2\sum_{j<k} \mathrm{Im}[U^*_{\beta k} U_{\alpha k} U_{\beta j} U^*_{\alpha j}] \sin(\Delta_{kj})$$

ultra-relativistic

phase differences

$$\frac{\Delta_{kj}}{2} \equiv \frac{\Delta m^2_{kj}}{4E} L = \pi \frac{L}{L_\odot}$$

$$\Delta m^2_{kj} = m^2_k - m^2_j$$

- We can define

oscillation length

$$L^{kj}_\odot = \frac{4\pi E}{\Delta m^2_{kj}}$$

Neutrino Flavor Oscillations in Vacuum

Basic Concepts

$$P_{\alpha\beta}(t) = \delta_{\alpha\beta} - 4\sum_{j<k} \mathrm{Re}[U^*_{\beta k}U_{\alpha k}U_{\beta j}U^*_{\alpha j}]\sin^2\left(\frac{\Delta_{kj}}{2}\right) + 2\sum_{j<k} \mathrm{Im}[U^*_{\beta k}U_{\alpha k}U_{\beta j}U^*_{\alpha j}]\sin(\Delta_{kj})$$

ultra-relativistic

phase differences

$$\frac{\Delta_{kj}}{2} \equiv \frac{\Delta m^2_{kj}}{4E}L = \pi\frac{L}{L_\odot} \qquad \Delta m^2_{kj} = m^2_k - m^2_j$$

- We can define

oscillation length

$$L^{kj}_\odot = \frac{4\pi E}{\Delta m^2_{kj}}$$

- if $L \ll L^{kj}_\odot$ the distance is too short for this phase difference to develop - no oscillation can be observer due it
- if $L \gg L^{kj}_\odot$ the distance is long enough for this phase for the system to oscillate many time $\overline{\sin^2\left(\frac{\Delta_{kj}}{2}\right)} \to \frac{1}{2}$ $\overline{\sin\left(\Delta_{kj}\right)} \to 0$
- only for $L \approx L^{kj}_\odot$ we can see the oscillation pattern due to this phase difference

Two Flavor Oscillations in Vacuum

simplest but still very useful case

$$\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} = U \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

U **is real**

only one phase difference $\Delta m_{21}^2 = m_2^2 - m_1^2$

only oscillation length $L_\odot = \dfrac{4\pi E}{\Delta m_{21}^2}$

Two Flavor Oscillations in Vacuum

simplest but still very useful case

$$\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} = U \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

U is real

only one phase difference $\Delta m_{21}^2 = m_2^2 - m_1^2$

only oscillation length $L_\odot = \dfrac{4\pi E}{\Delta m_{21}^2}$

$\Delta m_{21}^2 = 2.5 \times 10^{-3}\ \mathrm{eV}^2$

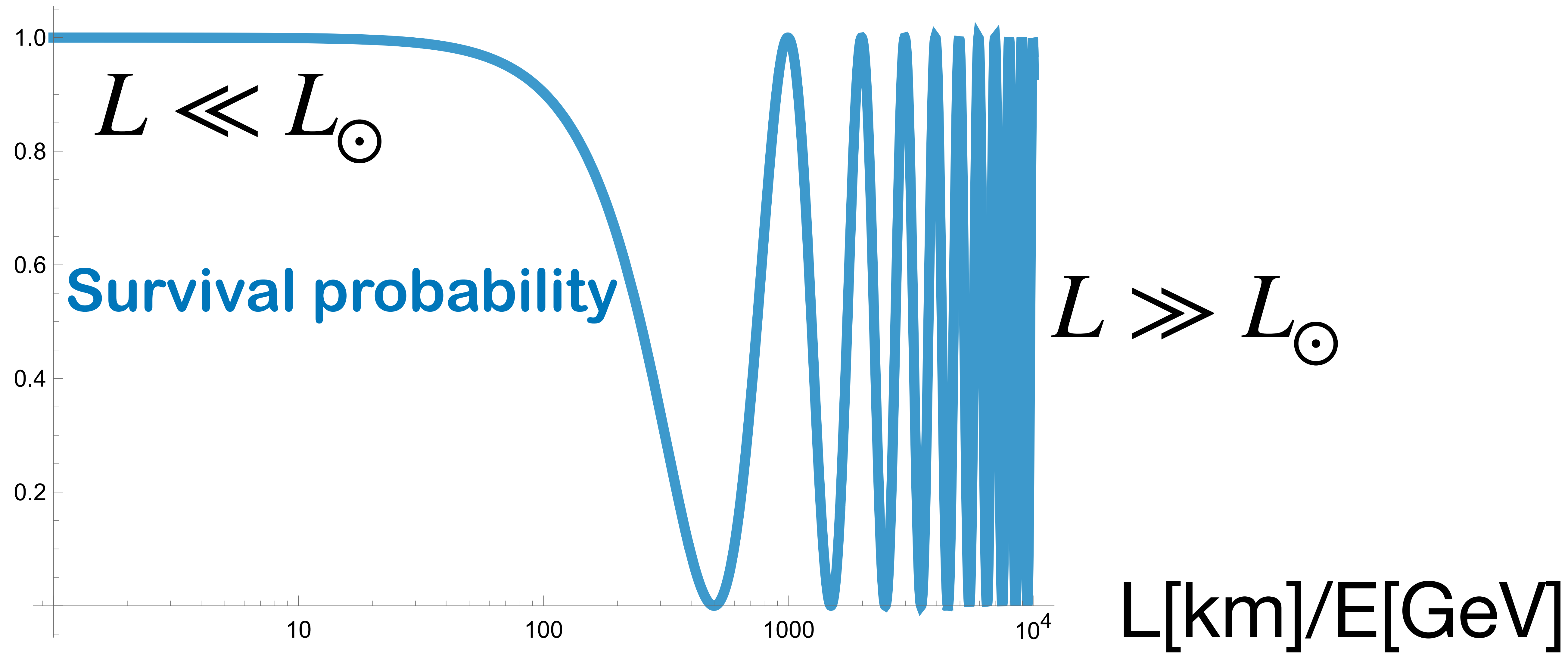
$\sin^2(2\theta) = 1.0$

Oscillation probability

$$P_{\alpha\beta}(t) = \sin^2(2\theta)\ \sin^2\left(\pi \frac{L}{L_\odot}\right)$$

Survival probability

$$P_{\alpha\alpha}(t) = 1 - \sin^2(2\theta)\ \sin^2\left(\pi \frac{L}{L_\odot}\right)$$



Two Flavor Oscillations in Vacuum

simplest but still very useful case

$$\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} = U \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

U **is real**

only one phase difference $\Delta m^2_{21} = m^2_2 - m^2_1$

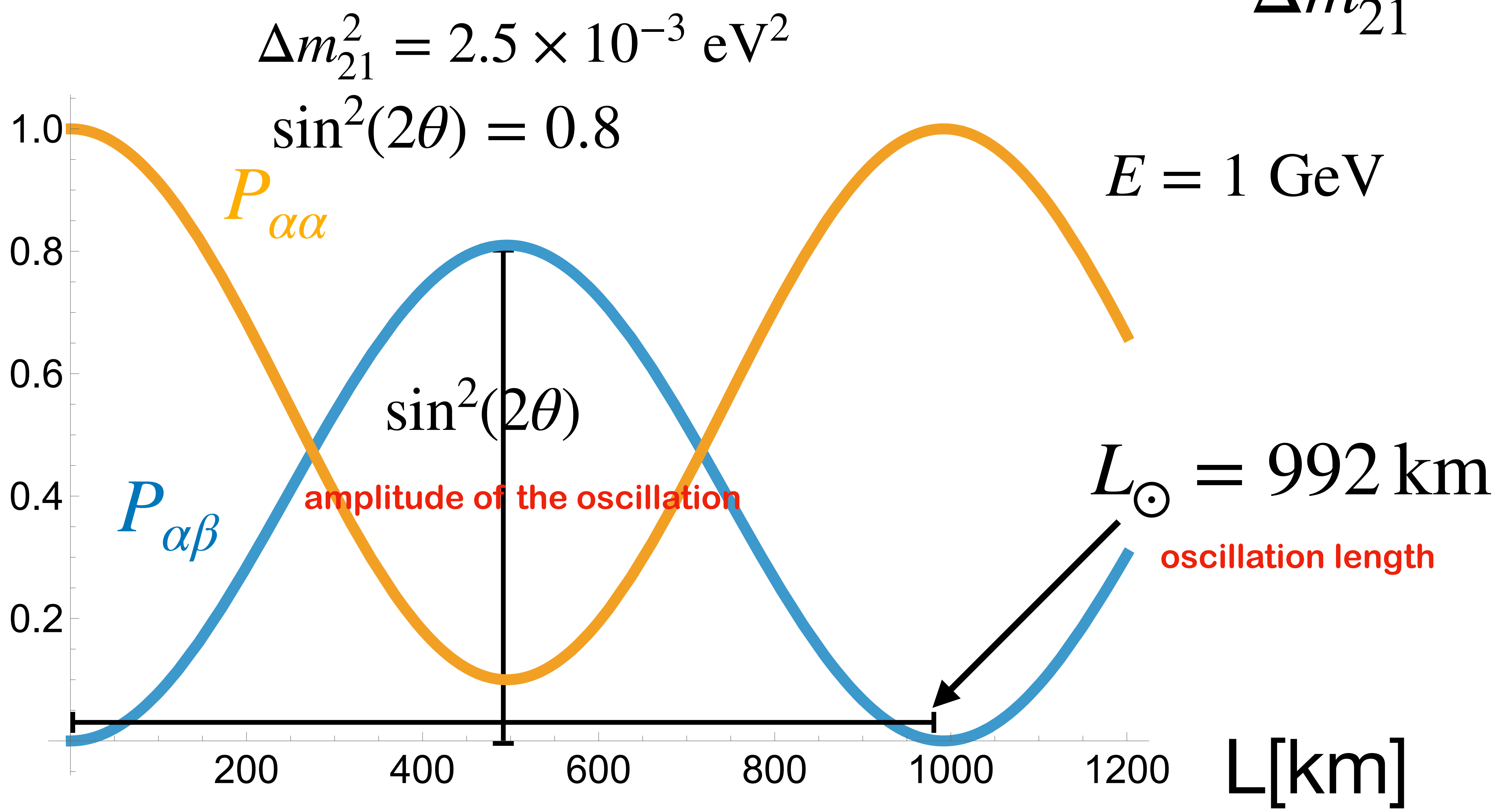
only oscillation length $L_\odot = \dfrac{4\pi E}{\Delta m^2_{21}}$

Oscillation probability

$$P_{\alpha\beta}(t) = \sin^2(2\theta)\ \sin^2\left(\pi \frac{L}{L_\odot}\right)$$

Survival probability

$$P_{\alpha\alpha}(t) = 1 - \sin^2(2\theta)\ \sin^2\left(\pi \frac{L}{L_\odot}\right)$$



Three Flavor Oscillations in Vacuum

We have 3 flavors in nature

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U^* \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

Unitary Matrix

$$UU^\dagger = I \quad \Longrightarrow \sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j} = \delta_{\alpha\beta} \quad \sum_{\alpha=e}^{\tau} U_{\alpha j} U^*_{\alpha k} = \delta_{jk}$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} V$$

Majorana Phases

Three Flavor Oscillations in Vacuum

We have 3 flavors in nature

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U^* \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

Unitary Matrix

$$UU^\dagger = I \quad \Longrightarrow \sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j} = \delta_{\alpha\beta} \quad \sum_{\alpha=e}^{\tau} U_{\alpha j} U^*_{\alpha k} = \delta_{jk}$$

$$\mathbf{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\imath\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\imath\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\imath\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\imath\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\imath\delta} & c_{23}c_{13} \end{pmatrix} V$$

Majorana Phases

3 mixing angles + 1 phase

$$c_{ij} \equiv \cos\theta_{ij} \quad s_{ij} \equiv \sin\theta_{ij} \quad \theta_{ij} \in [0, \pi/2] \quad \delta \in [0, 2\pi]$$

$V = \mathrm{diag}(\alpha_1, 1, \alpha_3)$ do not enter oscillation

Oscillations are the same for DIRAC & MAJORANA neutrinos !

