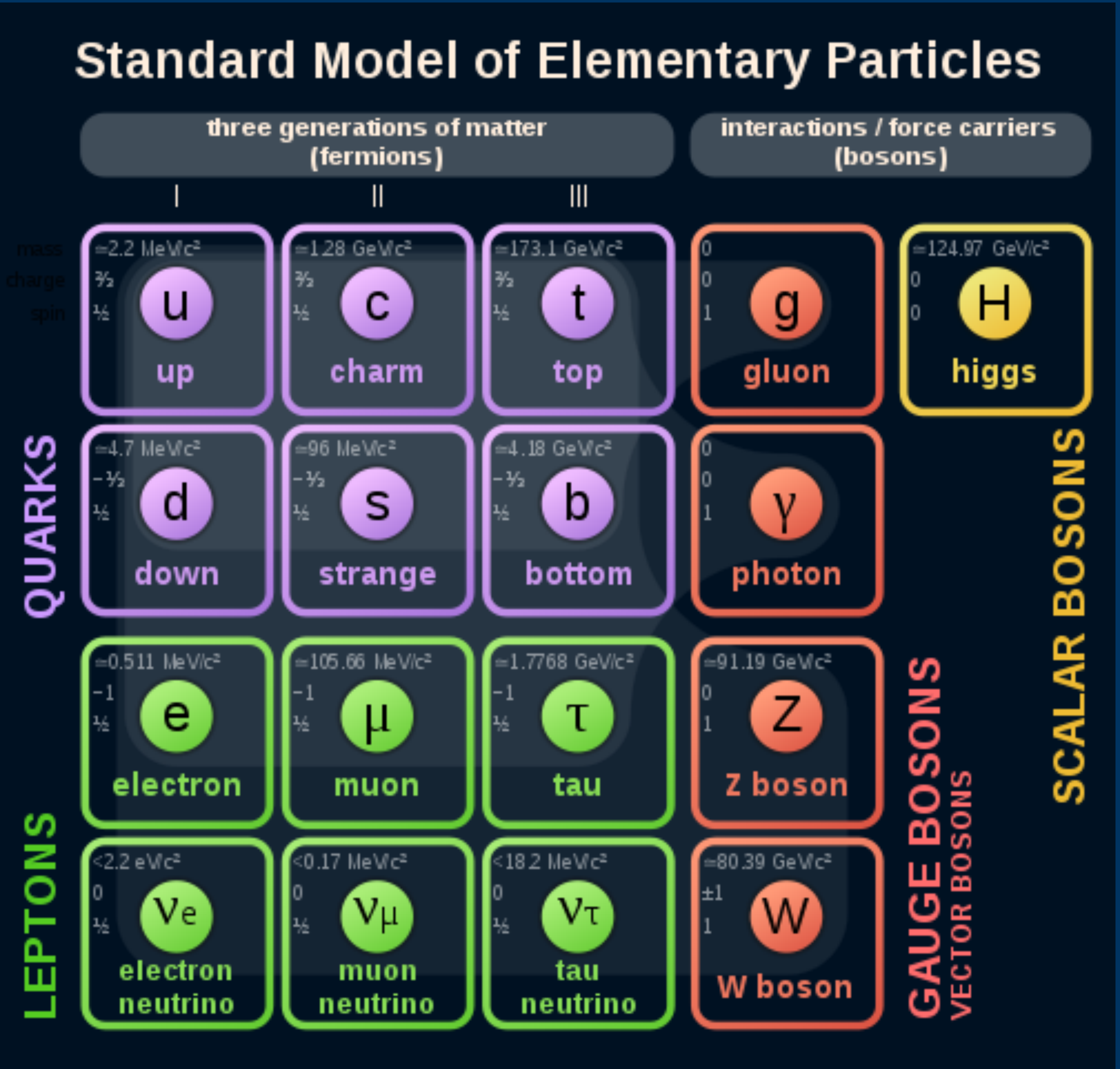
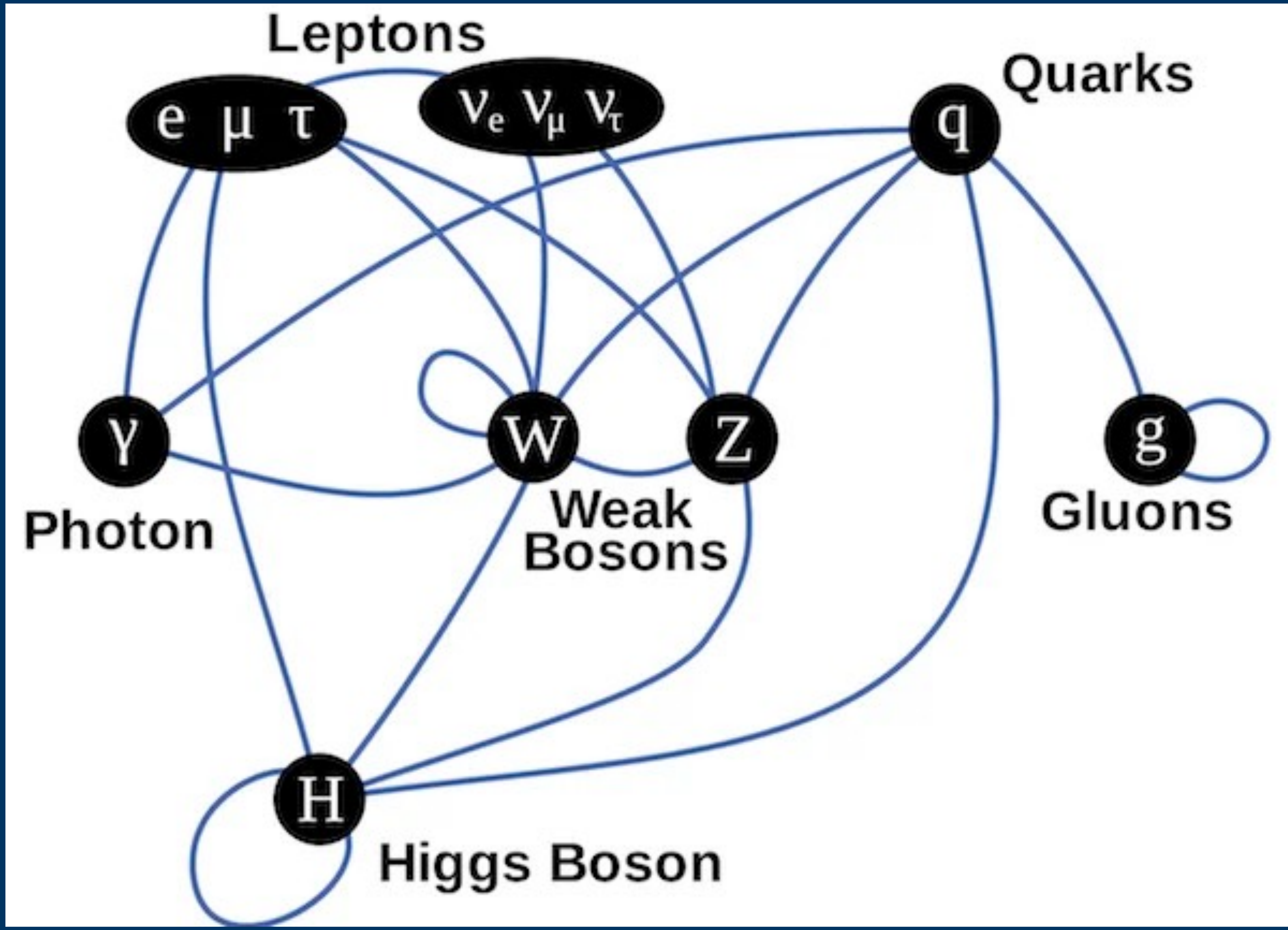
The Standard Model

A brief review

July 22, 2026 15th IDPASC Summer School Oscar Éboli

SM aims to describe the dynamics of the elementary particles:

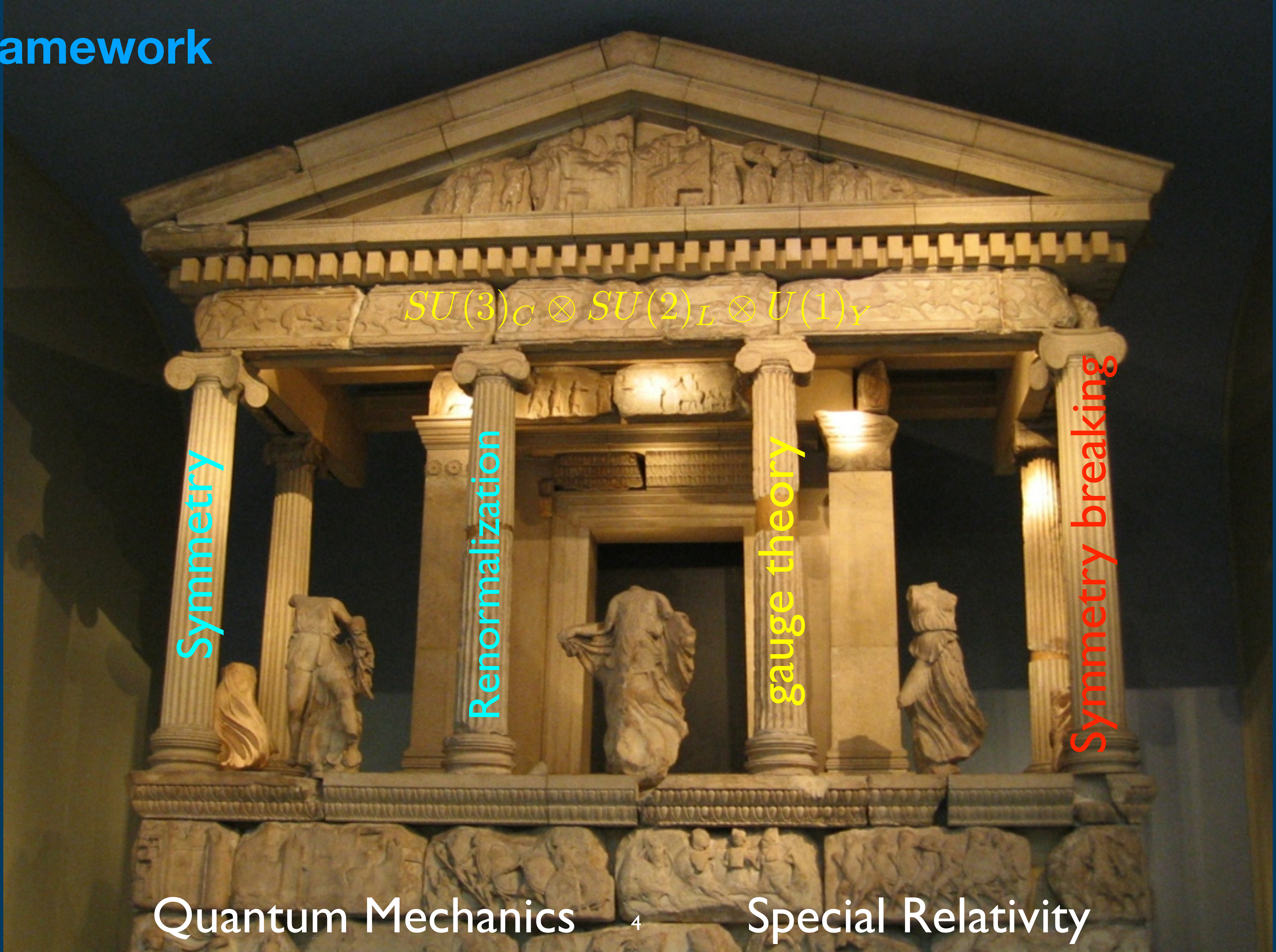




spoiler alert: SM explains available data

Theoretical Tools

Basic framework



Basic framework: gauge theory

- M multiplet under $SU(N)$: $M \to M' = GM = \exp(ig\alpha_a T^a)\ M$

  i.e. $M'_j = G_{jk} M_k$ and $\overline{M}' = \overline{M} G^\dagger$

- Start with a free theory invariant under global $SU(N)$, i.e. α_a constant:

  $\mathscr{L}_0 = \overline{M}' i\gamma^\mu \partial_\mu M' = \overline{M} G^\dagger i\gamma^\mu \partial_\mu G M$

- To impose invariance under local transformations: $\partial_\mu \to D_\mu = \partial_\mu + igT^a b^a_\mu$

- such that $D'_\mu M' = G D_\mu M$ leading to $T^a b'^a_\mu = G(T^a b^a_\mu) G^{-1} + \dfrac{i}{g}(\partial_\mu) G^{-1}$

- $\mathscr{L}_0 \to \mathscr{L} = \overline{M} i\gamma^\mu D_\mu M = \overline{M} i\gamma^\mu \partial_\mu M - g\ b^a_\mu\ \ \overline{M}\gamma^\mu T^a M$