Three Flavor Oscillations in Vacuum

We have 3 flavors in nature

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U^* \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

Unitary Matrix

$$UU^\dagger = I \quad \Longrightarrow \sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j} = \delta_{\alpha\beta} \quad \sum_{\alpha=e}^{\tau} U_{\alpha j} U^*_{\alpha k} = \delta_{jk}$$

$$\mathbf{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\imath\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\imath\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\imath\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\imath\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\imath\delta} & c_{23}c_{13} \end{pmatrix} V$$

Majorana Phases

3 mixing angles + 1 phase

$$c_{ij} \equiv \cos\theta_{ij} \quad s_{ij} \equiv \sin\theta_{ij} \quad \theta_{ij} \in [0, \pi/2] \quad \delta \in [0, 2\pi]$$

$$V = \mathrm{diag}(\alpha_1, 1, \alpha_3)$$

$$\Delta m^2_{31} = \Delta m^2_{32} - \Delta m^2_{21}$$

only two independent

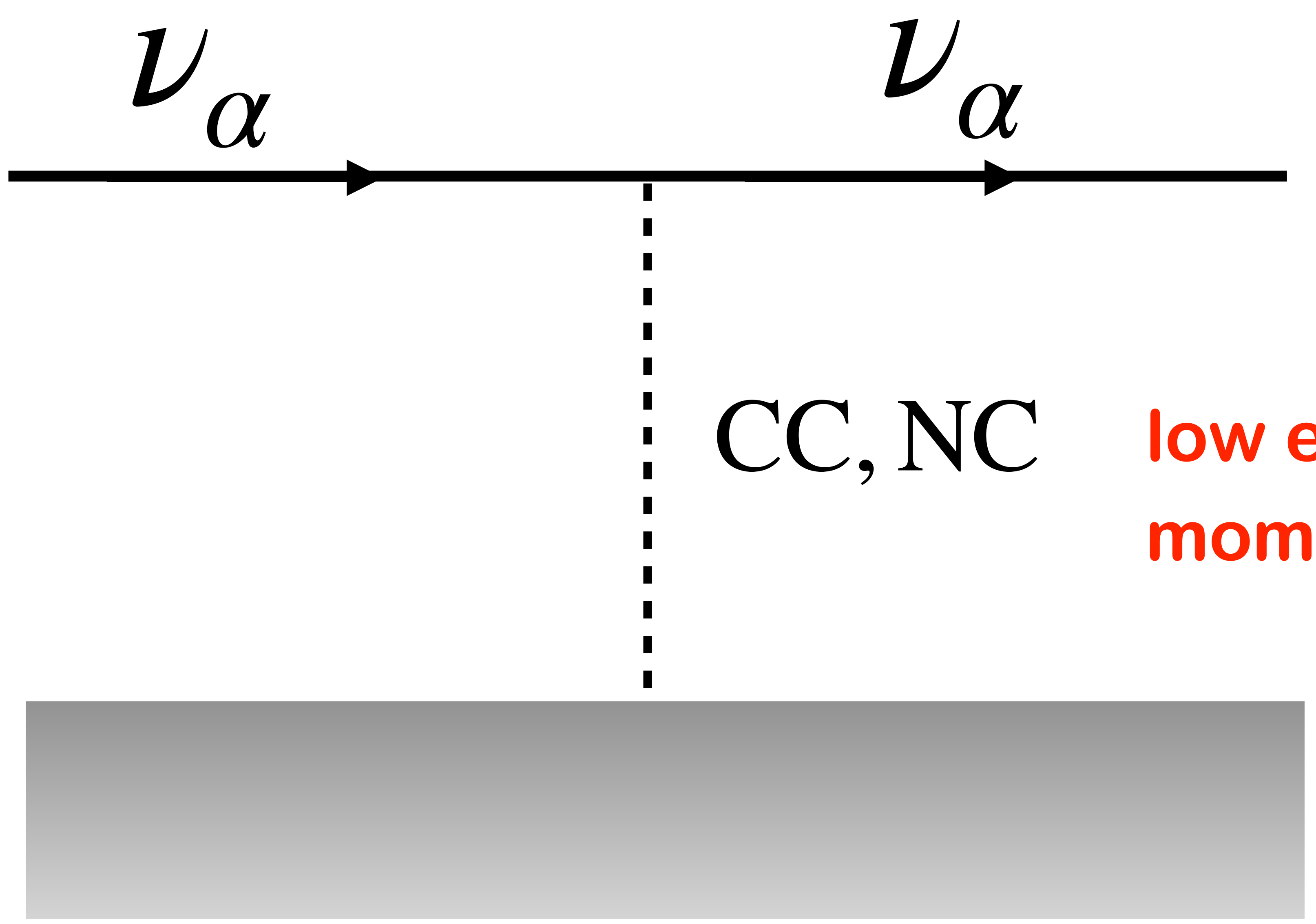
Neutrino Flavor Oscillations in Matter

Basic Concepts

incoherent processes (capture, finite angle scattering) $\sigma \propto G_F^2$

coherent forward scattering effects $\propto G_F$ (a lot larger!)

they lead to a tiny effective potential for neutrinos in matter



low energy limit
momentum transfer $\ll M_W, M_Z$

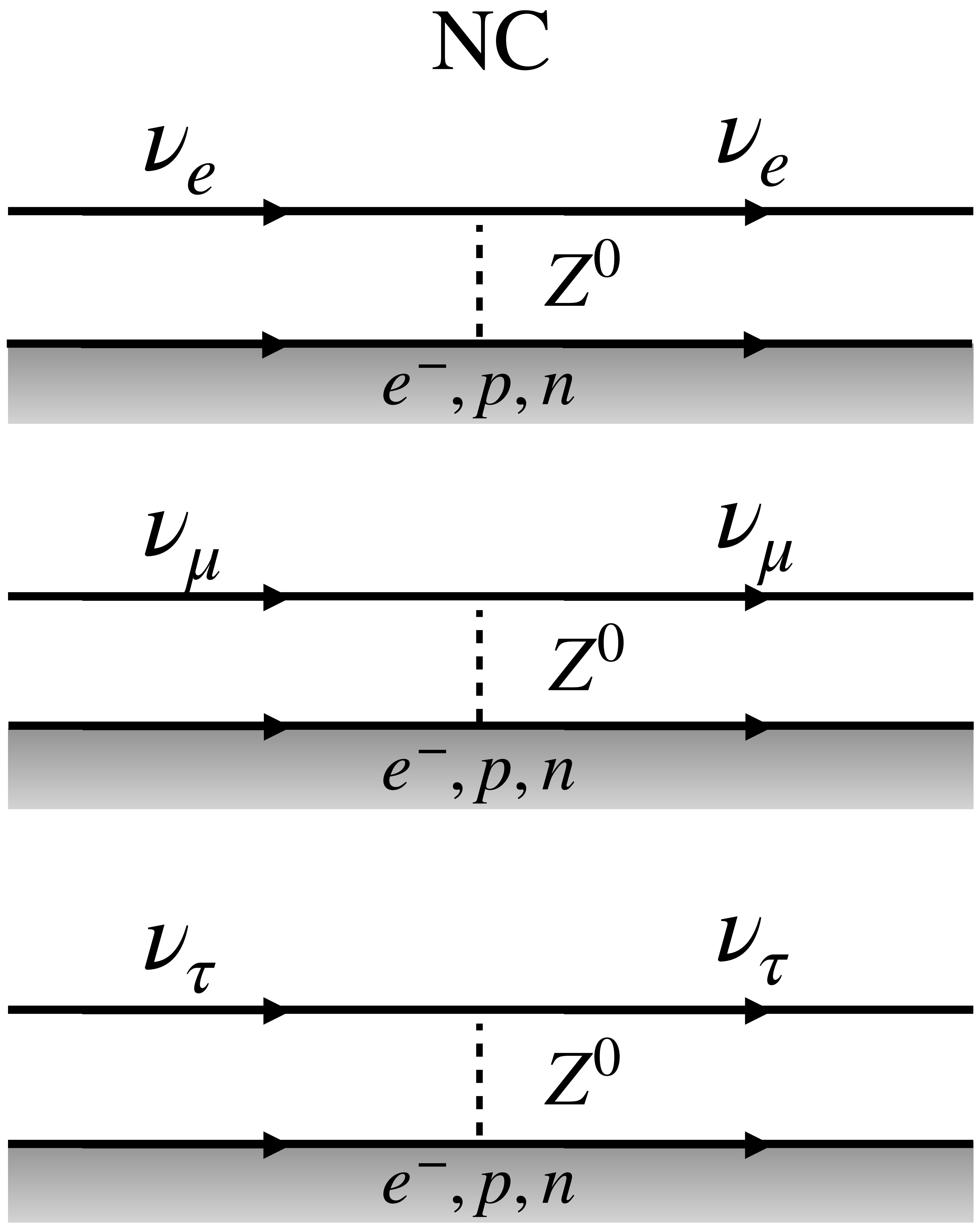
ordinary matter (electrons, nucleons)

[L. Wolfenstein (1978)]

[S. Mikheyev and A. Yu Smirnov (1985)]