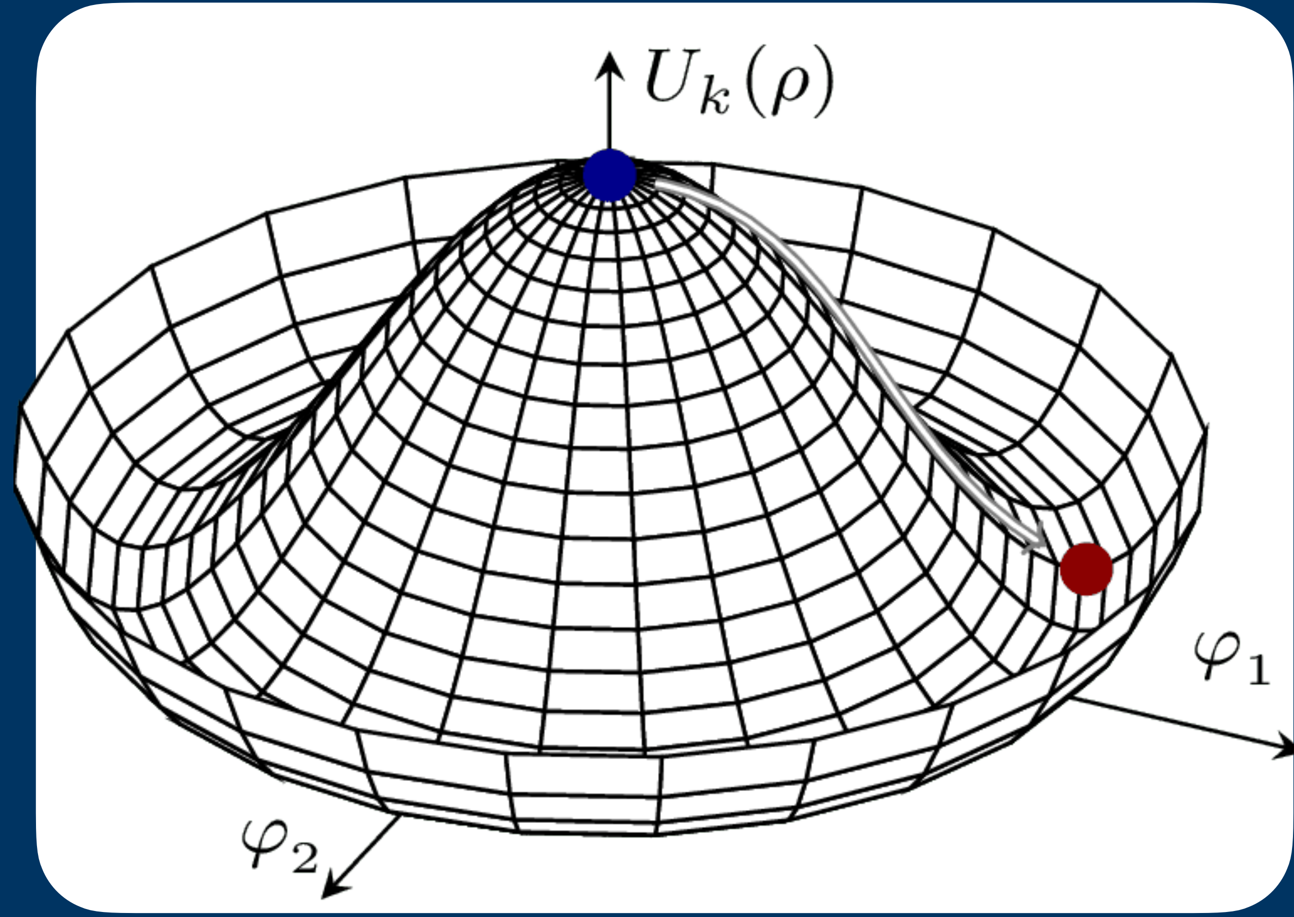
Basic framework: gauge theory

- Total lagrangian density: $\mathscr{L} = \overline{M} i\gamma^\mu \partial_\mu M - g \ b^a_\mu \ \ \overline{M}\gamma^\mu T^a M - \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$
- $F^a_{\mu\nu} T^a = \frac{1}{ig}[D_\mu, D_\nu] = \partial_\mu(b^a_\nu T^a) - \partial_\nu(b^a_\mu T^a) + i \ g \ [b^a_\mu T^a, b^c_\nu T^c]$

Remarks

- The symmetry forbids mass terms $m^2 b^a_\mu b^{a\,\mu}$
- Gauge invariance fixes the FFV interactions
- Gauge invariance fixes the VVV and $VVVV$ interactions
- There is a single coupling g!

Basic framework: symmetry breaking sector (SSB)



- There are massive gauge bosons! The symmetry must be broken!
- Simplest form introduce scalar whose vev is non-vanishing $\langle \varphi^a \rangle \neq 0$
- $\mathscr{L}_{\mathrm{SSB}} = (D_\nu \varphi^a)^* D^\nu \varphi^a - \mu^2 \varphi^{a\,*} \varphi^a - \lambda(\varphi^{a\,*}\varphi^a)^2$ with $\mu^2 < 0$
- For each independent broken generator $T^c \langle \varphi \rangle \neq 0$ there is a Nambu-Goldstone boson (GB)
- Miracle for gauge theories: GB's disappear and gauge bosons become massive
- Remarks:
- The number of degree of freedom is preserved
- Gauge boson masses are proportional to $g\,|\langle \varphi \rangle|$

Basic framework: model building recipe (70's)

1. Choose the local gauge symmetry
2. Choose the matter content and its representation(s)
3. Choose the symmetry breaking sector
4. Verify whether theoretical and experimental constraints are satisfied; if not go back to 1.

STANDARD MODEL

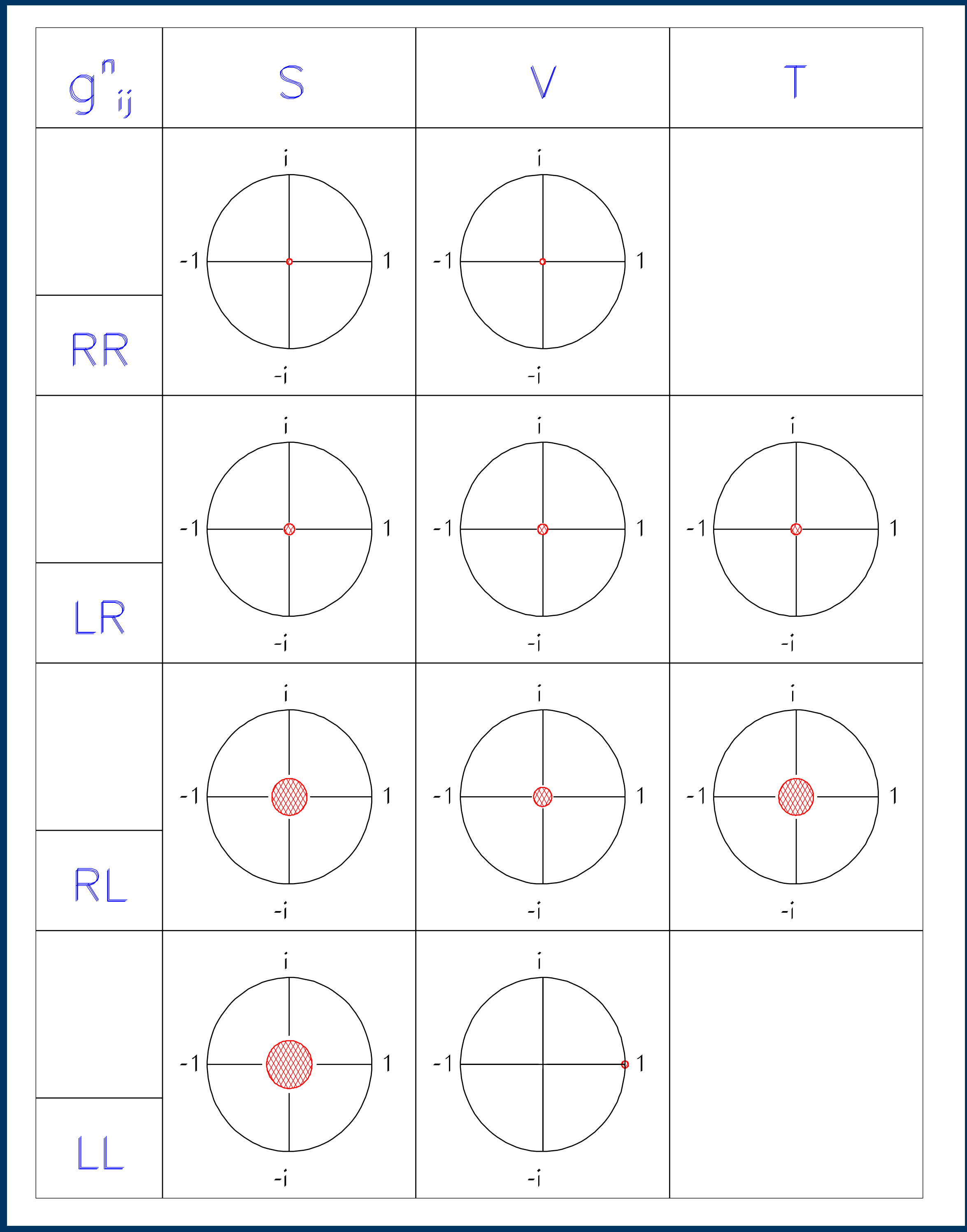
Standard Model: Gauge sector

- There are four electroweak gauge bosons: W^+ , W^- , Z , γ
- Minimal model need 4 symmetry generators
- Choice: $SU(2)_L \otimes U(1)_Y$
- Gauge bosons: W^j_μ with $j = 1{,}2{,}3$ ($SU(2)_L$) and B_μ ($U(1)_Y$) with couplings g and g'
- Kinetic lagrangian density: $\mathscr{L}_{\text{gauge}} = -\dfrac{1}{4}\, B_{\mu\nu}B^{\mu\nu} - \dfrac{1}{4}\, W^a_{\mu\nu}W^{a\,\mu\nu}$
- with $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\nu - g\epsilon^{abc}W^b_\mu W^c_\nu$ and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

this determines the VVV and VVVV vertices

Standard Model: matter (leptons)

- We need to choose the matter representations
- Lorentz transformation of multiplet members must be the same
- We know that charged currents are left-handed
- We define $P_L = \dfrac{1-\gamma^5}{2}$ and $P_R = \dfrac{1+\gamma^5}{2}$
- Degrees of freedom: $e_{L(R)} = P_{L(R)}\, e$ and $\nu_{e\,L} = P_L\, \nu_e$
- $SU(2)_L$ representations: $L = \begin{pmatrix} \nu_{e\,L} \\ e_L \end{pmatrix}$ and $R = e_R$
- $U(1)_Y$ representations from $Q = T^3 + \dfrac{Y}{2} \implies Y_L = -1$ and $Y_R = -2$



$\bar{\psi}O_1\psi \quad \bar{\psi}O_2\psi$

Standard Model: matter (leptons)

- $SU(2)_L$ representations: $L = \begin{pmatrix} \nu_{e\,L} \\ e_L \end{pmatrix}$ and $R = e_R$

- $U(1)_Y$ representations from $Q = T^3 + \dfrac{Y}{2}$ we have $Y_L = -1$ and $Y_R = -2$

- Covariant derivatives: $D_\mu L = (\partial_\mu + ig'\dfrac{Y}{2}\,B_\mu + ig\dfrac{\sigma^a}{2}\,W^a_\mu)\,L$ and $D_\mu R = (\partial_\mu + ig'\dfrac{Y}{2}\,B_\mu)\,R$

- Lagrangian: $\mathscr{L} = i\bar{L}\gamma^\mu D_\mu L + i\bar{R}\gamma^\mu D_\mu R$

- Charged current $\left(W^\pm_\mu = \dfrac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}} \right)$: $-\dfrac{g}{\sqrt{2}}\,\left[\bar{\nu}_e\,\gamma^\mu P_L\,e\,W^+_\mu + \bar{e}\,\gamma^\mu P_R\,\nu_e\,W^-_\mu\right]$

- Neutral current: $-\dfrac{g}{2}\left(\bar{\nu}_{e\,L}\gamma^\mu\nu_{e\,L} - \bar{e}_L\gamma^\mu e_L\right)W^3_\mu + \dfrac{g'}{2}\left(\bar{\nu}_{e\,L}\gamma^\mu\nu_{e\,L} + \bar{e}_L\gamma^\mu e_L + 2\bar{e}_R\gamma^\mu e_R\right)B_\mu$

Standard Model: symmetry breaking sector (SSB)

- Before SSB the charged leptons are massless since $SU(2)_L$ forbids $\bar{e}_L e_R$
- To generate masses we must break the $SU(2)_L \otimes U(1)_Y$ symmetry
- Minimal model: complex scalar doublet $\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$ with $Y_\varphi = +1$
- $\mathscr{L}_{\mathrm{SSB}} = |D_\mu \varphi|^2 - \mu^2 |\varphi|^2 - \lambda |\varphi|^4$ with $\mu^2 < 0$ and $\langle \varphi^\dagger \varphi \rangle = -\dfrac{\mu^2}{\lambda} = \dfrac{v^2}{2}$
- Let's choose the vacuum $\langle \varphi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

Standard Model: symmetry breaking sector (SSB)

- $\mathscr{L}_{\text{SSB}} = |D_\mu \varphi|^2 - \mu^2 |\varphi|^2 - \lambda |\varphi|^4$
- Let's choose the vacuum $\langle \varphi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$
- $\dfrac{\sigma^a}{2}\langle \varphi \rangle \neq 0$ and $Y\langle \varphi \rangle \neq 0$ however $Q\langle \varphi \rangle = \dfrac{1}{2}(\sigma^3 + Y)\langle \varphi \rangle = 0$
- Therefore $SU(2)_L \otimes U(1)_Y$ is broken to $U(1)_{\text{em}}$
- 3 broken generators $\implies$ 3 goldstone bosons $\implies$ 3 massive gauge bosons
- Unitary gauge $\varphi = \begin{pmatrix} 0 \\ (v+H)/\sqrt{2} \end{pmatrix}$

Standard Model: symmetry breaking sector (SSB)

- $\mathscr{L}_{\rm SSB} = |D_\mu \varphi|^2 - \mu^2 |\varphi|^2 - \lambda |\varphi|^4$
- Unitary gauge $\varphi = \begin{pmatrix} 0 \\ (v+H)/\sqrt{2} \end{pmatrix}$
- $\mathscr{L}_{\rm SSB} = \dfrac{1}{2}(\partial_\mu H \partial^\mu H - 2\lambda v^2 H^2) + \dfrac{v^2}{8}(g^2 W_\mu^+ W^{-\mu} + (g' B_\mu - g W_\mu^3)^2) + \ldots$

to be diagonalized

- $M_W = \dfrac{gv}{4}$ and $M_H = \sqrt{2\lambda}\ v$
- Neutral mass eigenstates: $Z_\mu = \dfrac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}}$ and $A_\mu = \dfrac{g' W_\mu^3 + g B_\mu}{\sqrt{g^2 + g'^2}}$

Standard Model: symmetry breaking sector (SSB)

- $\mathscr{L}_{\text{SSB}} = \frac{1}{2}(\partial_\mu H \partial^\mu H - 2\lambda v^2 H^2) + \frac{v^2}{8}(g^2 W_\mu^+ W^{-\mu} + (g' B_\mu - g W_\mu^3)^2) + \ldots$
- $M_W = \frac{gv}{2}$ and $M_H = \sqrt{2\lambda}\ v$
- Neutral mass eigenstates: $Z_\mu = \frac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}}$ and $A_\mu = \frac{g' W_\mu^3 + g B_\mu}{\sqrt{g^2 + g'^2}}$
- So $M_Z = \sqrt{g^2 + g'^2}\ \frac{v}{2}$
- Writing $g = \frac{e}{\sin \theta_W}$ and $g' = \frac{e}{\cos \theta_W}$ we have $M_Z = \frac{M_W}{\cos \theta_W}$

Standard Model: symmetry breaking sector (SSB)

- Neutral current: $-\frac{g}{2}\left(\bar{\nu}_{eL}\gamma^\mu\nu_{eL} - \bar{e}_L\gamma^\mu e_L\right) W^3_\mu + \frac{g'}{2}\left(\bar{\nu}_{eL}\gamma^\mu\nu_{eL} + \bar{e}_L\gamma^\mu e_L + 2\bar{e}_R\gamma^\mu e_R\right)\ B_\mu$
- Neutrinos interact only with the Z: $-\frac{g}{2\cos\theta_W}\ \overline{\nu_{eL}}\gamma^\mu\nu_{eL}Z_\mu$
- Electron interactions: $-eQ\ \bar{e}\gamma^\mu e A_\mu - \frac{g}{2\cos\theta_W}\ \bar{e}\gamma^\mu(g_V - g_A\gamma^5)eZ_\mu$
- with $g_V = T^3_e - 2\sin^2\theta_W Q$ and $g_A = T^3_e$
- Electron mass: $\mathscr{L}_Y = -\,Y_e\ [\bar{R}(\varphi^\dagger L) + (\bar{L}\varphi)R]$
- In the unitarity gauge: $\mathscr{L}_Y = -\,Y_e\ \frac{v+H}{\sqrt{2}}\ (\bar{e}_L e_R + \bar{e}_R e_L)$ with $m_e = \frac{Y_e v}{\sqrt{2}}$

Standard Model: quark sector

- Quark multiplets are $Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, u_R, d_R$
- Quantum numbers: $Y_{u_R} = \frac{4}{3}, Y_{d_R} = -\frac{2}{3}, Y_Q = \frac{1}{3}$
- Quark-gauge sector: $\mathscr{L}_Q = i\bar{u}_R\gamma^\mu D_\mu u_R + i\bar{d}_R\gamma^\mu D_\mu d_R + i\bar{Q}_L\gamma^\mu D_\mu Q_L$
- CC interactions: $\mathscr{L}_{\mathrm{CC}} = -\frac{g}{\sqrt{2}}\bar{u}_L\gamma^\mu d_L W_\mu^+ + h.c.$
- NC interactions: $\mathscr{L}_{\mathrm{NC}} = -eQ_q\bar{q}\,\gamma^\mu q\,A_\mu - \frac{g}{2\cos\theta_W}\bar{q}\gamma^\mu(g_V^q - g_A^q\gamma^5)q\,Z_\mu$
- $g_V^q = T_q^3 - 2\sin^2\theta_W\,Q_q$ and $g_A^q = T_q^3$

Standard Model: quark sector

- Quark multiplets are $Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$, u_R , d_R
- Quark-gauge sector: $\mathscr{L}_Q = i\bar{u}_R\gamma^\mu D_\mu u_R + i\bar{d}_R\gamma^\mu D_\mu d_R + i\bar{Q}_L\gamma^\mu D_\mu Q_L$
- To write the mass term we need the doublet $\tilde{\varphi} = i\sigma^2\varphi^*$ with $Y_{\tilde{\varphi}} = -1$
- $\mathscr{L}_{Yq} = -Y_u[\bar{u}_R(\tilde{\varphi}^\dagger Q_L) + (\bar{Q}_L\tilde{\varphi})u_R] \ + \ Y_d\,[\bar{d}_R(\varphi^\dagger Q_L) + (\bar{Q}_L\varphi)d_R]$
- Unitary gauge: $-Y_u \dfrac{v+H}{\sqrt{2}}\,(\bar{u}_L u_R + \bar{u}_R u_L) - Y_d\,\dfrac{v+H}{\sqrt{2}}\,(\bar{d}_L d_R + \bar{d}_R d_L)$

Standard Model: 3 families

- Three copies of the one family gauge-fermion interactions
- For massless neutrinos the lepton sector is just the repetition of the first family
- Important consequence: neutral current interactions do not change the lepton flavor
- Most general mass term for quarks:

$$\mathscr{L}_{Yq} = - \sum_{j,k=1,2,3} \left\{ Y_{u\,jk}\, \bar{u}^j_R (\tilde{\varphi}^\dagger Q^k_L) + Y_{d\,jk}\, \bar{d}^j_R (\varphi^\dagger Q^k_L) \right\} + h.c.$$

- After SSB the mass matrix is $\displaystyle\sum_{j,k=1,2,3} \left[\bar{u}^j_R M^{jk}_u u^k_L + \bar{d}^j_R M^{jk}_d d^k_L \right] + \text{h.c.}$

- The matrices $M_{u,d}$ are not diagonal, general.

Standard Model: 3 families

- We diagonalize the mass matrices M_u and M_d by making the transformations (show it!)

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \to A_L^{-1} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \qquad \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L \to B_L^{-1} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L$$

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_R \to A_R^{-1} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R \qquad \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R \to B_R^{-1} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R$$

such that $A_R M_d A_L^{-1} = \mathrm{diag}(m_d\ m_s\ m_b\ )$ $\quad B_R M_u B_L^{-1} = \mathrm{diag}\,(m_u\ m_c\ m_t)$

- The charged current interactions are modified to

$$\mathcal{L}_{\mathrm{CC}} \quad \propto \left(\overline{u_L}\ \overline{c_L}\ \overline{t_L}\right)_j \gamma^\mu V_{jk} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}_k W_\mu^+ + \text{ h.c.}$$

CKM

$$V = B_L A_L^{-1}$$

Standard Model: 3 families

- $$\mathscr{L}_{Yq} = -\sum_{j,k=1,2,3} \left\{ Y_{u\,jk}\ \bar{u}^j_R(\tilde{\varphi}^\dagger Q^k_L) + Y_{d\,jk}\ \bar{d}^j_R(\varphi^\dagger Q^k_L) \right\} + h.c.$$

- The neutral current does not change flavor: $\sum_{j \in \text{all q}} \bar{q}_j \gamma^\mu (g^j_V - g^j_A \gamma^5) q_j\ Z_\mu$

  where q are mass eigenstates.

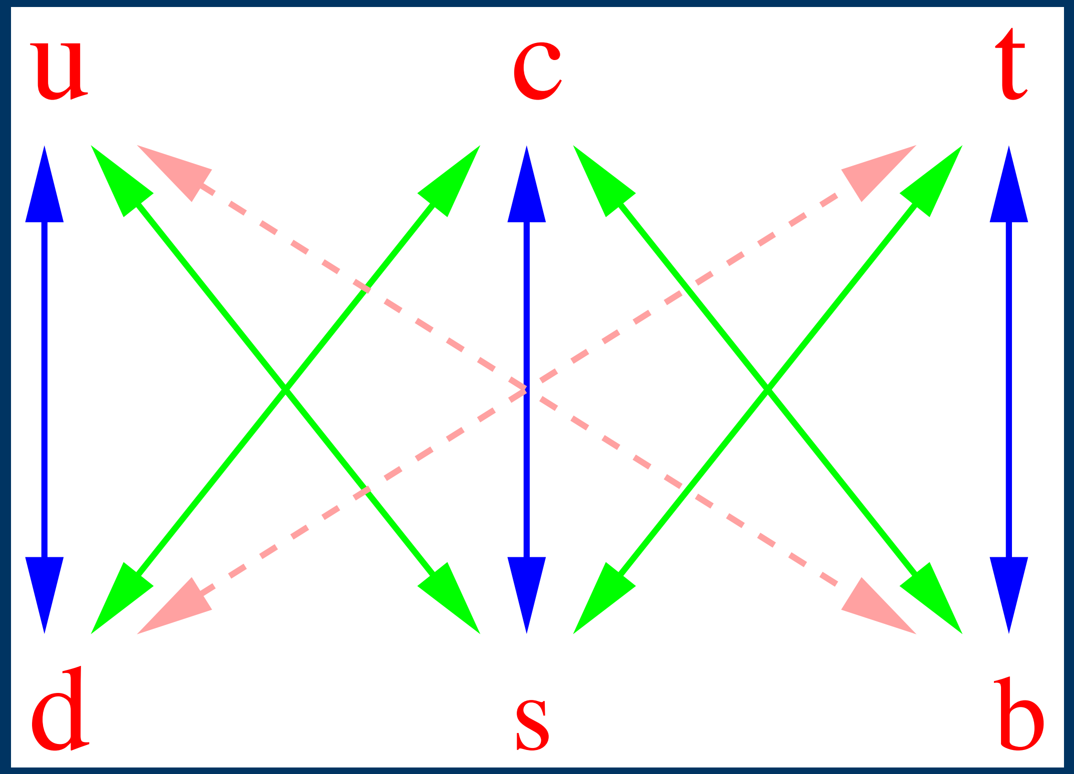
- Charged currents are modified: $\sum_{jk} (\bar{u}_L\ \bar{c}_L\ \bar{t}_L)\gamma^\mu V_{jk} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W^+_\mu$ + h.c.

- V is the Cabibbo-Kobayashi-Maskawa (CKM) matrix. It is unitary

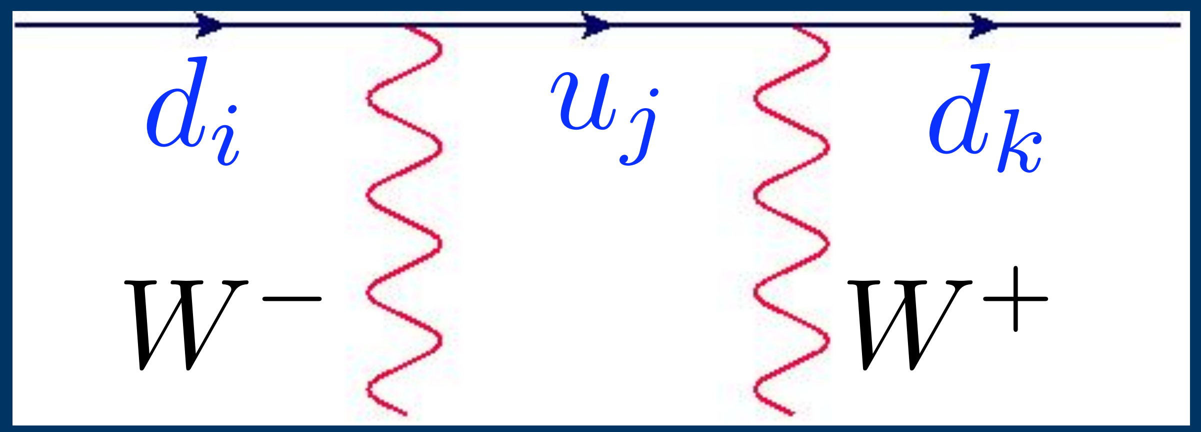
Standard Model: 3 families

- CC couples to different families:
- Example: $\mathrm{Br}(K^+_{u\bar{s}} \to W^+ \to \mu^+\nu) = (63.56 \pm 0.11)\,\%$
- While flavor changing neutral currents (FCNC) are suppressed:

$$\mathrm{Br}(K^+_{u\bar{s}} \to \pi^+_{u\bar{d}}\nu\bar{\nu}) = (1.33 \pm 0.30) \times 10^{-10}$$



- Higher order corrections can induce FCNC at the level $\mathcal{O}(G_F \alpha m_q^2/M_W^2)$



mass independent term $V_{kj}V^\dagger_{ji} = \delta_{ki}$

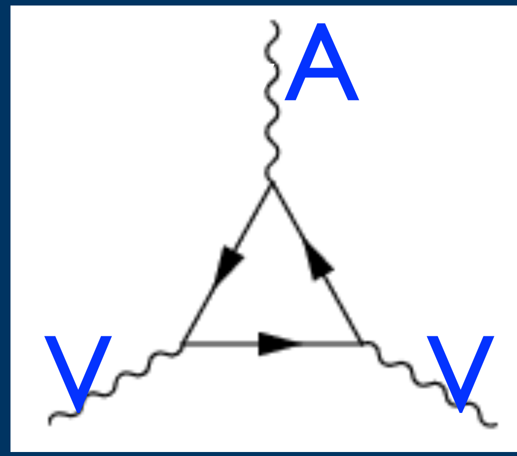
mass dependent term $V_{kj}V^\dagger_{ji}\,\frac{m_{qj}}{M_W}$

Standard Model: 3 families

- CP violation is related to the CKM matrix!
- An $n \times n$ unitary matrix has n^2 real parameters: $\dfrac{n(n-1)}{2}$ angles and $\dfrac{n(n+1)}{2}$ phases
- We can remove $2n-1$ phases by refining phases: $\begin{array}{l} d_{jL} \to e^{i\beta_j^d} d_{jL} \\ u_{jL} \to e^{i\beta_j^u} u_{jL} \end{array} \implies V_{jk} \to V_{jk} e^{i(\beta_k^d - \beta_j^u)}$
- So we are left with $\dfrac{(n-1)(n-2)}{2}$ phases.
- In order to have CP violation the number of generations must be greater or equal to 3

Standard Model: additional properties

- Anomaly cancelation: quantum effects might spoil current conservation!
- Writing the gauge boson interaction to fermions as $gX^a_\mu(\bar{R}\gamma^\mu T^a R + \bar{L}\gamma^\mu T^a L)$
- The axial anomaly is proportional do $A^{abc} = A^{abc}_+ - A^{abc}_-$ with $A^{abc}_\pm = \mathrm{Tr}\left[\left\{T^a_\pm, T^b_\pm\right\} T^c_\pm\right]$
- In the SM, $T^a = \sigma^a$ or Y
- $SU(2)^3 \implies \mathrm{Tr}(\{\sigma^a, \sigma^b\}\sigma^c) = 2\delta_{ab}\mathrm{Tr}(\sigma^c) = 0$ so there is $SU(2)$ anomaly



$\overline{\psi}\gamma^\mu\gamma_5\psi$

- $SU(2)^2U(1) \implies \mathrm{Tr}(\{\sigma^a, \sigma^b\}Y) = 2\delta_{ab}\mathrm{Tr}(Y)$, however,
- $\mathrm{Tr}(Y) = \sum_{\text{leptons}} Y + \sum_{\text{quarks}} Y = (-1 \times 2 - 2) + 3(2 \times \frac{1}{3} + \frac{4}{3} - \frac{2}{3}) = 0$

- The SM is anomaly free due to the choice of the fermion representations!