Neutrino Flavor Oscillations in Matter

Basic Concepts

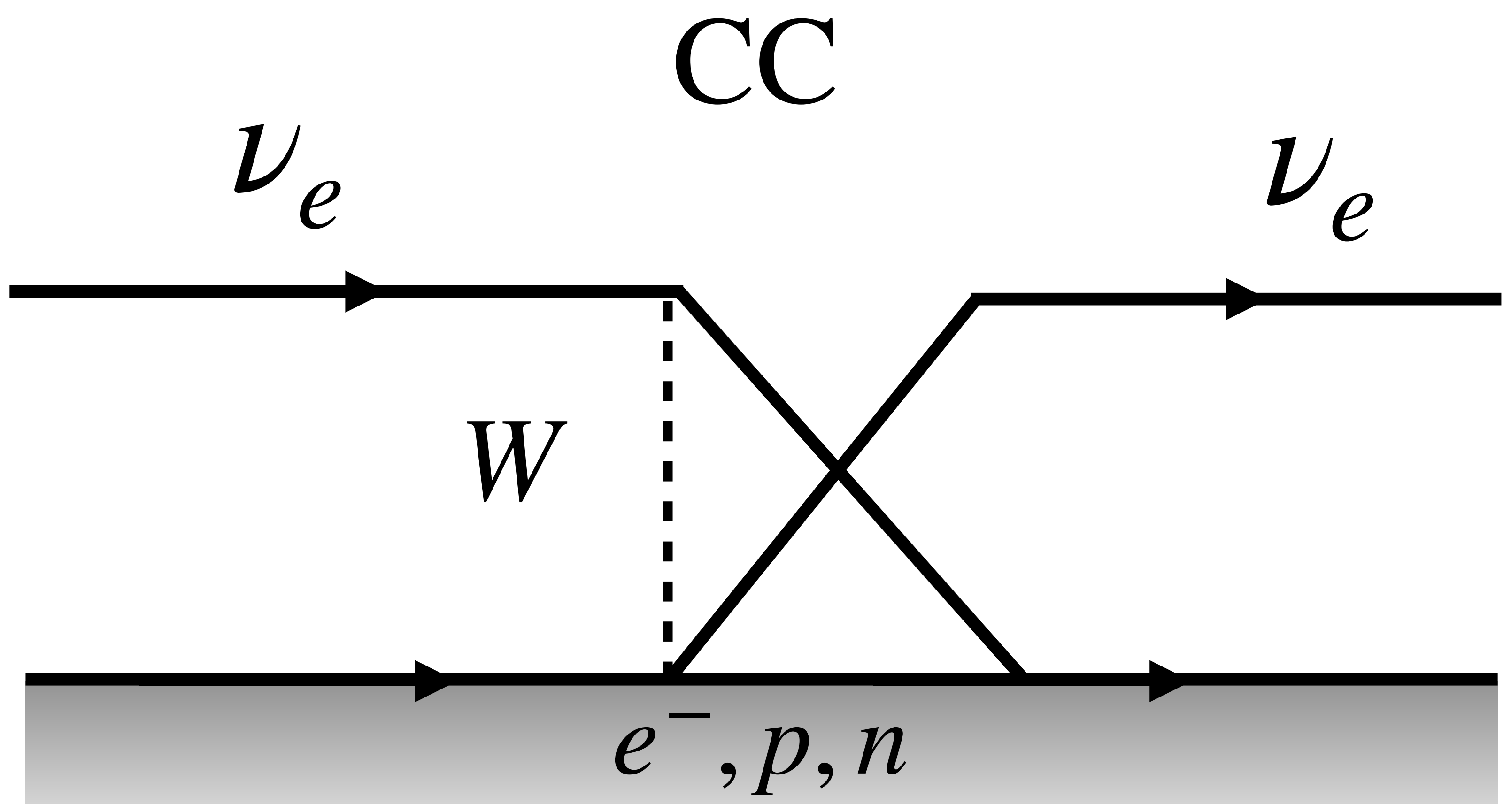


affects all neutrinos in the same way

gives rise to an unobservable common phase

Neutrino Flavor Oscillations in Matter

Basic Concepts



for a homogeneous and isotropic medium (Earth, Sun)

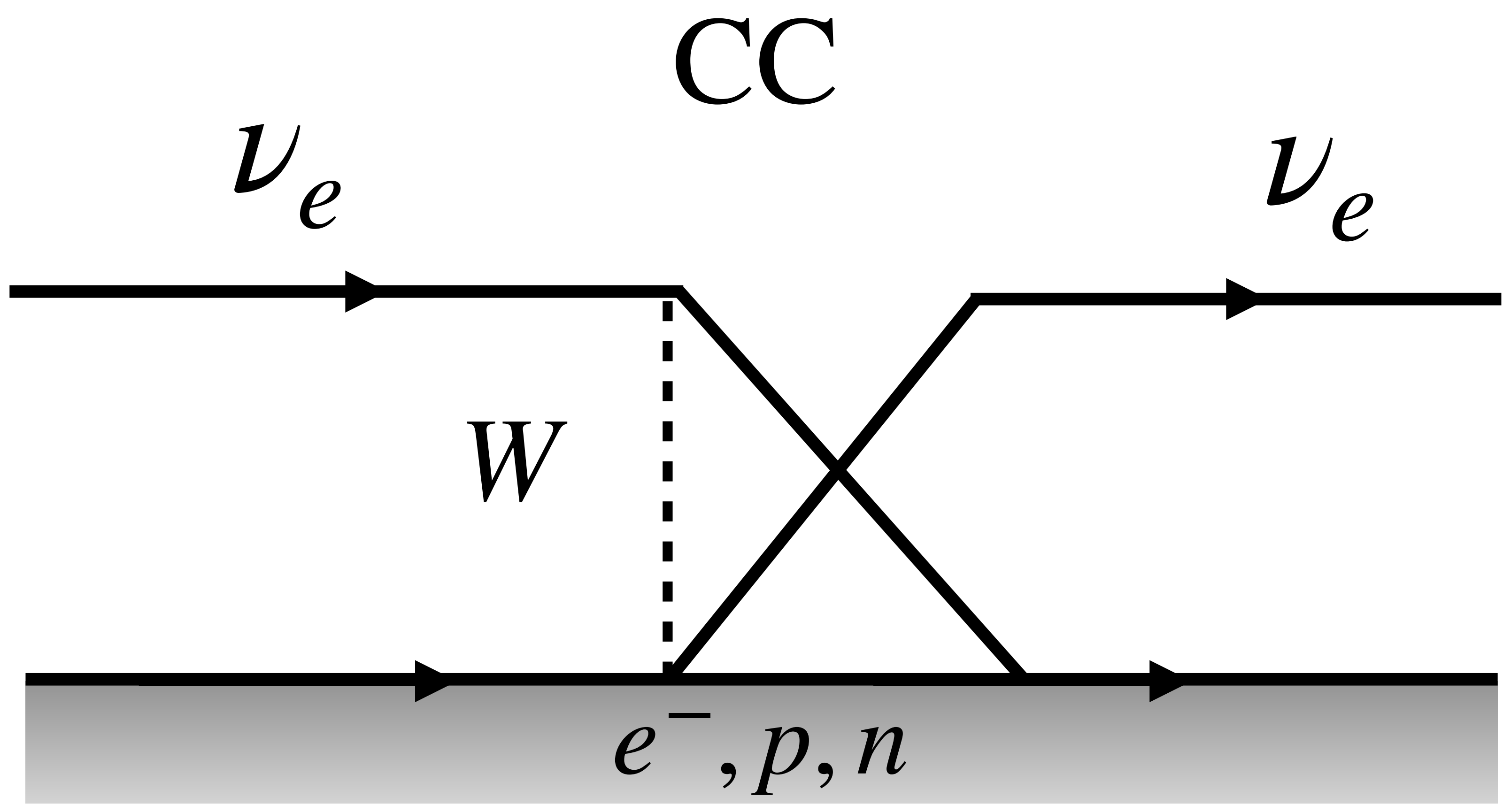
$$V_{cc}(r) = \sqrt{2} G_F n_e(r)$$

electron number density

singles out ν_e

Neutrino Flavor Oscillations in Matter

Basic Concepts



for a homogeneous and isotropic medium (Earth, Sun)

$$V_{\rm cc}(r) = \sqrt{2} G_F \, n_e(r)$$

electron number density

singles out ν_e

and $\overline{\nu}_e$: $V_{\rm cc}(r) \rightarrow -V_{\rm cc}(r)$

gives rise to an extra phase difference among the flavors

Neutrino Flavor Oscillations in Matter

order of magnitude of the effect

electron fraction　　matter density

$$V_{\rm cc}(r) = \sqrt{2} G_F \, n_e(r) \simeq 7.6 \left(\frac{n_e}{n_p + n_n} \right) \left(\frac{\rho}{\rm g/cm^3} \right) \times 10^{-14} \, \rm eV$$