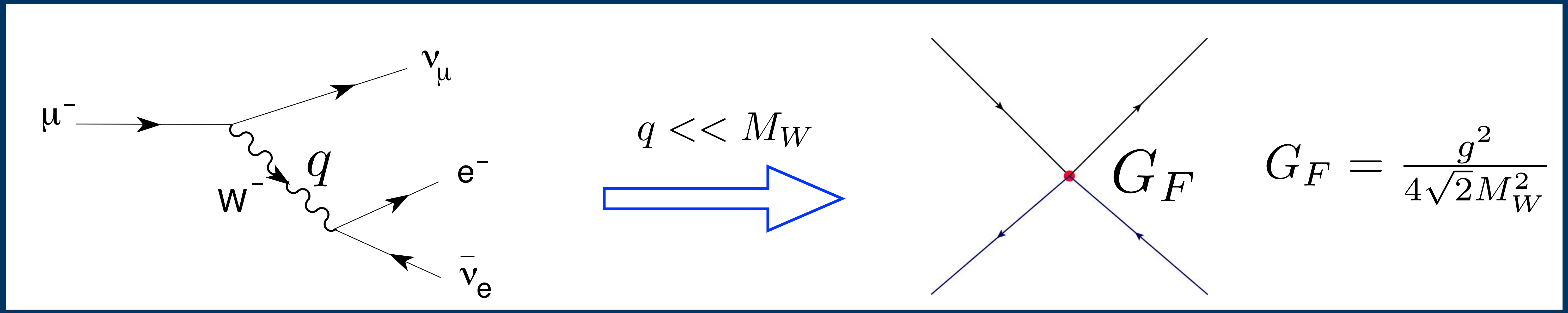
Standard Model: additional properties

- Accidental symmetries: Baryon number is automatically conserved in the SM
- Terms of the form $\bar{q}Xq'$ are invariant under $q^j_{L,R} \to e^{i\theta_B}\, q^j_{L,R}$
- Terms like $\bar{u}_R \varphi^\dagger L$ are not $SU(2)_L \otimes U(1)_Y$ invariant!
- For massless neutrinos, the SM is invariant under $L_j \to e^{i\theta_j} L_j$ and $\ell^j_R \to e^{i\theta_j}\, \ell^j_R$
- The accidental global symmetry group of the SM is $U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau \otimes U(1)_B$
- For instance, $\dfrac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to all)} < 1.5 \times 10^{-13}$
- For massive neutrinos, just the total lepton number is conserved

PHENOMENOLOGY

Phenomenology: muon decay

- Just one 3-body decay is allowed: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$



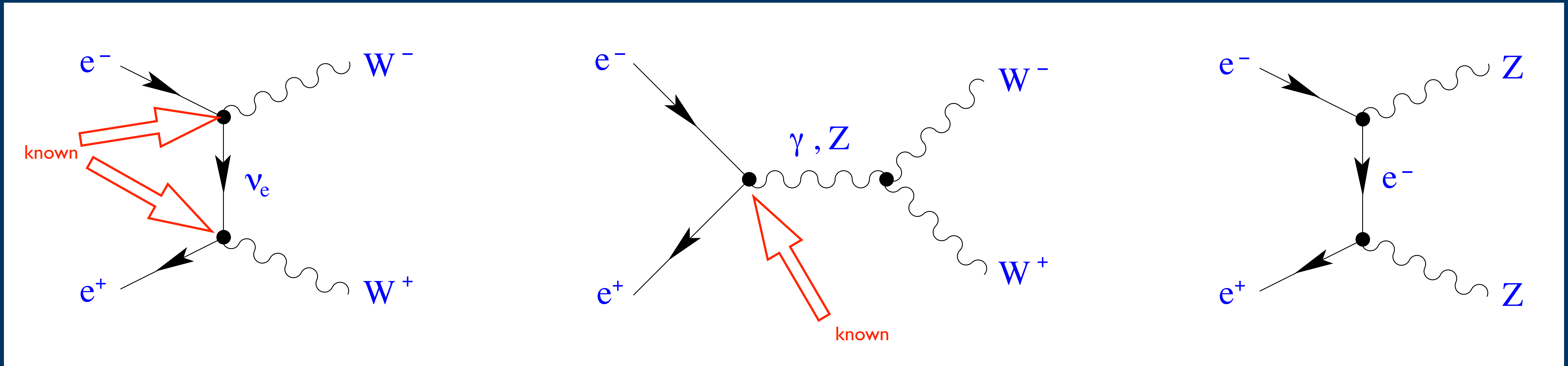
- This is process is used to determine G_F: $\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3} \times f\left(\frac{m_e^2}{m_\mu^2}\right) (1 + \delta_{\mathrm{RC}})$

- From the muon lifetime: $G_F = 1.1663785(6) \times 10^{-5}$ GeV^{-2}

- G_F is used as input parameter in choices of input parameters to analyses

Phenomenology: triple gauge couplings

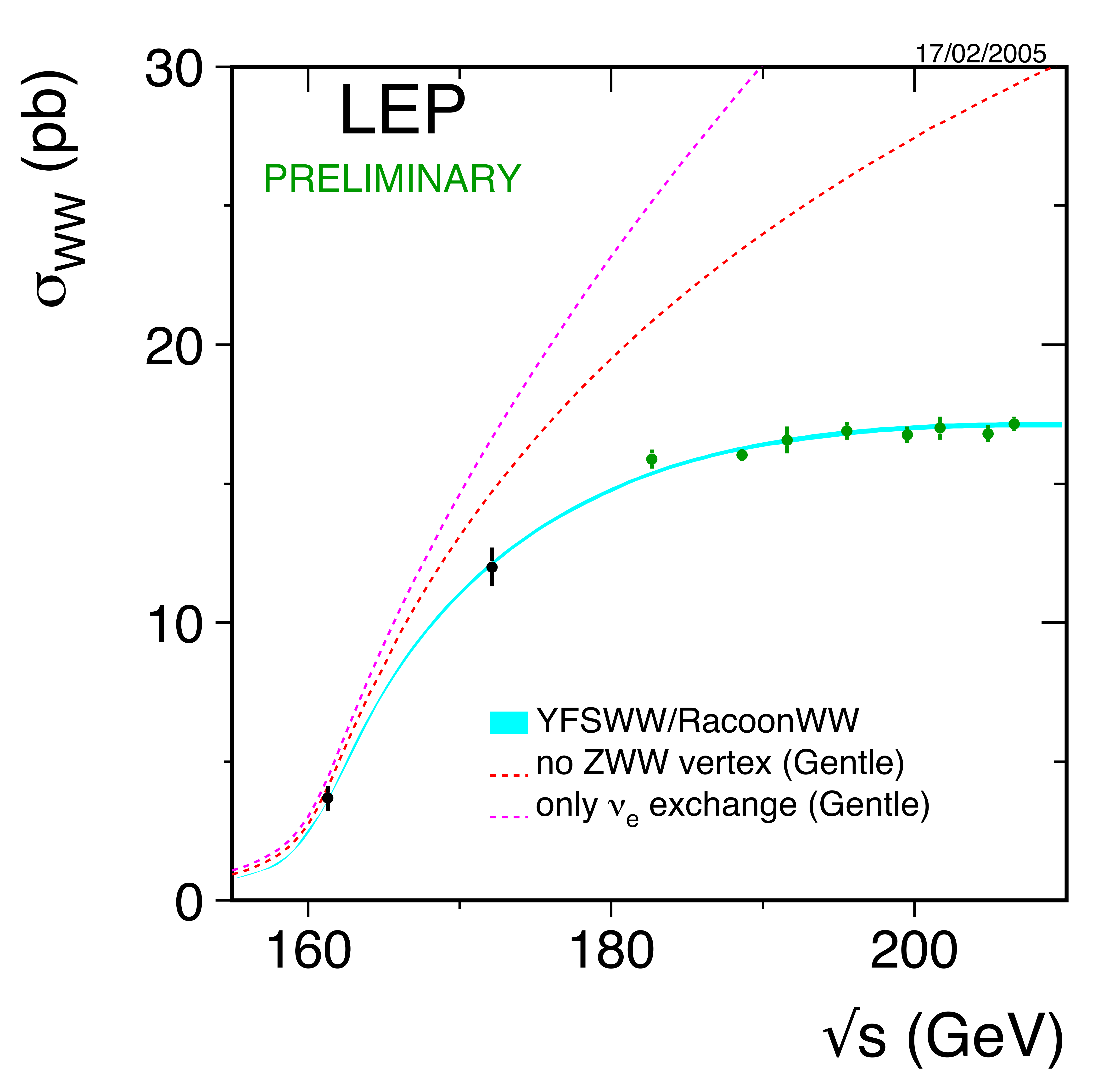
- VVV couplings are determined in gauge theories
- LEP II tested $e^+e^- \to W^+W^-$



- This reaction related FVV and VVV couplings!
- Without γW^+W^- and ZW^+W^- interactions the cross section grows as $\sigma \simeq \dfrac{G_F^2 s}{12\pi}$
- We need all diagrams to avoid unitarity violation

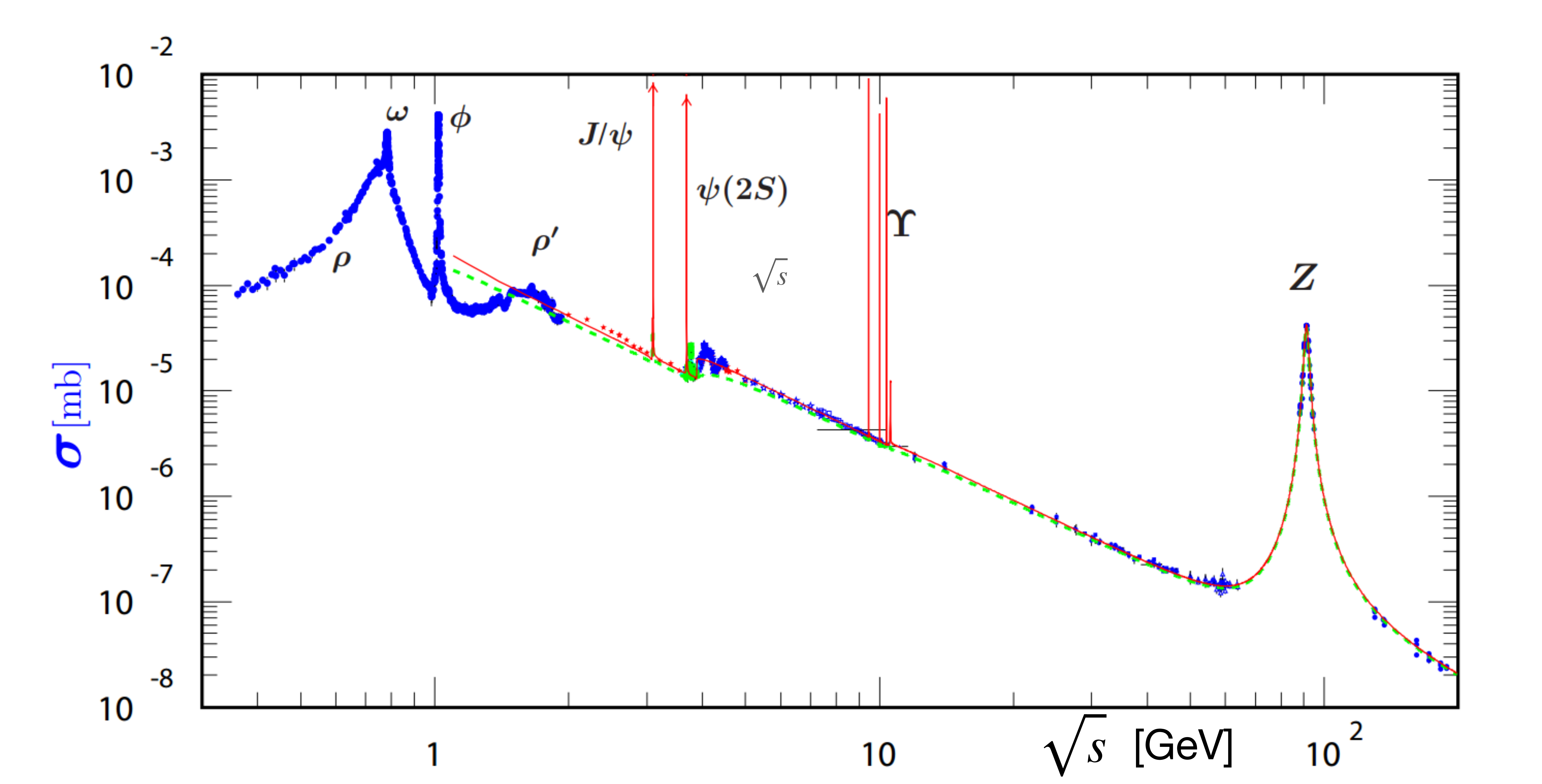
Phenomenology: triple gauge couplings

- LEP II tested $e^+e^- \to W^+W^-$
- The experimental results is



Phenomenology: electroweak precision observables (EWPO)

- LEP I measured with high accuracy the Z couplings to fermions



Phenomenology: electroweak precision observables (EWPO)

- LEP I measured the Z couplings to fermions with high accuracy
- The SM model is a constant field theory, so we can calculate higher order corrections!
- The $Z\bar{f}f$ coupling: $(\sqrt{2}G_F M_Z^2)^{1/2}\ \sqrt{\rho_f}\ \left[\left(T_f^3 - 2Q_f \sin^2\theta_W \kappa_f\right)\gamma^\mu - T_f^3\gamma^\mu\gamma^5\right]\ Z_\mu$

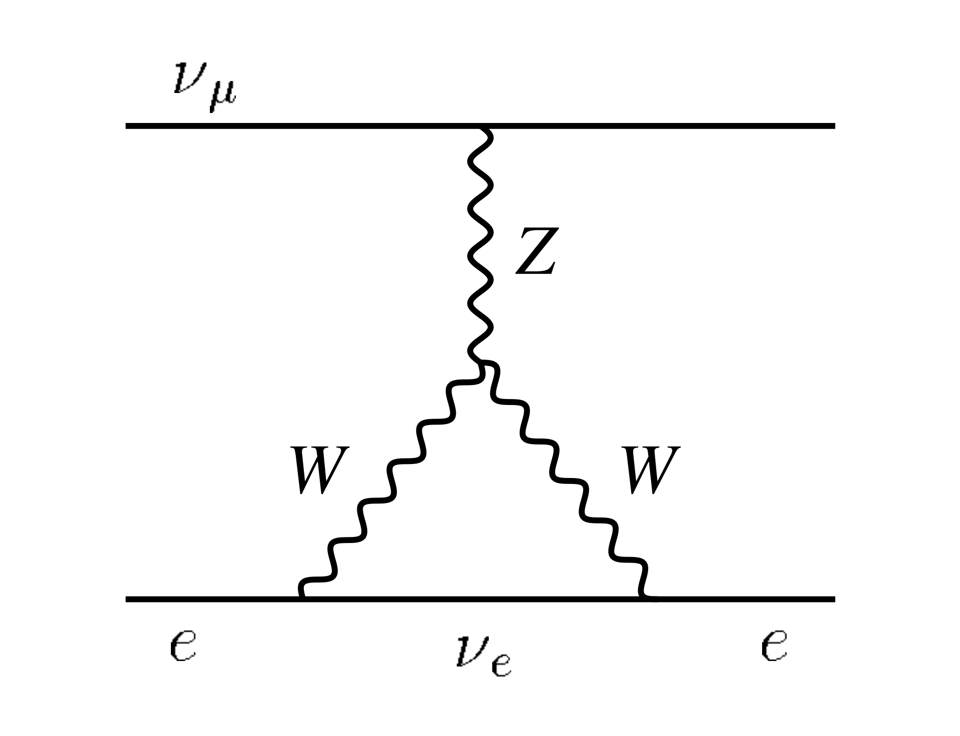
  $\sqrt{s}$
- At tree level: $\rho_f = 1$ and $\kappa_f = 1$, however, higher order corrections modify these values
- We define $\sin^2\theta_{\text{eff}} = \kappa_f \sin^2\theta_W$
- Due to the experimental precision higher order corrections are needed

Phenomenology: electroweak precision observables (EWPO)

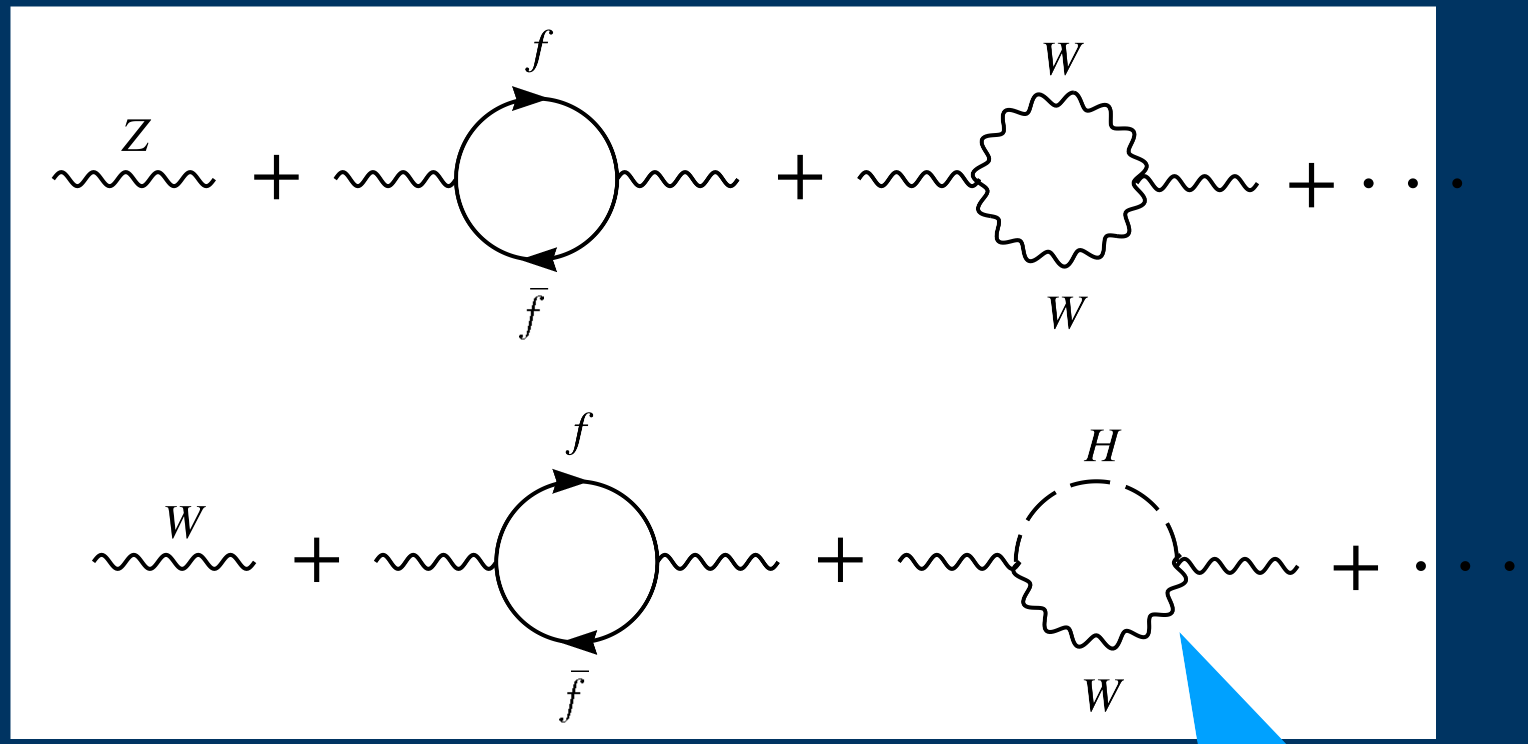
- The $Z\bar{f}f$ coupling: $(\sqrt{2}G_F M_Z^2)^{1/2}\sqrt{\rho_f}\left[\left(T_f^3 - 2Q_f\sin^2\theta_W\kappa_f\right)\gamma^\mu - T_f^3\gamma^\mu\gamma^5\right] Z_\mu$

- There many contributions: vertex corrections; running of α from to M_z; etc

- The main contributions come from gauge boson self-energies and they are universal



$\sqrt{s}$



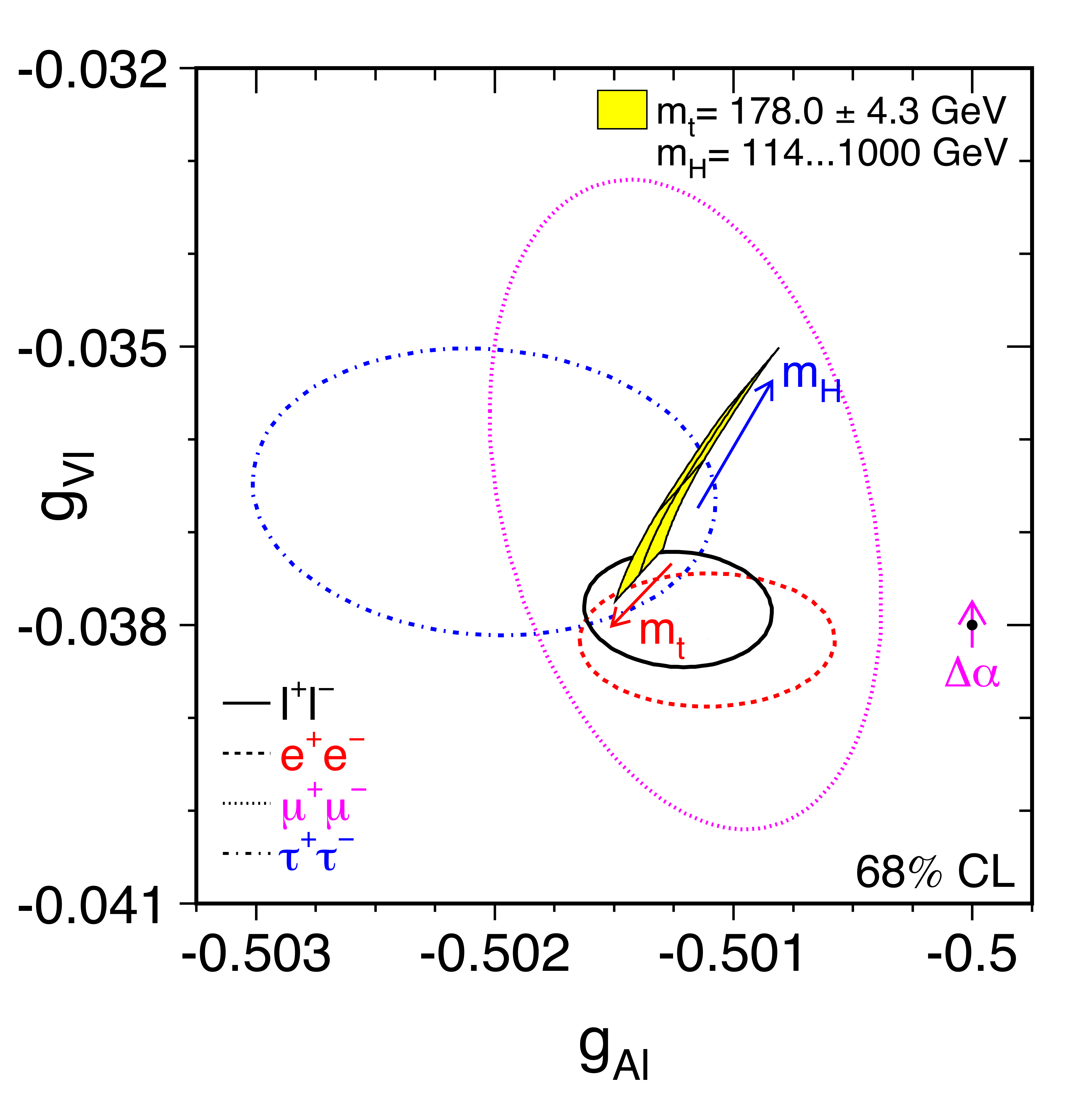
- Important fact: $(\Delta\rho_f)_{\text{univ}} \propto m_t^2$ and $(\Delta\kappa_f)_{\text{univ}} \propto m_t^2$

$\propto \ln(m_H)$

- There is an indirect dependence on the top and Higgs masses!