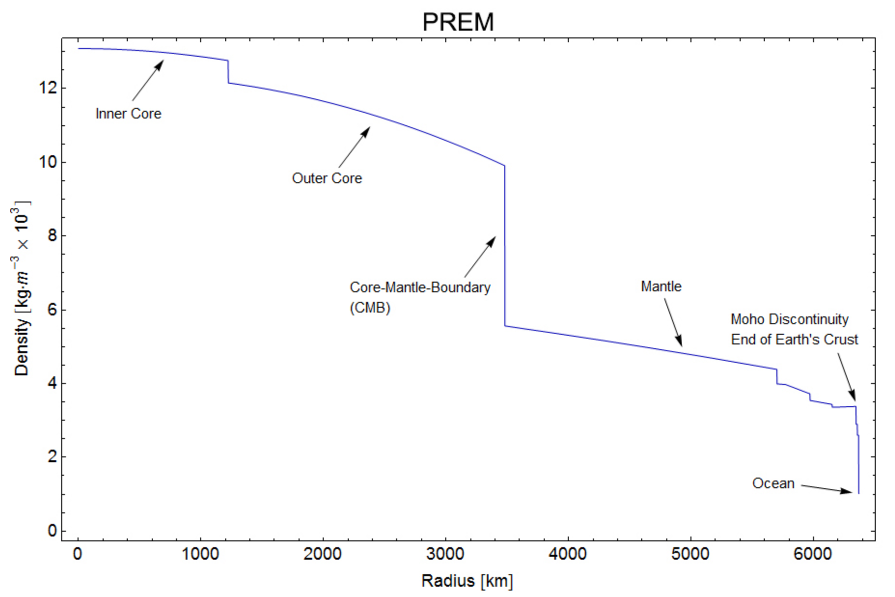
electron fraction $\sim \dfrac{1}{2}$

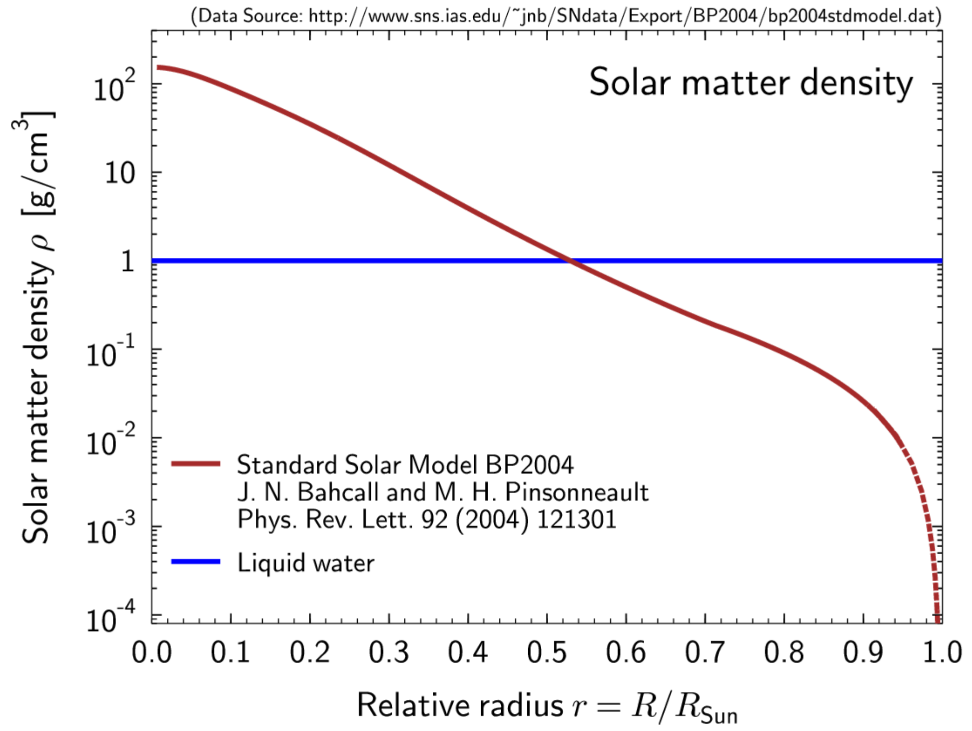
$\rho = 3\,\rm g/cm^3 \implies V_{\rm cc} \sim 1 \times 10^{-13}\,\rm eV$ — Earth's crust

$\rho = 10\,\rm g/cm^3 \implies V_{\rm cc} \sim 4 \times 10^{-13}\,\rm eV$ — Earth's core

$\rho = 100\,\rm g/cm^3 \implies V_{\rm cc} \sim 4 \times 10^{-12}\,\rm eV$ — Sun's core

$\rho = 10^{14}\,\rm g/cm^3 \implies V_{\rm cc} \sim 4\,\rm eV$ — Supernova





Neutrino Flavor Oscillations in Matter

For 3 flavors

We have to solve the Schrödinger evolution equation

$$i\frac{d}{dr}\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\\|\nu_\tau\rangle\end{pmatrix} = H\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\\|\nu_\tau\rangle\end{pmatrix}$$

for the Hamiltonian

matter contribution (dynamics)

$$H(r) = \underbrace{\frac{1}{2E}U\begin{pmatrix}m_1^2 & & \\ & m_2^2 & \\ & & m_3^2\end{pmatrix}U^\dagger}_{\text{vacuum contribution}} + \begin{pmatrix}V_{\mathrm{cc}}(r) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

Neutrino Flavor Oscillations in Matter

For 3 flavors

It is useful to write this like this

$$H(r) = \frac{1}{2E}\left[U \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} \overset{A_{cc}(r)}{\boxed{2EV_{cc}(r)}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]$$

Neutrino Flavor Oscillations in Matter

For 3 flavors

It is useful to write this like this

$$H(r) = \frac{1}{2E}\left[U \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} \overset{A_{cc}(r)}{\boxed{2EV_{cc}(r)}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]$$

$$\Delta m^2 \sim 2 \times 10\,\mathrm{MeV} \times 4 \times 10^{-12}\,\mathrm{eV} \sim 8 \times 10^{-5}\,\mathrm{eV}^2$$ 10 MeV ν @ Sun's core

$$\Delta m^2/(2EV_{cc}) \sim \mathcal{O}(1)$$

$$\Delta m^2 \sim 2 \times 10\,\mathrm{GeV} \times 10^{-13}\,\mathrm{eV} \sim 2 \times 10^{-3}\,\mathrm{eV}^2$$ 10 GeV ν @ Earth's crust

Neutrino Flavor Oscillations in Matter

For 3 flavors

This Hamiltonian can be diagonal at each point r of the evolution

$$H(r) = \frac{1}{2E}\left[U\begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}U^\dagger + \begin{pmatrix} A_{\rm cc}(r) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]$$

instantaneous mass eigenstates

$$= \frac{1}{2E}\widetilde{U}(r)\begin{pmatrix} \tilde{m}_1{}^2(r) & & \\ & \tilde{m}_2{}^2(r) & \\ & & \tilde{m}_3{}^2(r) \end{pmatrix}\widetilde{U}^\dagger(r)$$

$$\begin{pmatrix} |\nu_e(r)\rangle \\ |\nu_\mu(r)\rangle \\ |\nu_\tau(r)\rangle \end{pmatrix} = \tilde{U}(r)\begin{pmatrix} |\widetilde{\nu_1}(r)\rangle \\ |\widetilde{\nu_2}(r)\rangle \\ |\widetilde{\nu_3}(r)\rangle \end{pmatrix}$$

Neutrino Flavor Oscillations in Matter

For 3 flavors

This Hamiltonian can be diagonal at each point r of the evolution

$$H(r) = \frac{1}{2E}\left[U\begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix}U^\dagger + \begin{pmatrix} A_{\mathrm{cc}}(r) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]$$

in generally, solve numerally

Neutrino Flavor Oscillations in Matter

For 3 flavors

This Hamiltonian can be diagonal at each point r of the evolution

$$H(r) = \frac{1}{2E}\left[U \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} A_{\rm cc}(r) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]$$

in generally, solve numerally

but there are useful analytic solutions

$dA_{cc}(r)/dr = 0$ constant matter density

$dA_{cc}(r)/dr$ is small

i.e. adiabatic transition

we will discuss

them for 2 flavors

Neutrino Flavor Oscillations in Constant Matter

For 2 flavors

$$i\frac{d}{dr}\begin{pmatrix}|\nu_e\rangle\\|\nu_x\rangle\end{pmatrix}=\begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta+V_{\mathrm{cc}} & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}|\nu_e\rangle\\|\nu_x\rangle\end{pmatrix}$$

$$\Delta m^2=\Delta m^2_{21}$$

in constant matter density

mixing in matter

$$\widetilde{U}=\begin{pmatrix}\cos\theta_{\mathrm{m}} & \sin\theta_{\mathrm{m}}\\ -\sin\theta_{\mathrm{m}} & \cos\theta_{\mathrm{m}}\end{pmatrix}$$

$$|\nu_e\rangle=\cos\theta_{\mathrm{m}}\,|\widetilde{\nu_1}\rangle+\sin\theta_{\mathrm{m}}\,|\widetilde{\nu_2}\rangle$$

$$|\nu_x\rangle=-\sin\theta_{\mathrm{m}}\,|\widetilde{\nu_1}\rangle+\cos\theta_{\mathrm{m}}\,|\widetilde{\nu_2}\rangle$$

Neutrino Flavor Oscillations in Constant Matter

For 2 flavors

$$i\frac{d}{dr}\begin{pmatrix}|\nu_e\rangle\\|\nu_x\rangle\end{pmatrix} = \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta + V_{\rm cc} & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}|\nu_e\rangle\\|\nu_x\rangle\end{pmatrix}$$

$$\Delta m^2 = \Delta m^2_{21}$$

in constant matter density

mixing in matter

$$\widetilde{U} = \begin{pmatrix}\cos\theta_{\rm m} & \sin\theta_{\rm m}\\ -\sin\theta_{\rm m} & \cos\theta_{\rm m}\end{pmatrix}$$

$$|\nu_e\rangle = \cos\theta_{\rm m}\,|\widetilde{\nu_1}\rangle + \sin\theta_{\rm m}\,|\widetilde{\nu_2}\rangle$$

$$|\nu_x\rangle = -\sin\theta_{\rm m}\,|\widetilde{\nu_1}\rangle + \cos\theta_{\rm m}\,|\widetilde{\nu_2}\rangle$$

mass squared splitting in matter

$$\Delta\widetilde{m}^2 = \sqrt{(\Delta m^2\cos 2\theta - 2E\,V_{\rm cc})^2 + (\Delta m^2\sin 2\theta)^2}$$

$$\sin^2\theta_{\rm m} = \frac{1}{2}\left(1 + \frac{(2EV_{\rm cc} - \Delta m^2\cos 2\theta)}{\Delta\widetilde{m}^2}\right)$$