Phenomenology: electroweak precision observables (EWPO)

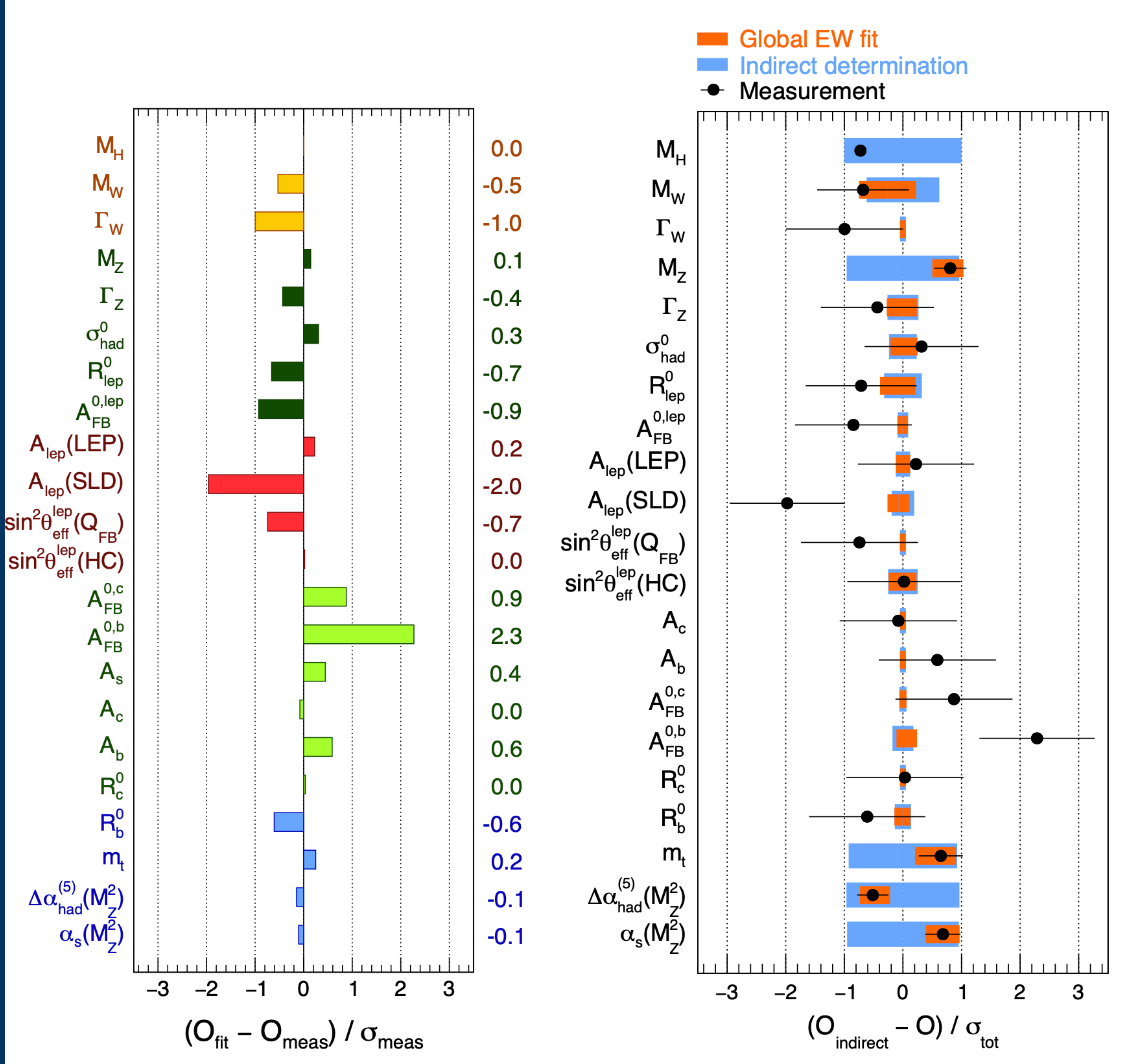
- Fermions couplings to the Z



- Consistent with universality

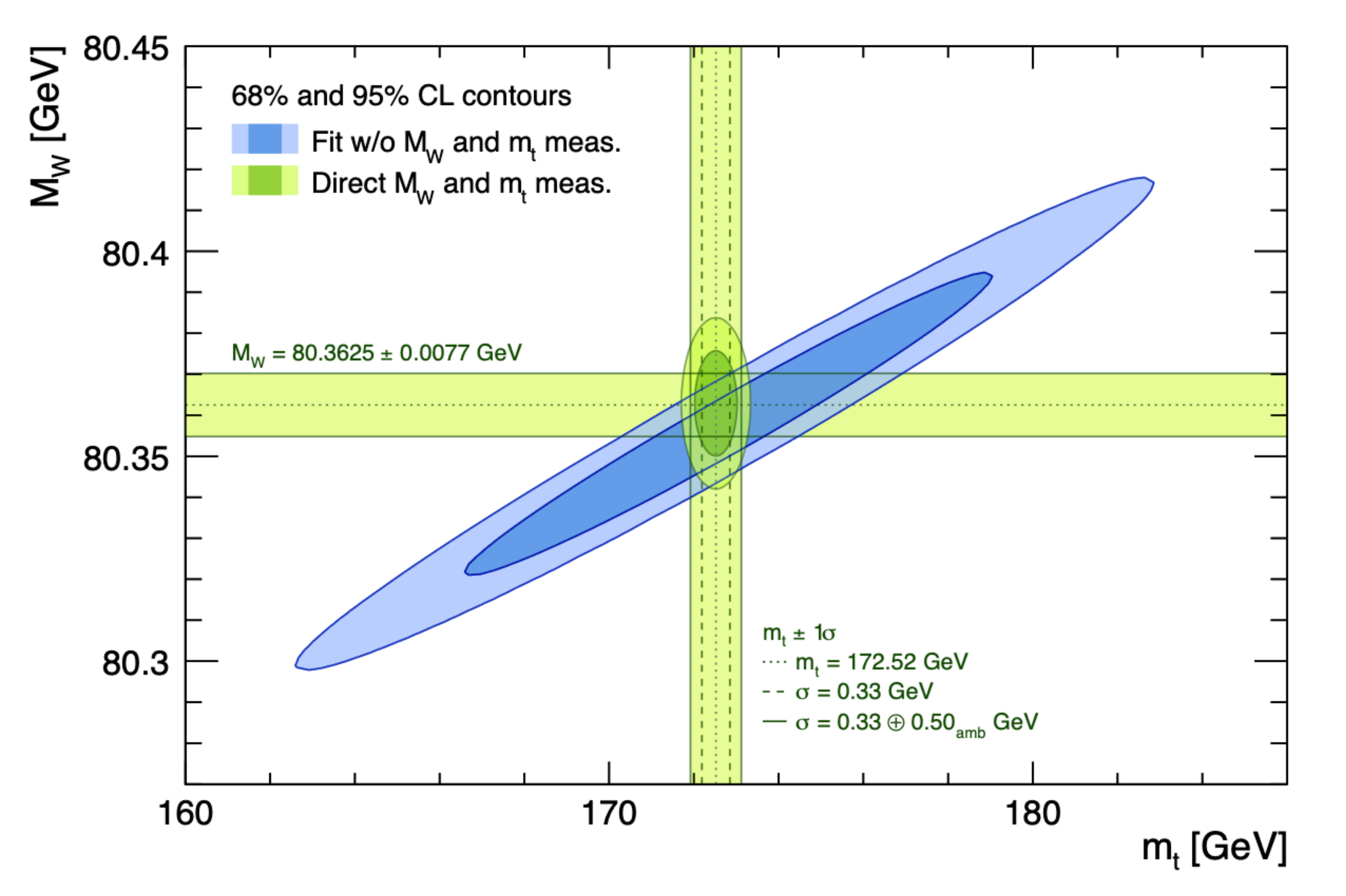
Phenomenology: electroweak precision observables (EWPO)

- Inputs and results to the fits
- The SM is doing very well



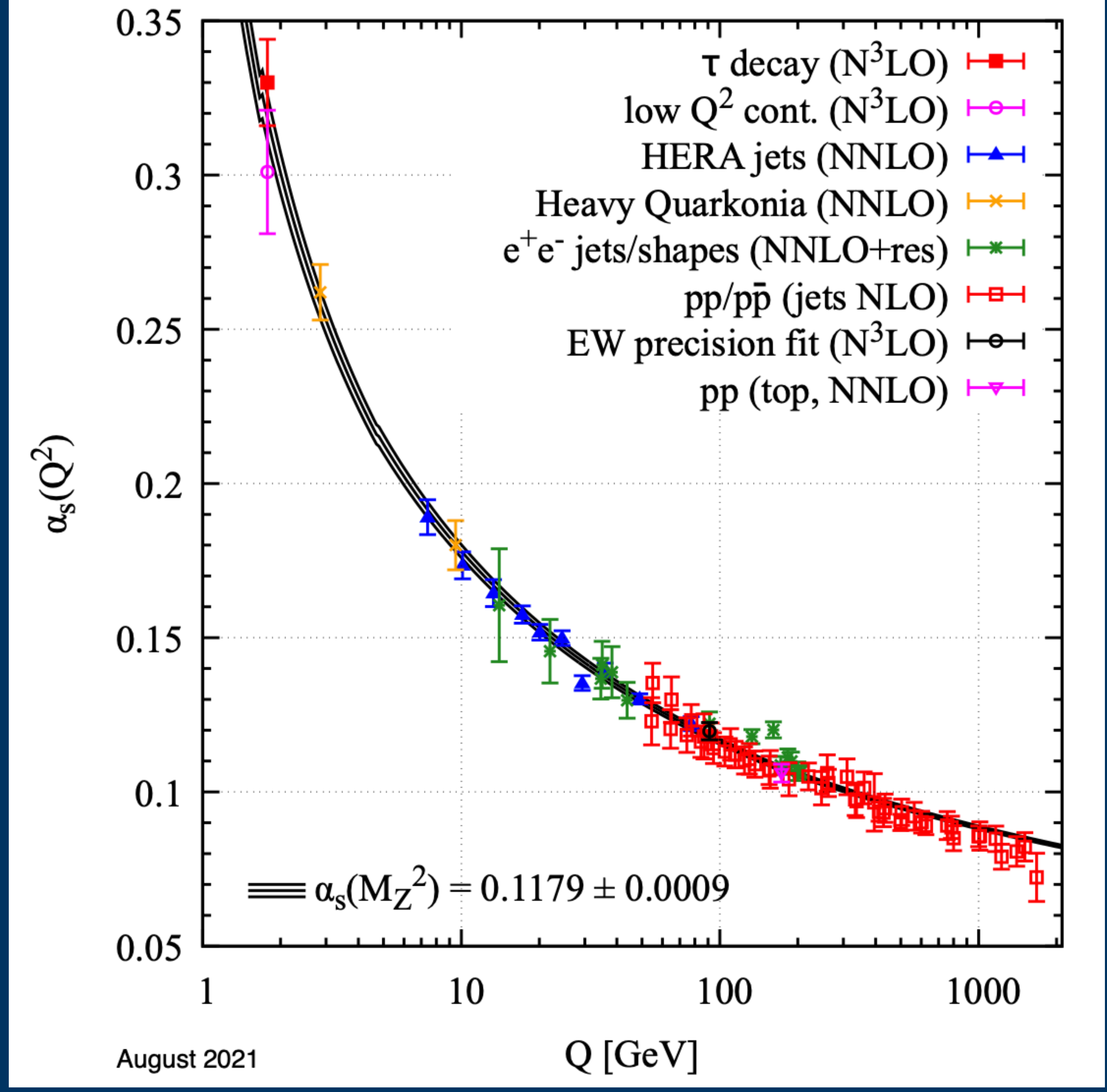
Phenomenology: electroweak precision observables (EWPO)

- Inputs and results to the fits



Final Remarks

Final remarks



- We saw only the electroweak sector of the SM
- Strong interactions are described by a $SU(3)_c$ gauge theory (QCD)
- $$\mathscr{L}_{\text{QCD}} = \sum_{j\in\text{quarks}} \bar{q}\gamma^\mu(\partial_\mu + ig_s\frac{\lambda^a}{2}g^a_\mu)q - \frac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu}$$

- The SM has a great success explaining the available data
- The SM does not answer all our questions!
- We have good reasons to extend it!

Basic framework: abelian gauge theory

- $\mathscr{L} = \bar{\Psi}(i\partial_\mu\gamma^\mu - m)\Psi$ is invariant under $\Psi \to \Psi' = e^{ie\alpha}\,\Psi$
- However, $\partial_\mu\Psi' = e^{ie\alpha(x)}\,(\partial_\mu\Psi + \Psi\;ie\partial_\mu\alpha)$
- Modify $\mathscr{L}$ to be invariant under $\Psi \to \Psi' = e^{ie\alpha(x)}\,\Psi$
- Simple solution: replace ∂_μ by $D_\mu = \partial_\mu - ieA_\mu$ with $A_\mu \to A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\alpha$
- We have introduced a new vector field (force) A_μ
- Dynamic of A_μ must respect the symmetry: $-ieF_{\mu\nu} = [D_\mu, D_\nu] = -ie(\partial_\mu A_\nu - \partial_\nu A_\mu)$
- $\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\partial_\mu\gamma^\mu - m)\Psi + eA_\mu\bar{\Psi}\gamma^\mu\Psi$

Basic framework: abelian gauge theory

- $\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\Psi}(i\partial_\mu\gamma^\mu - m)\Psi + eA_\mu\overline{\Psi}\gamma^\mu\Psi$
- $\mathscr{L}$ is invariant under $\Psi \to \Psi' = e^{ie\alpha(x)}\,\Psi$ and $A_\mu \to A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\alpha$

Remarks:

- Gauge invariance determines the vertex AFF
- The gauge symmetry forbids $M^2A_\mu A^\mu$
- There is some freedom left: $\overline{\Psi}\sigma^{\mu\nu}\Psi\ F_{\mu\nu}$
- At point point we can require the model to be renormalizable
- Works pretty well for QED!