if $2EV_{\rm cc} \gg \Delta m^2\cos 2\theta$

$\sin^2\theta_m \to 1 \quad |\nu_\alpha\rangle \to |\widetilde{\nu}_2\rangle$

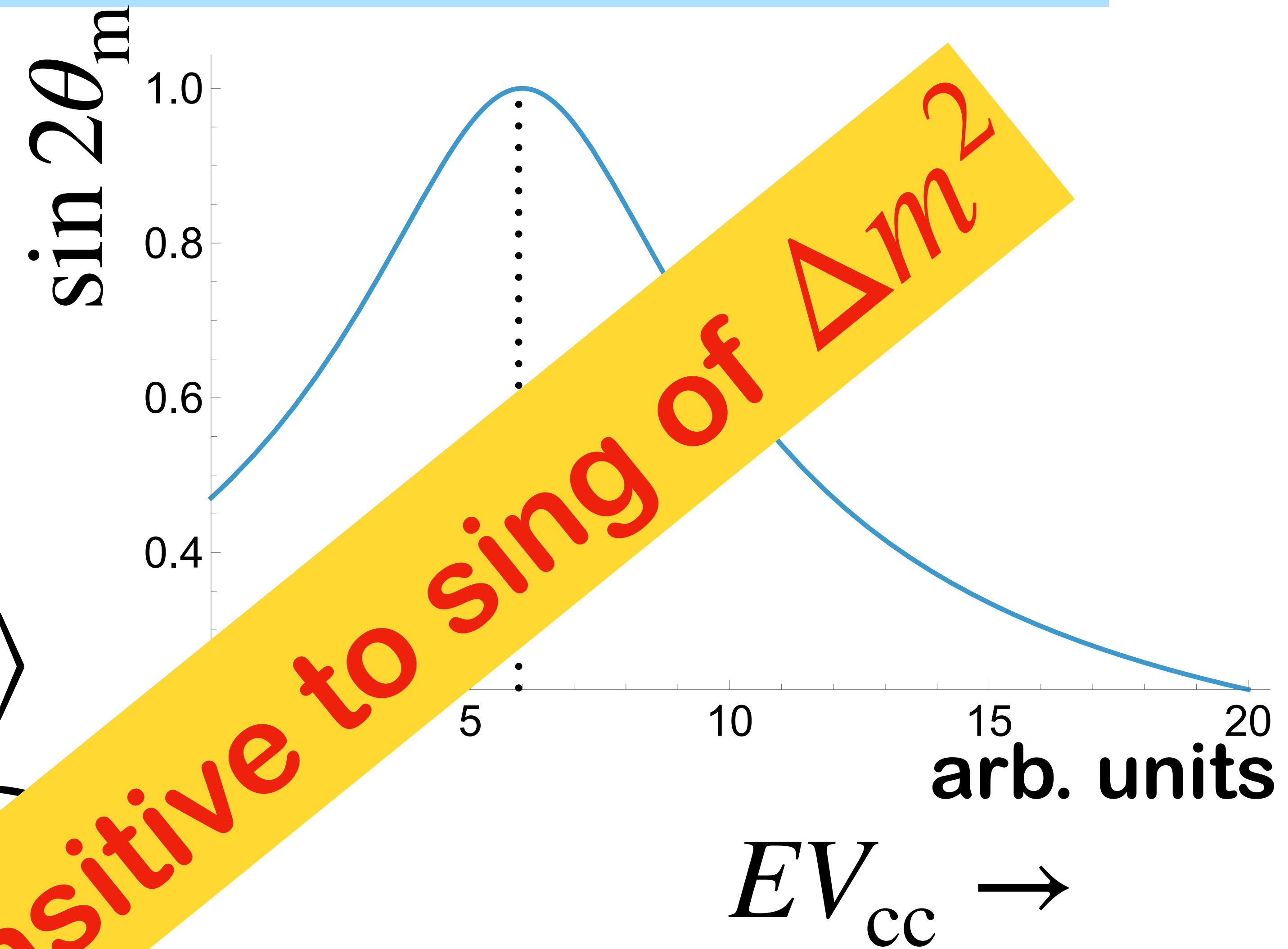
if $2EV_{\rm cc} \ll \Delta m^2\cos 2\theta$

we recover vacuum solution

Neutrino Flavor Oscillations in Constant Matter

For 2 flavors

$$i\frac{d}{dr}\begin{pmatrix}|\nu_e\rangle \\ |\nu_x\rangle\end{pmatrix} = \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta + V_{\rm cc} & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}|\nu_e\rangle \\ |\nu_x\rangle\end{pmatrix}$$



mixing in matter

$$\widetilde{U} = \begin{pmatrix}\cos\theta_{\rm m} & \sin\theta_{\rm m} \\ -\sin\theta_{\rm m} & \cos\theta_{\rm m}\end{pmatrix}$$

$$|\nu_e\rangle = \cos\theta_{\rm m}\,|\widetilde{\nu_1}\rangle + \sin\theta_{\rm m}\,|\widetilde{\nu_2}\rangle$$

$$|\nu_x\rangle = -\sin\theta_{\rm m}\,|\widetilde{\nu_1}\rangle + \cos\theta_{\rm m}\,|\widetilde{\nu_2}\rangle$$

mass squared splitting in matter

$$\Delta\widetilde{m}^2 = \sqrt{(\Delta m^2\cos 2\theta - 2E\,V_{\rm cc})^2 + (\Delta m^2\sin 2\theta)^2}$$

mixing in matter

$$\sin 2\theta_{\rm m} = \frac{\sin 2\theta}{\sqrt{(\cos 2\theta - 2EV_{\rm cc}/\Delta m^2)^2 + \sin^2 2\theta}}$$

Matter Effects are sensitive to sing of Δm^2

MSW resonance condition

$$\Delta m^2\cos 2\theta = 2E\,V_{\rm cc}$$

maximal mixing in matter

$$\theta_m = 45^\circ$$

happens either if $\Delta m^2, \cos 2\theta > 0$ or $\Delta m^2, \cos 2\theta < 0$

Adiabatic Transition in Matter

For 2 flavors

$$i\frac{d}{dr}\begin{pmatrix}|\nu_e\rangle \\ |\nu_x\rangle\end{pmatrix} = H\begin{pmatrix}|\nu_e\rangle \\ |\nu_x\rangle\end{pmatrix} \qquad H = \frac{1}{2E}\widetilde{U}(r)\begin{pmatrix}\tilde{m}_1{}^2(r) & \\ & \tilde{m}_2{}^2(r)\end{pmatrix}\widetilde{U}^\dagger(r)$$

$$\begin{pmatrix}|\nu_e(r)\rangle \\ |\nu_x(r)\rangle\end{pmatrix} = \widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu_1}(r)\rangle \\ |\widetilde{\nu_2}(r)\rangle\end{pmatrix}$$

Adiabatic Transition in Matter

For 2 flavors

$$i\frac{d}{dr}\begin{pmatrix}|\nu_e\rangle\\|\nu_x\rangle\end{pmatrix} = H\begin{pmatrix}|\nu_e\rangle\\|\nu_x\rangle\end{pmatrix} \qquad H = \frac{1}{2E}\widetilde{U}(r)\begin{pmatrix}\tilde{m}_1{}^2(r) & \\ & \tilde{m}_2{}^2(r)\end{pmatrix}\widetilde{U}^\dagger(r)$$

$$\begin{pmatrix}|\nu_e(r)\rangle\\|\nu_x(r)\rangle\end{pmatrix} = \widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu_1}(r)\rangle\\|\widetilde{\nu_2}(r)\rangle\end{pmatrix}$$

$$\Rightarrow \quad i\frac{d}{dr}\widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu}_1\rangle\\|\widetilde{\nu}_2\rangle\end{pmatrix} = \frac{1}{2E}\widetilde{U}(r)\begin{pmatrix}\tilde{m}_1{}^2(r) & \\ & \tilde{m}_2{}^2(r)\end{pmatrix}\widetilde{U}^\dagger(r)\widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu}_1\rangle\\|\widetilde{\nu}_2\rangle\end{pmatrix}$$

Adiabatic Transition in Matter

For 2 flavors

$$i\frac{d}{dr}\begin{pmatrix}|\nu_e\rangle \\ |\nu_x\rangle\end{pmatrix} = H\begin{pmatrix}|\nu_e\rangle \\ |\nu_x\rangle\end{pmatrix} \qquad H = \frac{1}{2E}\widetilde{U}(r)\begin{pmatrix}\tilde{m}_1{}^2(r) & \\ & \tilde{m}_2{}^2(r)\end{pmatrix}\widetilde{U}^\dagger(r)$$

$$\begin{pmatrix}|\nu_e(r)\rangle \\ |\nu_x(r)\rangle\end{pmatrix} = \widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu_1}(r)\rangle \\ |\widetilde{\nu_2}(r)\rangle\end{pmatrix}$$

$$\Rightarrow \quad i\frac{d}{dr}\widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle\end{pmatrix} = \frac{1}{2E}\widetilde{U}(r)\begin{pmatrix}\tilde{m}_1{}^2(r) & \\ & \tilde{m}_2{}^2(r)\end{pmatrix}\widetilde{U}^\dagger(r)\widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle\end{pmatrix}$$

$$i\frac{d}{dr}\widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle\end{pmatrix} = \frac{1}{2E}\widetilde{U}(r)\begin{pmatrix}\tilde{m}_1{}^2(r) & \\ & \tilde{m}_2{}^2(r)\end{pmatrix}\begin{pmatrix}|\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle\end{pmatrix}$$

Adiabatic Transition in Matter

For 2 flavors

$$i\frac{d}{dr}\widetilde{U}(r)\begin{pmatrix} |\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle \end{pmatrix} = \frac{1}{2E}\widetilde{U}(r)\begin{pmatrix} \tilde{m}_1{}^2(r) & \\ & \tilde{m}_2{}^2(r) \end{pmatrix}\begin{pmatrix} |\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle \end{pmatrix} \qquad \textbf{(1)}$$

slowly varying matter density (Sun)

if $\frac{d}{dr}\widetilde{U}(r) \simeq 0$

$$i\frac{d}{dr}\widetilde{U}(r)\begin{pmatrix} |\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle \end{pmatrix} \simeq i\,\widetilde{U}(r)\frac{d}{dr}\begin{pmatrix} |\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle \end{pmatrix} \qquad \textbf{(2)}$$

Adiabatic Transition in Matter

For 2 flavors

$$i\frac{d}{dr}\widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle\end{pmatrix} = \frac{1}{2E}\widetilde{U}(r)\begin{pmatrix}\tilde{m}_1{}^2(r) & \\ & \tilde{m}_2{}^2(r)\end{pmatrix}\begin{pmatrix}|\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle\end{pmatrix} \quad \textbf{(1)}$$

slowly varying matter density (Sun)

if $\frac{d}{dr}\widetilde{U}(r) \simeq 0$ $\quad i\frac{d}{dr}\widetilde{U}(r)\begin{pmatrix}|\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle\end{pmatrix} \simeq i\,\widetilde{U}(r)\frac{d}{dr}\begin{pmatrix}|\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle\end{pmatrix}$ **(2)**

$\widetilde{U}(r)^\dagger \times$ **(1) using (2)**

$$\boxed{i\frac{d}{dr}\begin{pmatrix}|\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle\end{pmatrix} = \frac{1}{2E}\begin{pmatrix}\tilde{m}_1{}^2(r) & \\ & \tilde{m}_2{}^2(r)\end{pmatrix}\begin{pmatrix}|\widetilde{\nu}_1\rangle \\ |\widetilde{\nu}_2\rangle\end{pmatrix}}$$

Equations decouple !

Mass eigenstates evolve independently

They do not mix!

Adiabatic Transition in Matter

For 2 flavors

Mass eigenstates only get a phase during propagation

$$|\widetilde{\nu}_1\rangle(r_f) = e^{-i\boxed{\int_{r_i}^{r_f}\left(\frac{\tilde{m}_1^2(r)}{2E}\right)dr}}|\widetilde{\nu}_1\rangle(r_i) \qquad \phi_1/2E$$

$$|\widetilde{\nu}_2\rangle(r_f) = e^{-i\boxed{\int_{r_i}^{r_f}\left(\frac{\tilde{m}_2^2(r)}{2E}\right)dr}}|\widetilde{\nu}_2\rangle(r_i) \qquad \phi_2/2E$$

Adiabatic Transition in Matter

For 2 flavors

Mass eigenstates only get a phase during propagation

$$|\widetilde{\nu}_1\rangle(r_f) = e^{-i\boxed{\int_{r_i}^{r_f}\left(\frac{\tilde{m}_1^2(r)}{2E}\right)dr}}|\widetilde{\nu}_1\rangle(r_i) \qquad |\widetilde{\nu}_2\rangle(r_f) = e^{-i\boxed{\int_{r_i}^{r_f}\left(\frac{\tilde{m}_2^2(r)}{2E}\right)dr}}|\widetilde{\nu}_2\rangle(r_i)$$

$\phi_1/2E$ $\qquad$ $\phi_2/2E$

$$\begin{pmatrix}|\nu_e(r_i)\rangle\\|\nu_x(r_i)\rangle\end{pmatrix} = \widetilde{U}(r_i)\begin{pmatrix}|\widetilde{\nu_1}(r_i)\rangle\\|\widetilde{\nu_2}(r_i)\rangle\end{pmatrix} = \begin{pmatrix}\cos\widetilde{\theta}(r_i) & \sin\widetilde{\theta}(r_i)\\-\sin\widetilde{\theta}(r_i) & \cos\widetilde{\theta}(r_i)\end{pmatrix}\begin{pmatrix}|\widetilde{\nu_1}(r_i)\rangle\\|\widetilde{\nu_2}(r_i)\rangle\end{pmatrix}$$

Flavor eigenstates @ the production point

$$\begin{pmatrix}|\nu_e(r_f)\rangle\\|\nu_x(r_f)\rangle\end{pmatrix} = \widetilde{U}(r_f)\begin{pmatrix}|\widetilde{\nu_1}(r_f)\rangle\\|\widetilde{\nu_2}(r_f)\rangle\end{pmatrix} = \begin{pmatrix}\cos\widetilde{\theta}(r_f) & \sin\widetilde{\theta}(r_f)\\-\sin\widetilde{\theta}(r_f) & \cos\widetilde{\theta}(r_f)\end{pmatrix}\begin{pmatrix}|\widetilde{\nu_1}(r_f)\rangle\\|\widetilde{\nu_2}(r_f)\rangle\end{pmatrix}$$

Flavor eigenstates @ the end of evolution

Adiabatic Transition in Matter

For 2 flavors

Mass eigenstates only get a phase during propagation

$$|\widetilde{\nu}_1\rangle(r_f) = e^{-i\boxed{\int_{r_i}^{r_f}\left(\frac{\tilde{m}_1^2(r)}{2E}\right)dr}}|\widetilde{\nu}_1\rangle(r_i) \qquad |\widetilde{\nu}_2\rangle(r_f) = e^{-i\boxed{\int_{r_i}^{r_f}\left(\frac{\tilde{m}_2^2(r)}{2E}\right)dr}}|\widetilde{\nu}_2\rangle(r_i)$$

$\phi_1/2E$ $\qquad$ $\phi_2/2E$

$$\begin{pmatrix}|\nu_e(r_i)\rangle\\|\nu_x(r_i)\rangle\end{pmatrix} = \widetilde{U}(r_i)\begin{pmatrix}|\widetilde{\nu_1}(r_i)\rangle\\|\widetilde{\nu_2}(r_i)\rangle\end{pmatrix} = \begin{pmatrix}\cos\widetilde{\theta}(r_i) & \sin\widetilde{\theta}(r_i)\\-\sin\widetilde{\theta}(r_i) & \cos\widetilde{\theta}(r_i)\end{pmatrix}\begin{pmatrix}|\widetilde{\nu_1}(r_i)\rangle\\|\widetilde{\nu_2}(r_i)\rangle\end{pmatrix}$$

Flavor eigenstates @ the production point

$$\begin{pmatrix}|\nu_e(r_f)\rangle\\|\nu_x(r_f)\rangle\end{pmatrix} = \widetilde{U}(r_f)\begin{pmatrix}|\widetilde{\nu_1}(r_f)\rangle\\|\widetilde{\nu_2}(r_f)\rangle\end{pmatrix} = \begin{pmatrix}\cos\widetilde{\theta}(r_f) & \sin\widetilde{\theta}(r_f)\\-\sin\widetilde{\theta}(r_f) & \cos\widetilde{\theta}(r_f)\end{pmatrix}\begin{pmatrix}|\widetilde{\nu_1}(r_f)\rangle\\|\widetilde{\nu_2}(r_f)\rangle\end{pmatrix}$$

Flavor eigenstates @ the end of evolution

$$= \begin{pmatrix}\cos\widetilde{\theta}(r_f) & \sin\widetilde{\theta}(r_f)\\-\sin\widetilde{\theta}(r_f) & \cos\widetilde{\theta}(r_f)\end{pmatrix}\begin{pmatrix}e^{-i\phi_1/2E} & 0\\0 & e^{-i\phi_2/2E}\end{pmatrix}\begin{pmatrix}\cos\widetilde{\theta}(r_i) & -\sin\widetilde{\theta}(r_i)\\\sin\widetilde{\theta}(r_i) & \cos\widetilde{\theta}(r_i)\end{pmatrix}\begin{pmatrix}|\nu_e(r_i)\rangle\\|\nu_x(r_i)\rangle\end{pmatrix}$$

Adiabatic Transition in Matter

For 2 flavors

Survival Propability of ν_e

$$P_{ee} = |\mathscr{A}_{ee}|^2 = |\langle \nu_e(r_f) | \nu_e(r_i) \rangle|^2 = |\cos\widetilde{\theta}(r_f)\, \cos\widetilde{\theta}(r_i) + e^{-i\frac{(\phi_2-\phi_1)}{2E}}\, \sin\widetilde{\theta}(r_f)\, \sin\widetilde{\theta}(r_i)|^2$$

the interference term average out

$\Delta\phi \gg E$ [S. Parke (1986)]

for solar neutrinos

$$\boxed{P_{ee} \simeq \cos^2\theta_{12}\, \cos^2\widetilde{\theta}(r_i) + \sin^2\theta_{12}\, \sin^2\widetilde{\theta}(r_i)}$$

vacuum value @ the exit of the Sun

How about the Mass Ordering?

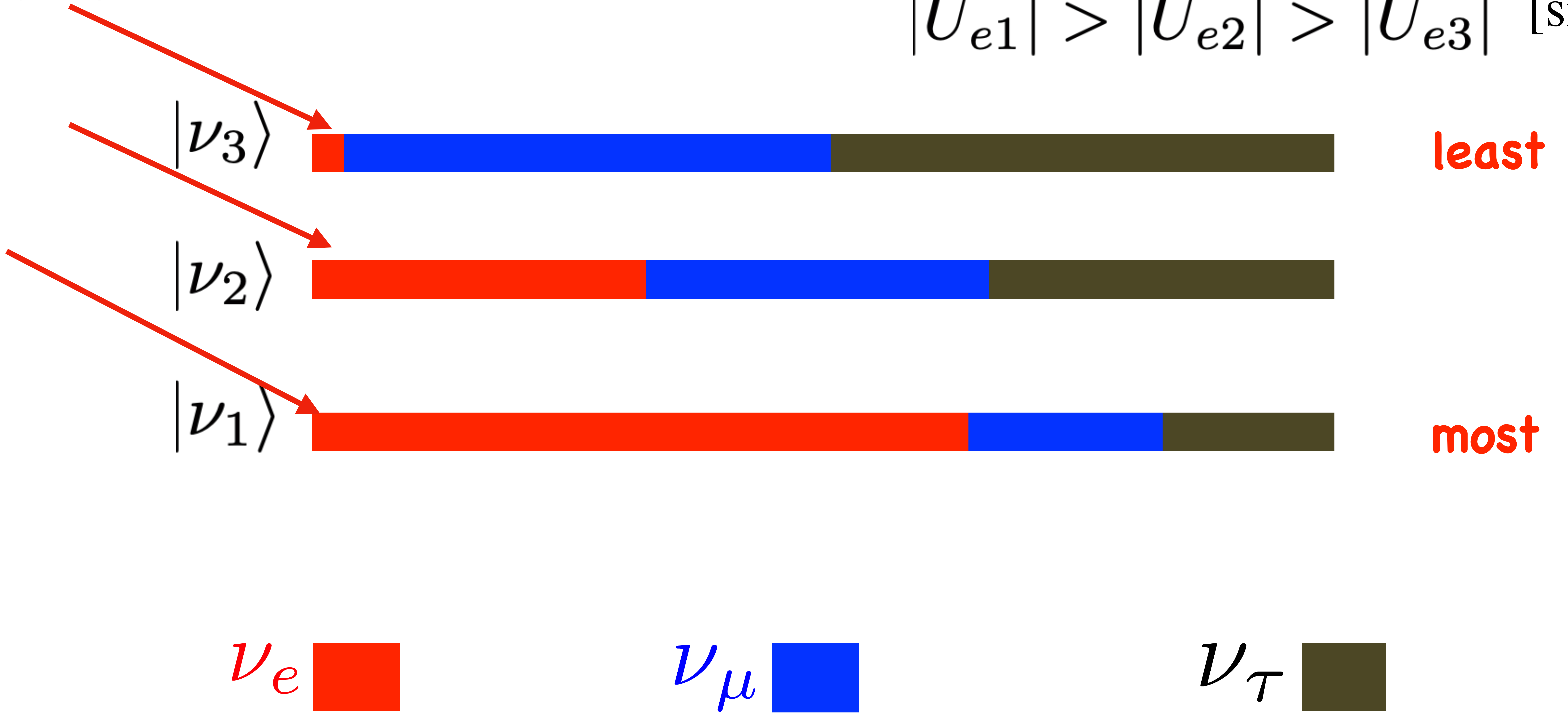
How can we Label the Mass Eigentates?