Basic framework: non-abelian gauge theory

- $M = \begin{pmatrix} M_1 \\ M_2 \end{pmatrix}$ with a free Lagrangian $\mathscr{L}_0 = \sum_{jk} \overline{M}_j i\gamma^\mu \partial_\mu \delta_{jk} M_k = \overline{M} i\gamma^\mu \partial_\mu M$
- Invariant under global $SU(2)$ transformation: $M \to M' = GM = \exp(ig\alpha_a t^a)\ M$
- $\overline{M}'\gamma^\mu \partial_\mu M' = \overline{M} G^\dagger \gamma^\mu \partial_\mu G M = \overline{M}\gamma^\mu \partial_\mu M$
- For a local transformation it isn't't: $\partial_\mu M' = G\partial_\mu M + (\partial_\mu G)M$
- To impose invariance under local transformations: $\partial_\mu \to D_\mu = \partial_\mu + igt^a b_\mu^a$
- such that $D'_\mu M' = GD_\mu M$ leading to $t^a b_\mu^{\prime a} = G(t^a b_\mu^a)G^{-1} + \frac{i}{g}(\partial_\mu)G^{-1}$

Basic framework: non-abelian gauge theory

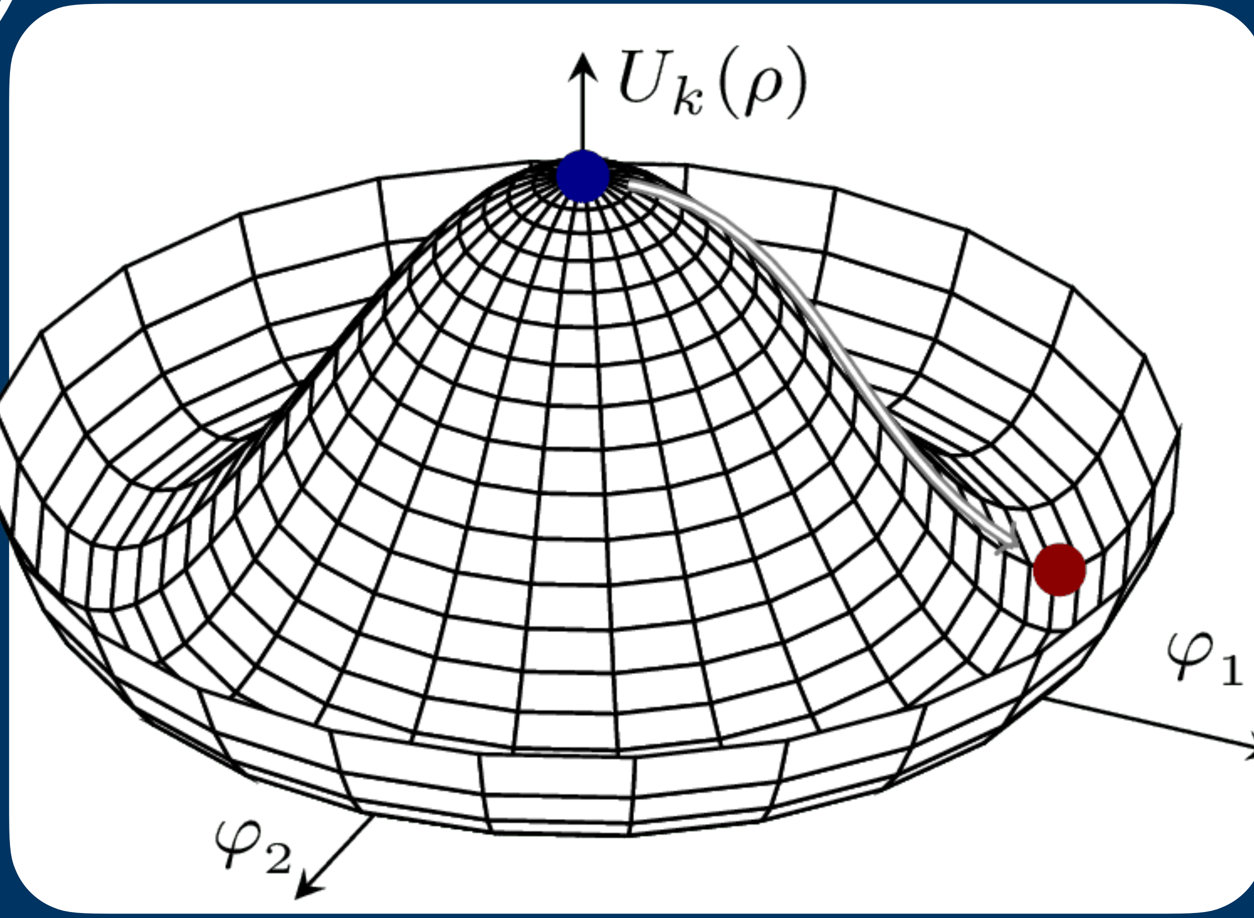
- $\partial_\mu \to D_\mu = \partial_\mu + igt^a b^a_\mu$
- $\mathscr{L}_0 \to \mathscr{L} = \overline{M} i\gamma^\mu D_\mu M = \overline{M} i\gamma^\mu \partial_\mu M - g\ b^a_\mu\ \ \overline{M}\gamma^\mu t^a M$
- b^a_μ dynamics using $F^a_{\mu\nu} = \frac{1}{ig}[D_\mu, D_\nu] = \partial_\mu(b^a_\nu t^a) - \partial_\nu(b^a_\mu) + i\ g\ [b^a_\mu t^a, b^c_\nu t^c]$
- Under a gauge transformation $t^a F'^a_{\mu\nu} = G(F^a_{\mu\nu})G^{-1}$
- Total lagrangian density: $\mathscr{L} = \overline{M} i\gamma^\mu \partial_\mu M - g\ b^a_\mu\ \ \overline{M}\gamma^\mu t^a M - \frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu}$

Remarks

- The symmetry forbids mass terms $m^2 b^a_\mu b^{a\,\mu}$
- Gauge invariance fixes the FFV, VVV and $VVVV$ interactions with coupling g!

Basic framework: symmetry breaking

- However $W^{\pm}$ and Z are massive!
- We need to break the symmetry, ie, the vacuum $|0\rangle$ is not invariant under Q : $Q|0\rangle \neq 0$
- Goldstone theorem: there is a massless particle for each broken generator
- Example: model invariant under $SO(2)$ with scalar doublet $\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$



- $\mathscr{L} = \frac{1}{2}\partial_\nu \varphi^a \partial^\nu \varphi^a - \frac{\mu^2}{2}\varphi^a\varphi^a - \frac{\lambda}{4}(\varphi^a\varphi^a)^2$
- for $\mu^2 < 0$ the vacuum satisfies $\langle \varphi^a \varphi^a \rangle = -\frac{\mu^2}{\lambda} \equiv v^2$
- Chossing the vacuum $\langle \varphi \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}$ and defining $\varphi = \langle \varphi \rangle + \begin{pmatrix} \eta \\ \chi \end{pmatrix}$ we have
- $\mathscr{L} = \frac{1}{2}\left[\partial_\mu \eta \partial^\mu \eta + 2\mu^2\eta^2\right] + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi + \ldots$,ie, there is a massless excitation χ

Basic framework: Higgs mechanism

- Let's break a gauge theory! Let's work out an example with $SU(2)$ gauge symmetry
- $\mathscr{L} = -\frac{1}{4} F^a_{\alpha\beta} F^{a\,\alpha\beta} + (D_\nu \phi^a)^\star D^\nu \phi^a - \mu^2 \phi^{a\star}\phi^a - \lambda(\phi^{a\star}\phi^a)^2$
- for $\mu^2 < 0$ the vacuum satisfies $\langle \phi^{a\star}\phi^a \rangle = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}$ so we choose $\langle \phi \rangle = \frac{v}{\sqrt{2}}$
- Notice that there are 3 Goldstone bosons since $\sigma^a \langle \phi \rangle \neq 0$
- Now we write $\phi = \exp(i\frac{\sigma^a \xi^a}{2v}) \begin{pmatrix} 0 \\ \frac{v+\eta}{2v} \end{pmatrix}$ where ξ^a are the Goldstone bosons
- To obtain the physical spectrum $\phi \to \phi' = \exp(-i\frac{\sigma^a \xi^a}{2v})\ \phi$

Basic framework: Higgs mechanism

- $\mathscr{L} = -\frac{1}{4} F^a_{\alpha\beta} F^{a\,\alpha\beta} + (D_\nu \phi^a)^\star D^\nu \phi^a - \mu^2 \phi^{a\,\star} \phi^a - \lambda(\phi^{a\,\star}\phi^a)^2$
- $\phi = \exp(i\frac{\sigma^a \xi^a}{2v}) \begin{pmatrix} 0 \\ \frac{v+\eta}{2v} \end{pmatrix}$
- To obtain the physical spectrum $\phi \rightarrow \phi' = \exp(-i\frac{\sigma^a \xi^a}{2v})\ \phi$
- Resulting in $\mathscr{L} = \frac{1}{2}\left(\partial_\mu \eta \partial^\mu \eta - 2\lambda v^2 \eta^2\right) - \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} - \frac{g^2 v^2}{8}\ \left(\,|\,b^1_\mu - i b^2_\mu\,|^2 + b^3_\mu b^{3\,\mu}\right)\ldots$
- All the massless particles disappeared and the gauge bosons got massive!

Basic framework: model building recipe (70's)

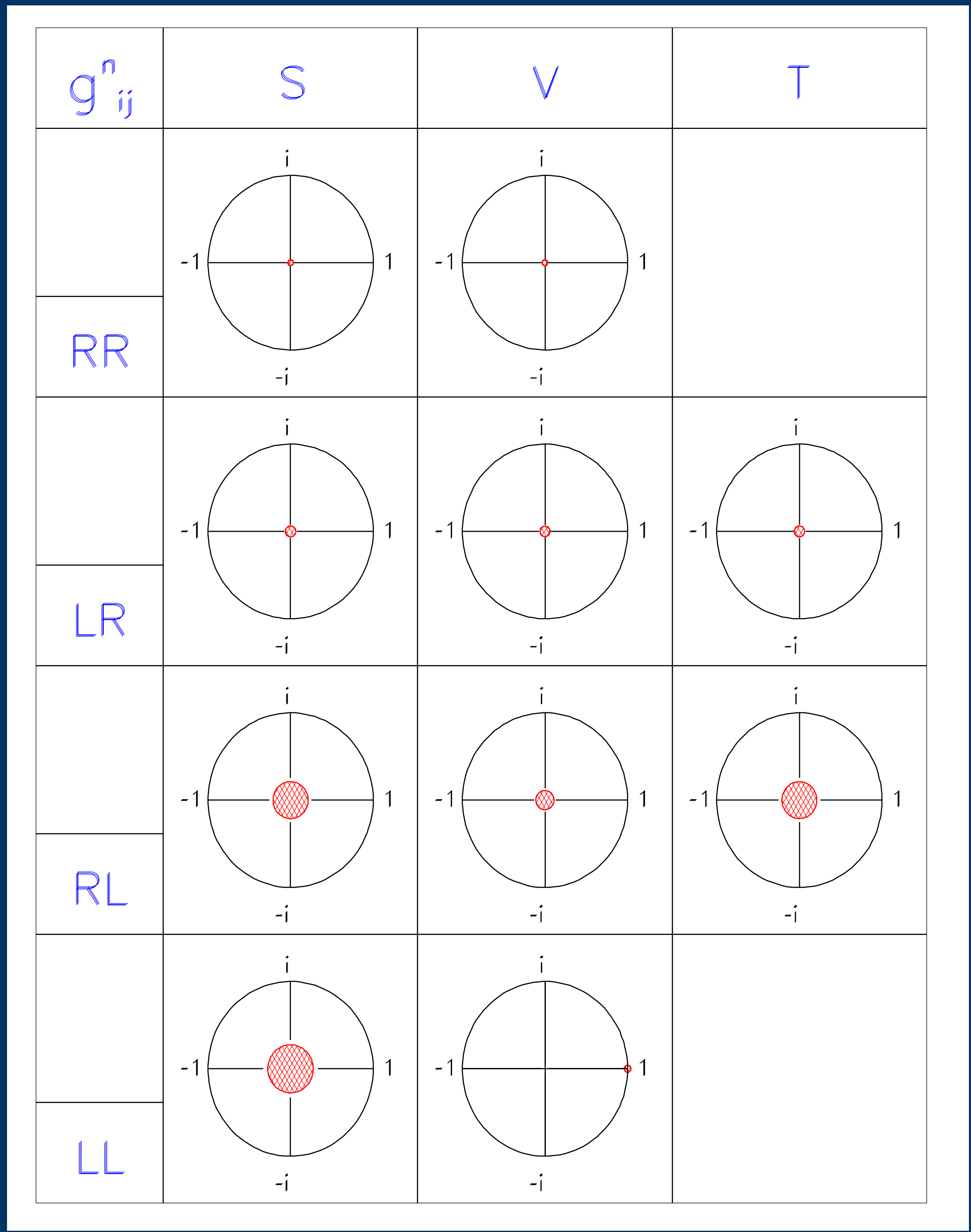
1. Choose the local gauge symmetry
2. Choose the matter content and its representation(s)
3. Choose the symmetry breaking sector
4. Verify whether theoretical and experimental constraints are satisfies; if not go back to 1.