Using data results

using $|\nu_e\rangle$ fraction

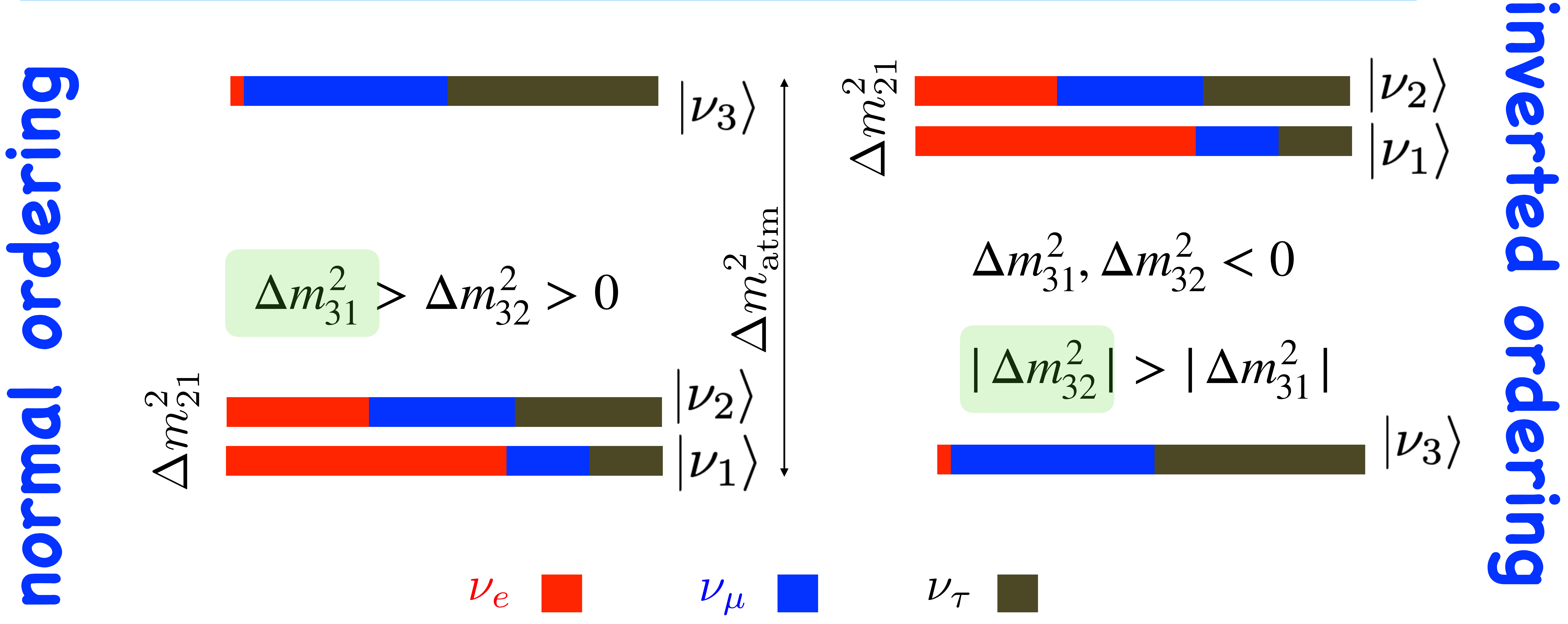
we know that (see Lecture II)

$$|U_{e1}| > |U_{e2}| > |U_{e3}| \quad [\sin^2\theta_{12} \simeq 0.3]$$



What is the neutrino mass Ordering?

We currently have two possibilities



Effect $\Delta m^2_{\rm atm}$ - scale
For Disappearance Experiments

[H. Nunokawa, S. J. Parke, RZF (2005)]

$$\nu_e \rightarrow \nu_e$$
$$\overline{\nu}_e \rightarrow \overline{\nu}_e$$

- for experiments where $\Delta_{21} \equiv \dfrac{\Delta m^2_{21} L}{4E} \ll 1$

in vacuum

$$P_{\nu_e \rightarrow \nu_e} = 1 - \sin^2 2\theta_{13} \sin^2 \Delta_{ee} + \mathcal{O}(\Delta^2_{21})$$

DayaBay/RENO

$$\Delta_{21}(\langle E \rangle \sim 4\,\mathrm{MeV}, \mathrm{L} \sim 1\,\mathrm{km}) \sim 2\,\%$$

$$\boxed{\Delta m^2_{ee} \equiv \Delta m^2_{31} \cos^2\theta_{12} + \Delta m^2_{32} \sin^2\theta_{21}}$$

effective scale for ν_e disappearance

$$\Delta m^2_{31}\,|^{\rm NO}_{e} = |\,\Delta m^2_{ee}\,| + \Delta m^2_{21} \sin^2\theta_{12}$$

$$|\,\Delta m^2_{32}\,|^{\rm IO}_{e} = |\,\Delta m^2_{ee}\,| + \Delta m^2_{21} \cos^2\theta_{12}$$

$\Delta m^2_{31}, \Delta m^2_{32}, \Delta m^2_{\rm ee}$

all equivalent

$$|\,\Delta m^2_{32}\,|^{\rm IO}_{e} - \Delta m^2_{31}\,|^{\rm NO}_{e} = \Delta m^2_{21} \cos 2\theta_{12}$$

Effect $\Delta m^2_{\rm atm}$ - scale

For Disappearance Experiments

[H. Nunokawa, S. J. Parke, RZF (2005)]

$$\nu_\mu \rightarrow \nu_\mu$$

$$\overline{\nu}_\mu \rightarrow \overline{\nu}_\mu$$

- for experiments where $\Delta_{21} \equiv \dfrac{\Delta m^2_{21} L}{4E} \ll 1$

in vacuum

$$P_{\nu_\mu \to \nu_\mu} = 1 - \sin^2 2\theta_{\rm eff} \sin^2 \Delta_{\mu\mu} + \mathcal{O}(\Delta^2_{21})$$

T2K

$$\Delta_{21}(\langle E \rangle \sim 0.6\,\text{GeV}, \text{L} \sim 295\,\text{km}) \sim 5\,\%$$

NOvA

$$\Delta_{21}(\langle E \rangle \sim 2.0\,\text{GeV}, \text{L} \sim 810\,\text{km}) \sim 4\,\%$$

$$\boxed{\Delta m^2_{\mu\mu} \equiv \Delta m^2_{31} \sin^2\theta_{12} + \Delta m^2_{32} \cos^2\theta_{21} + \sin\theta_{13} \sin 2\theta_{12} \tan\theta_{23} \cos\delta\, \Delta m^2_{21}}$$

effective scale for ν_μ disappearance

$$\Delta m^2_{31}\,|^{\rm NO}_\mu = |\,\Delta m^2_{\mu\mu}\,| + \Delta m^2_{21}(\cos^2\theta_{12} - \sin\theta_{13}\cos\delta^{\rm NO})$$

$$|\,\Delta m^2_{32}\,|^{\rm IO}_\mu = |\,\Delta m^2_{\mu\mu}\,| + \Delta m^2_{21}(\sin^2\theta_{12} + \sin\theta_{13}\cos\delta^{\rm IO})$$

$\Delta m^2_{31}, \Delta m^2_{32}, \Delta m^2_{\mu\mu}$

all equivalent

$$\Delta m^2_{31}\,|^{\rm NO}_\mu - |\,\Delta m^2_{32}\,|^{\rm IO}_\mu = \Delta m^2_{21}\,(\cos 2\theta_{12} - 2\sin\theta_{13}\,\overline{\cos\delta})$$

Mass Ordering Sum Rule

For Disappearance Experiments

[S. J. Parke, RZF (2024) arXiv:2404.0873]

@ the moment no disappearance experiment has any sensitivity to the ordering

few $\%$

$$\boxed{(\Delta m^2_{31}|^{NO}_{\mu} - \Delta m^2_{31}|^{NO}_{e}) + (|\Delta m^2_{32}|^{IO}_{e} - |\Delta m^2_{32}|^{IO}_{\mu}) = (2\cos 2\theta_{12} - 2\sin\theta_{13}\overline{\cos\delta})\,\Delta m^2_{21}}$$

$$(2.4 - 0.9\,\overline{\cos\delta})\,\%\,|\Delta m^2_{atm}|$$

$\Delta m^2_{31}|^{NO}_{\mu} = \Delta m^2_{31}|^{NO}_{e}$ if NO is true $\Delta m^2_{31}|^{NO}_{T2K+NOvA} = (2.516 \pm 0.031) \times 10^{-3}\mathrm{eV}^2$

$$\sim 1.2\,\%$$

$|\Delta m^2_{32}|^{IO}_{e} = |\Delta m^2_{32}|^{IO}_{\mu}$ if IO is true $|\Delta m^2_{32}|^{IO}_{T2K+NOvA} = (2.485 \pm 0.031) \times 10^{-3}\mathrm{eV}^2$

[http://www.nufit.org - v52.fig-chisq]-dma.pdf]

Mass Ordering Sum Rule

For Disappearance Experiments

[S. J. Parke, RZF (2024) arXiv:2404.0873]

@ the moment no disappearance experiment has any sensitivity to the ordering

few %

$$\boxed{(\Delta m^2_{31}|^{NO}_{\mu} - \Delta m^2_{31}|^{NO}_{e}) + (|\Delta m^2_{32}|^{IO}_{e} - |\Delta m^2_{32}|^{IO}_{\mu}) = (2\cos 2\theta_{12} - 2\sin\theta_{13}\overline{\cos\delta})\,\Delta m^2_{21}}$$

$$(2.4 - 0.9\,\overline{\cos\delta})\,\%\,|\Delta m^2_{atm}|$$

$\Delta m^2_{31}|^{NO}_{\mu} = \Delta m^2_{31}|^{NO}_{e}$ if NO is true

Daya Bay

$$\sim 2.4\,\%$$

$|\Delta m^2_{32}|^{IO}_{e} = |\Delta m^2_{32}|^{IO}_{\mu}$ if IO is true

[http://www.nufit.org - v52.fig-chisq]-dma.pdf]

Disappearance Experiments Synergy

Neutrino Mass Ordering

$$\Delta m^2_{31}|^{NO}_{\mu} > |\Delta m^2_{32}|^{IO}_{\mu}$$

T2K/NOνA

$$\Delta m^2_{31}|^{NO}_{\mu} - |\Delta m^2_{32}|^{IO}_{\mu} = \Delta m^2_{21}(\cos 2\theta_{12} - 2\sin\theta_{13}\overline{\cos\delta})$$

Daya Bay/RENO/DC

T2K/NOνA

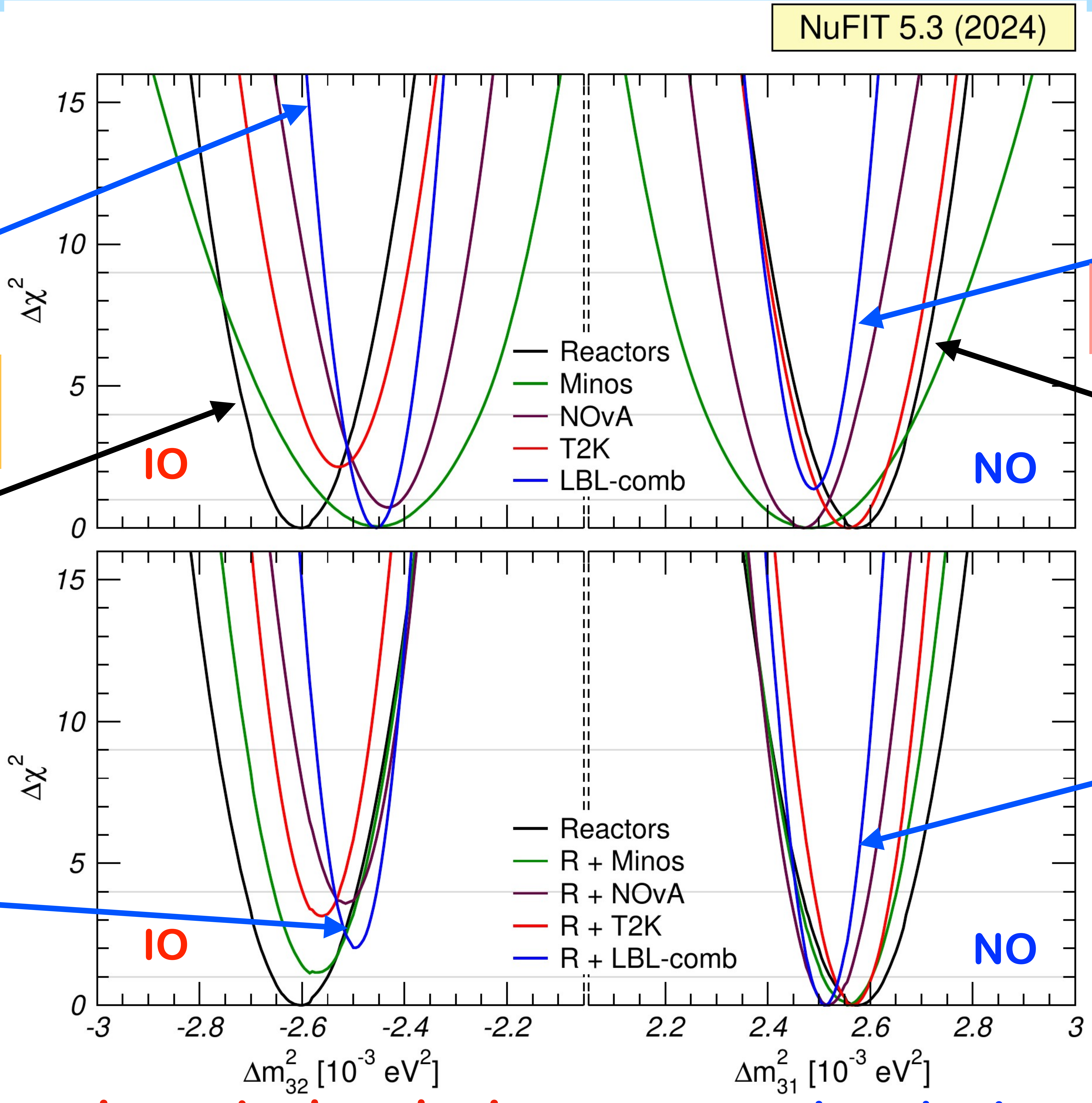
$$|\Delta m^2_{32}|^{IO}_{e} > \Delta m^2_{31}|^{NO}_{e}$$

Daya Bay/RENO/DC

$$|\Delta m^2_{32}|^{IO}_{e} - \Delta m^2_{31}|^{NO}_{e} = \Delta m^2_{21}\cos 2\theta_{12}$$

LBL+Reactors

LBL+Reactors



inverted ordering

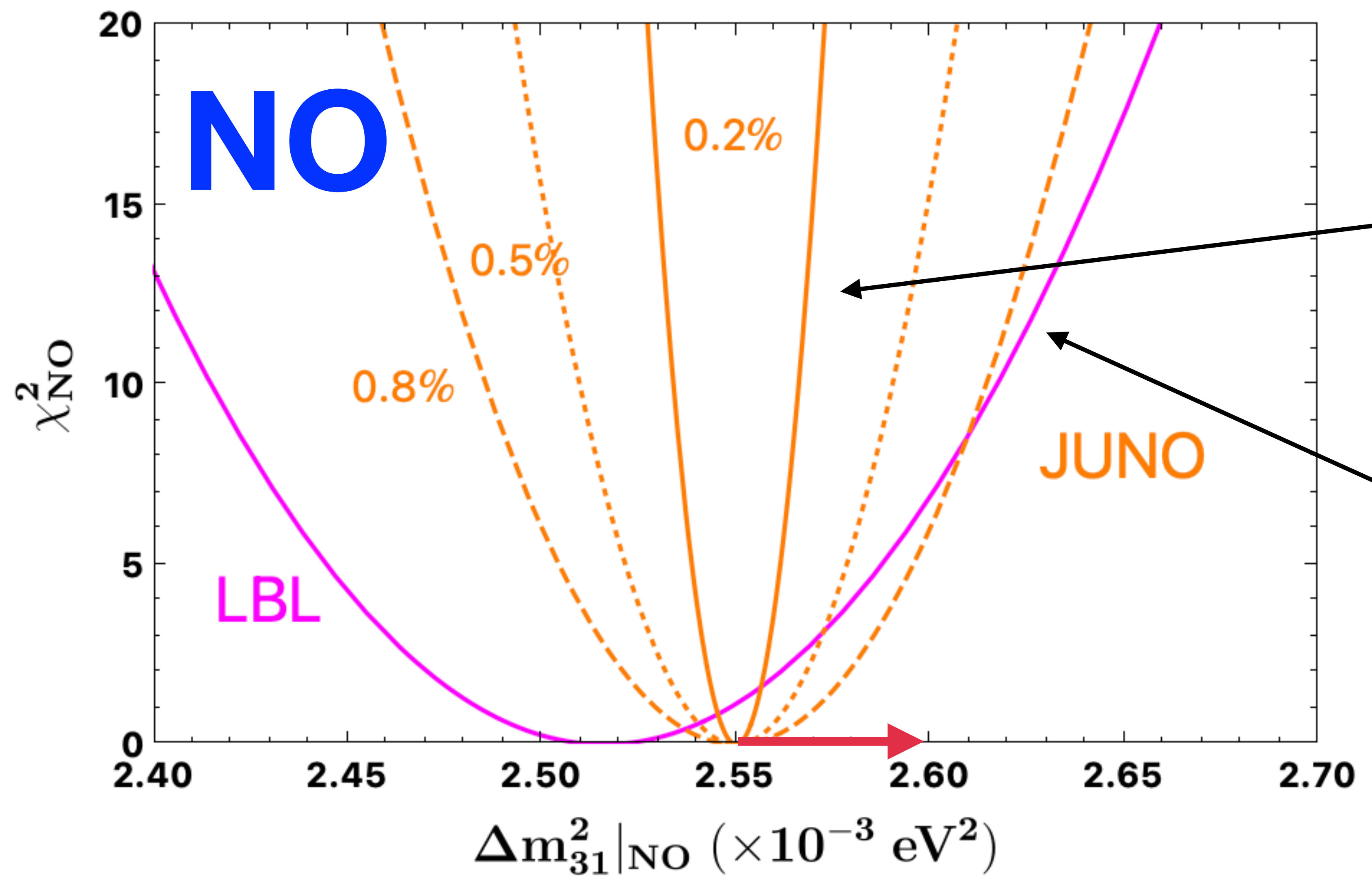
normal ordering

JUNO SUB-% PRECISION

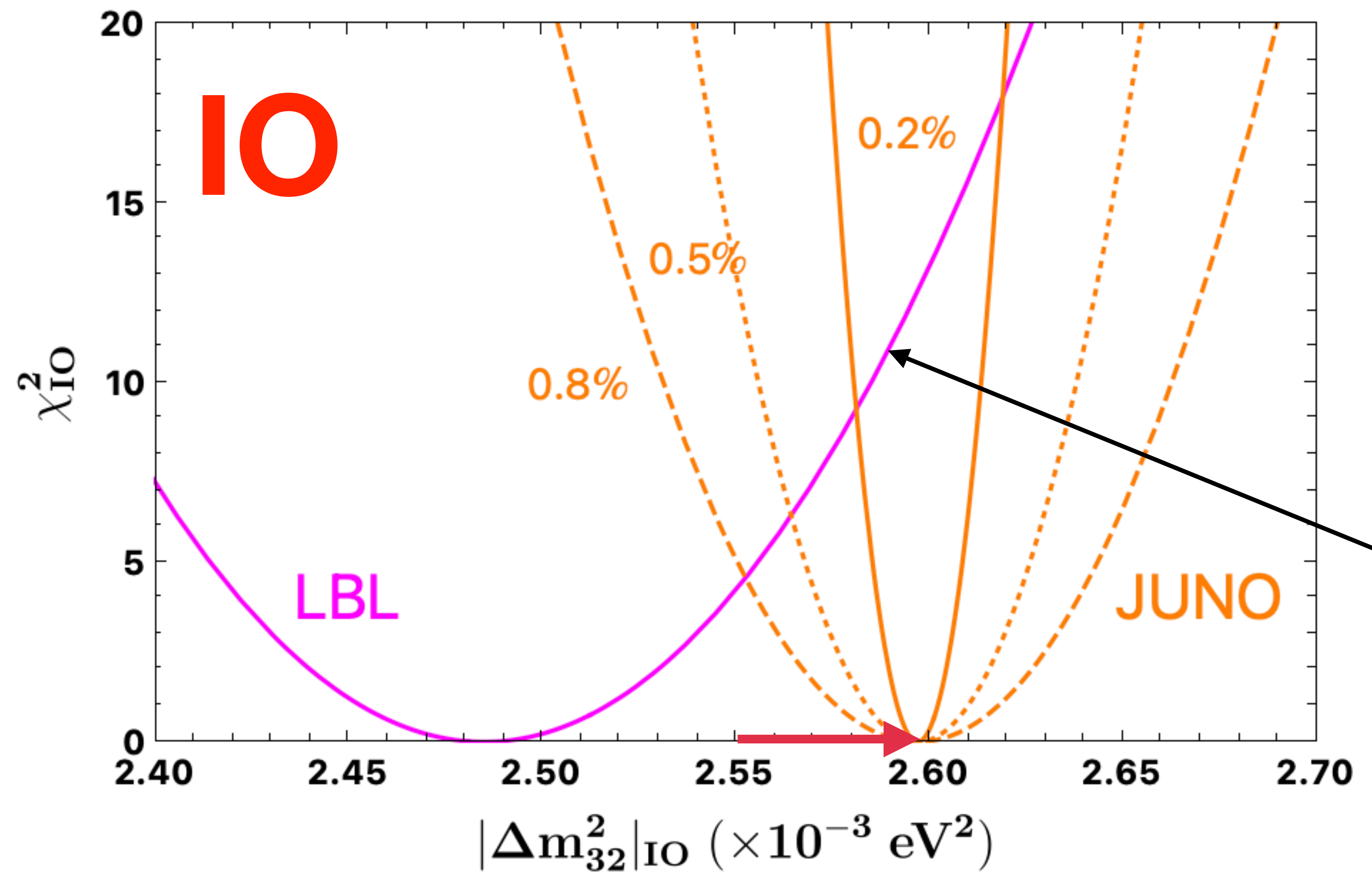
assuming NO is true

$$|\Delta m^2_{32}|^{IO}_{JUNO} = \Delta m^2_{31}|^{NO}_{JUNO} + 4.7 \times 10^{-5} eV^2$$

[S. J. Parke, RZF (2024) arXiv:2404.0873]



$$\Delta m^2_{31}|^{NO}_{T2K+NOvA} = (2.516 \pm 0.031) \times 10^{-3} eV^2$$



$$|\Delta m^2_{32}|^{IO}_{T2K+NOvA} = (2.485 \pm 0.031) \times 10^{-3} eV^2$$