Standard Model: Gauge sector

- There are four electroweak gauge bosons: W^+ , W^- , Z , γ
- Minimal model need 4 symmetry generators
- Choice: $SU(2) \otimes U(1)$
- Gauge bosons: W^j_μ with $j = 1,2,3$ ($SU(2)$) and B_μ ($((1))$) with couplings g and g'
- Kinetic lagrangian density: $\mathscr{L}_{\text{gauge}} = -\dfrac{1}{4} B_{\mu\nu}B^{\mu\nu} - \dfrac{1}{4} W^a_{\mu\nu}W^{a\,\mu\nu}$
- with $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\nu - g\epsilon^{abc}W^b_\mu W^c_\nu$ and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

this determines the VVV and VVVV vertices

Standard Model: matter (leptons)



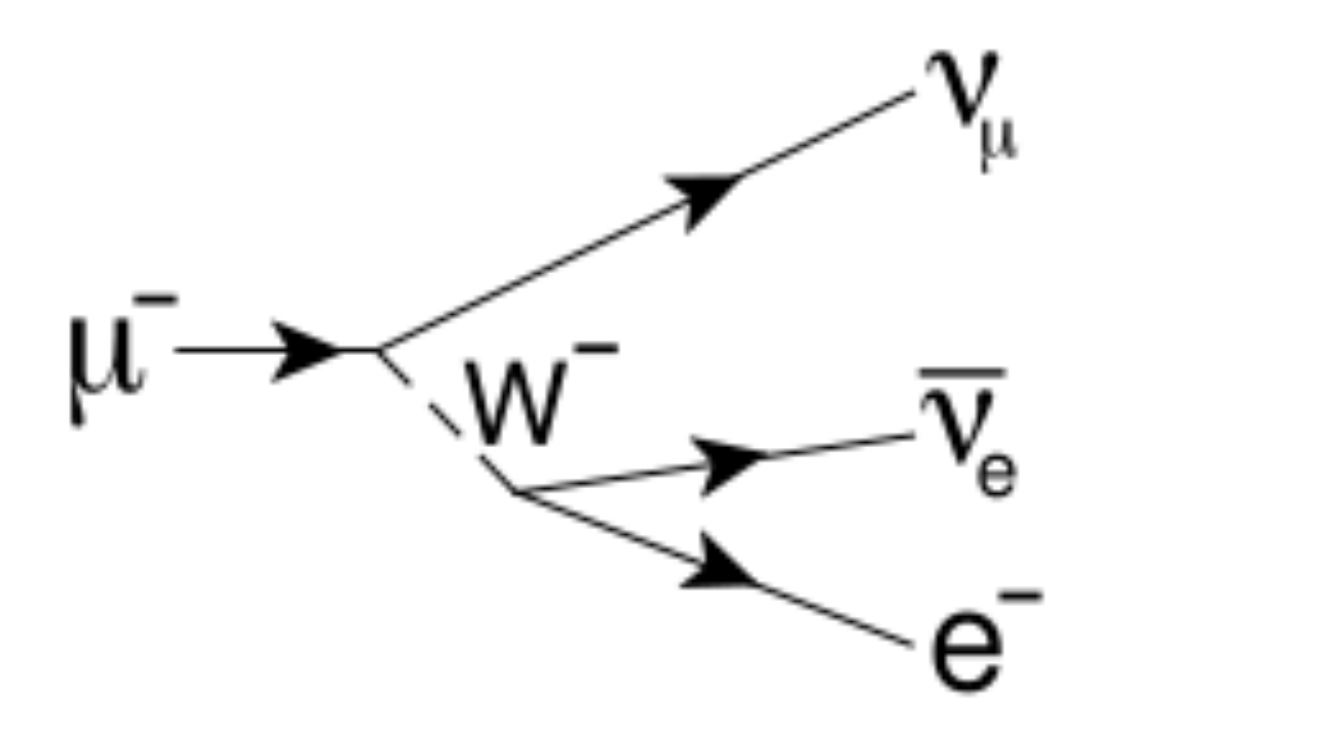
- We need to chose the matter representations
- Lorentz transformation of multiplet members must be the same
- We now that charged currents are left-handed
- We define $P_L = \dfrac{1-\gamma^5}{2}$ and $P_R = \dfrac{1+\gamma^5}{2}$
- Degrees of freedom: $e_{L(R)} = P_{L(R)}\ e$ and $\nu_{e\,L} = P_L\ \nu_e$
- $SU(2)_L$ representations: $L = \begin{pmatrix} \nu_{e\,L} \\ e_L \end{pmatrix}$ and $R = e_R$
- $U(1)_Y$ representations from $Q = T^3 + \dfrac{Y}{2}$ we have $Y_L = -1$ and $Y_R = -2$

Standard Model: matter (leptons)

- $SU(2)_L$ representations: $L = \begin{pmatrix} \nu_{e\,L} \\ e_L \end{pmatrix}$ and $R = e_R$
- $U(1)_Y$ representations from $Q = T^3 + \dfrac{Y}{2}$ we have $Y_L = -1$ and $Y_R = -2$
- Covariant derivatives: $D_\mu L = (\partial_\mu + ig'\dfrac{Y}{2}\,B_\mu + ig\dfrac{\sigma^a}{2}\,W^a_\mu)\,L$ and $D_\mu R = (\partial_\mu + ig'\dfrac{Y}{2}\,B_\mu)\,R$
- Lagrangian: $\mathscr{L} = i\bar{L}\gamma^\mu D_\mu L + i\bar{R}\gamma^\mu D_\mu R$
- Charged current $\left(W^\pm_\mu = \dfrac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}}\right)$: $-\dfrac{g}{\sqrt{2}}\left[\bar{\nu}_e\,\gamma^\mu P_L\,e\,W^+_\mu + \bar{e}\,\gamma^\mu P_R\,\nu_e\,W^-_\mu\right]$
- Neutral current: $-\dfrac{g}{2}\left(\bar{\nu}_{e\,L}\gamma^\mu\nu_{e\,L} - \bar{e}_L\gamma^\mu e_L\right)W^3_\mu + \dfrac{g'}{2}\left(\bar{\nu}_{e\,L}\gamma^\mu\nu_{e\,L} + \bar{e}_L\gamma^\mu e_L + 2\bar{e}_R\gamma^\mu e_R\right)B_\mu$

Standard Model: matter (leptons)

- Lagrangian: $\mathscr{L} = i\bar{L}\gamma^\mu D_\mu L + i\bar{R}\gamma^\mu D_\mu R$
- Before SSB the charged leptons are massless since $SU(2)_L$ forbids $\bar{e}_L e_R$
- Charged current $\left(W^\pm_\mu = \dfrac{W^1_\mu \mp i W^2_\mu}{\sqrt{2}} \right)$: $-\dfrac{g}{\sqrt{2}} \left[\bar{\nu}_e \, \gamma^\mu P_L \, e \, W^+_\mu + \bar{e} \, \gamma^\mu P_R \, \nu_e \, W^-_\mu \right]$
- The Fermi constant is given by $G_F = \dfrac{g^2}{4\sqrt{2}M_W^2}$



Standard Model: symmetry breaking sector (SSB)

- To generate masses we must break the $SU(2)_L \otimes U(1)_Y$ symmetry
- Minimal model: complex scalar doublet $\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$
- From $Q = T^3 + \frac{Y}{2}$ it follows that $Y_\varphi = +1$
- $\mathscr{L}_{\mathrm{SSB}} = |D_\mu \varphi|^2 - \mu^2 |\varphi|^2 - \lambda |\varphi|^4$ with $\mu^2 < 0$ and $\langle \varphi^\dagger \varphi \rangle = -\frac{\mu^2}{\lambda} = \frac{v^2}{2}$
- Let's choose the vacuum $\langle \varphi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

Standard Model: symmetry breaking sector (SSB)

- $\mathscr{L}_{\text{SSB}} = |D_\mu \varphi|^2 - \mu^2 |\varphi|^2 - \lambda |\varphi|^4$
- Let's choose the vacuum $\langle \varphi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$
- $\dfrac{\sigma^a}{2}\langle \varphi \rangle \neq 0$ and $Y\langle \varphi \rangle \neq 0$ however $Q\langle \varphi \rangle = \dfrac{1}{2}(\sigma^3 + Y)\langle \varphi \rangle = 0$
- Therefore $SU(2)_L \otimes U(1)_Y$ is broken to $U(1)_{\text{em}}$
- 3 broken generators $\implies$ 3 goldstone bosons $\implies$ 3 massive gauge bosons
- Unitary gauge $\varphi = \begin{pmatrix} 0 \\ (v + H)/\sqrt{2} \end{pmatrix}$

Standard Model: symmetry breaking sector (SSB)

- $\mathscr{L}_{\mathrm{SSB}} = |D_\mu \varphi|^2 - \mu^2 |\varphi|^2 - \lambda |\varphi|^4$
- Unitary gauge $\varphi = \begin{pmatrix} 0 \\ (v+H)/\sqrt{2} \end{pmatrix}$
- $\mathscr{L}_{\mathrm{SSB}} = \frac{1}{2}(\partial_\mu H \partial^\mu H - 2\lambda v^2 H^2) + \frac{v^2}{8}(g^2 W_\mu^+ W^{-\mu} + (g' B_\mu - g W_\mu^3)^2) + \ldots$
- $M_W = \frac{gv}{4}$ and $M_H = \sqrt{2\lambda}\; v$
- Neutral mass eigenstates: $Z_\mu = \frac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}}$ and $A_\mu = \frac{g' W_\mu^3 + g B_\mu}{\sqrt{g^2 + g'^2}}$

Standard Model: symmetry breaking sector (SSB)

- $\mathscr{L}_{\text{SSB}} = \frac{1}{2}(\partial_\mu H \partial^\mu H - 2\lambda v^2 H^2) + \frac{v^2}{8}(g^2 W_\mu^+ W^{-\mu} + (g' B_\mu - g W_\mu^3)^2) + \ldots$

  to be diagonalized

- $M_W = \frac{gv}{2}$ and $M_H = \sqrt{2\lambda}\ v$

- Neutral mass eigenstates: $Z_\mu = \frac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}}$ and $A_\mu = \frac{g' W_\mu^3 + g B_\mu}{\sqrt{g^2 + g'^2}}$

- So $M_Z = \sqrt{g^2 + g'^2}\ \frac{v}{2}$

- Writing $g = \frac{e}{\sin\theta_W}$ and $g' = \frac{e}{\cos\theta_W}$ we have $M_Z = \frac{M_W}{\cos\theta_W}$

Standard Model: symmetry breaking sector (SSB)

- Neutral current: $-\frac{g}{2}\left(\bar{\nu}_{e\,L}\gamma^\mu\nu_{e\,L} - \bar{e}_L\gamma^\mu e_L\right) W^3_\mu + \frac{g'}{2}\left(\bar{\nu}_{e\,L}\gamma^\mu\nu_{e\,L} + \bar{e}_L\gamma^\mu e_L + 2\bar{e}_R\gamma^\mu e_R\right)$
- Neutrinos interact only with the Z: $-\frac{g}{2\cos\theta_W}\ \overline{\nu_{e\,L}}\gamma^\mu\nu_{e\,L}Z_\mu$
- Electron interactions: $-eQ\ \bar{e}\gamma^\mu A_\mu - \frac{g}{2\cos\theta_W}\ \bar{e}\gamma^\mu(g_v - g_A\gamma^5)eZ_\mu$
- with $g_V = T^3_e - 2\sin^2\theta_W Q$ and $g_A = T^3_e$
- Electron mass: $\mathscr{L}_Y = -\,Y_e\ [\bar{R}(\varphi^\dagger L) + (\bar{L}\varphi)R]$
- In the unitarity gauge: $\mathscr{L}_Y = -\,Y_e\ \frac{v+H}{\sqrt{2}}\ (\bar{e}_L e_R + \bar{e}_R e_L)$ with $m_e = \frac{Y_e v}{\sqrt{2}}$

Standard Model: quark sector

- Quark multiplets are $Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$, u_R , d_R
- Quantum numbers: $Y_{u_R} = \frac{4}{3}$, $Y_{d_R} = -\frac{2}{3}$, $Y_Q = \frac{1}{3}$
- Quark-gauge sector: $\mathscr{L}_Q = i\bar{u}_R\gamma^\mu D_\mu u_R + i\bar{d}_R\gamma^\mu D_\mu d_R + i\bar{Q}_L\gamma^\mu D_\mu Q_L$
- CC interactions: $\mathscr{L}_{\mathrm{CC}} = -\frac{g}{\sqrt{2}}\ \bar{u}_L\gamma^\mu d_L W_\mu^+ + h.c.$
- NC interactions: $\mathscr{L}_{\mathrm{NC}} = -eQ_q\bar{q}\ \gamma^\mu q\ A_\mu - \frac{g}{2\cos\theta_W}\ \bar{q}\gamma^\mu(g_V^q - g_A^q\gamma^5)q\ Z_\mu$
- $g_V^q = T_q^3 - 2\sin^2\theta_W\ Q_q$ and $g_A^q = T_q^3$

Standard Model: quark sector

- Quark multiplets are $Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$, u_R , d_R
- Quark-gauge sector: $\mathscr{L}_Q = i\bar{u}_R\gamma^\mu D_\mu u_R + i\bar{d}_R\gamma^\mu D_\mu d_R + i\bar{Q}_L\gamma^\mu D_\mu Q_L$
- To write the mass term we need the doublet $\tilde{\varphi} = i\sigma^2\varphi^*$ with $Y_{\tilde{\varphi}} = -1$
- $\mathscr{L}_{Yq} = -Y_u[\bar{u}_R(\tilde{\varphi}^\dagger Q_L) + (\bar{Q}_L\tilde{\varphi})u_R] \; + \; Y_d\,[\bar{d}_R(\varphi^\dagger Q_L) + (\bar{Q}_L\varphi)d_R]$
- Unitary gauge: $-Y_u \dfrac{v+H}{\sqrt{2}}(\bar{u}_L u_R + \bar{u}_R u_l) - Y_d \dfrac{v+H}{\sqrt{2}}(\bar{d}_L d_R + \bar{d}_R d_l)$

Standard Model: 3 families

- For massless neutrinos the lepton sector is just the repetition of the first family
- Important consequence: neutral current interactions do not change the lepton flavor
- Three copies of the one family gauge-fermion interactions
- Most general mass term for quarks:

$$\mathscr{L}_{Yq} = - \sum_{j,k=1,2,3} \left\{ Y_{u\,jk}\, \bar{u}^j_R (\tilde{\varphi}^\dagger Q^k_L) + Y_{d\,jk}\, \bar{d}^j_R (\varphi^\dagger Q^k_L) \right\} + h.c.$$

- After SSB the mass matrix is